SCE 594: Special Topics in Intelligent Automation & Robotics

Lecture 13: Dynamics of Fixed-base Manipulators I

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Outline

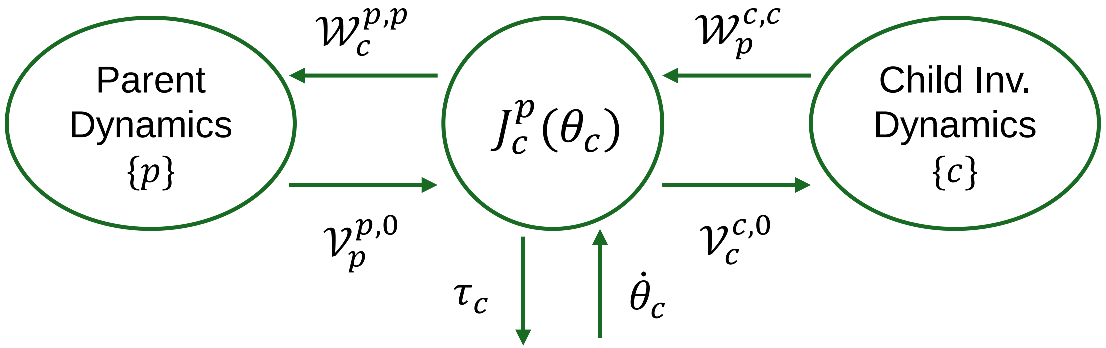
- Parent-Child Dynamic Model
- Pendulum dynamics

Parent-Child Dynamic Model

- Relative pose $$H_c^p(\theta_c) = H_c^p(0)e^{\tilde{S}_c^{c,p}\theta_c}$$

- Joint Transformation $$J_c^p(\theta_c) = \begin{pmatrix} 0 & -\mathrm{Ad}^{\mathsf{T}}_{H_p^c(\theta_c)} & 0 \\ \mathrm{Ad}_{H_p^c(\theta_c)} & 0 & S_c^{c,p} \\ 0 & \left(S_c^{c,p}\right)^{\mathsf{T}} & 0 \end{pmatrix}$$

$$\boxed{\begin{pmatrix} \mathcal{W}_c^{p,p} \\ \mathcal{V}_c^{c,0} \\ \tau_c \end{pmatrix} = J_c^p(\theta_c) \begin{pmatrix} \mathcal{V}_p^{p,0} \\ \mathcal{W}_p^{c,c} \\ \dot{\theta}_c \end{pmatrix}}$$



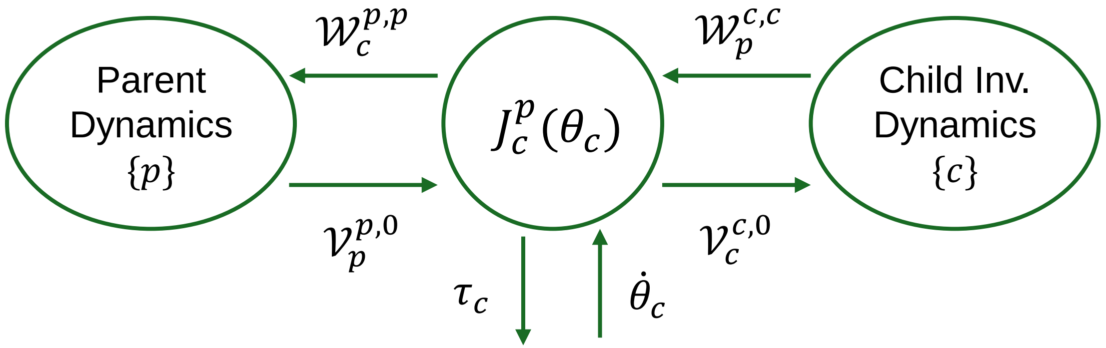
Parent-Child Dynamic Model

- Parent Dynamics

$$\mathfrak{I}^{p,p}\dot{\mathcal{V}}_p^{p,0} = \mathrm{ad}^{\top}_{\mathcal{V}_p^{p,0}}\left(\mathfrak{I}^{p,p}\mathcal{V}_p^{p,0}\right) + \mathcal{W}_c^{p,p}$$

- Child Inverse Dynamics

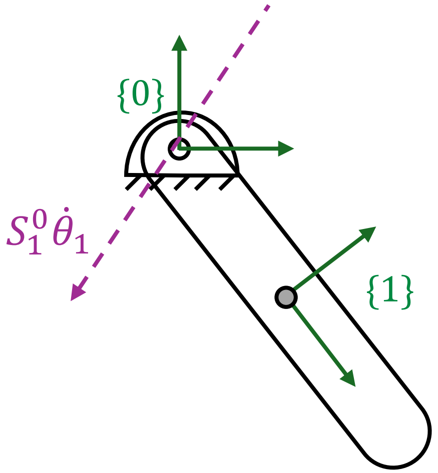
$$\mathcal{W}_p^{c,c} = \mathfrak{I}^{c,c}\dot{\mathcal{V}}_c^{c,0} - \mathrm{ad}^{\top}_{\mathcal{V}_c^{c,0}}\left(\mathfrak{I}^{c,c}\mathcal{V}_c^{c,0}\right)$$



$\dot{\mathcal{V}}_c^{c,0}$ can be computed analytically

Pendulum dynamics

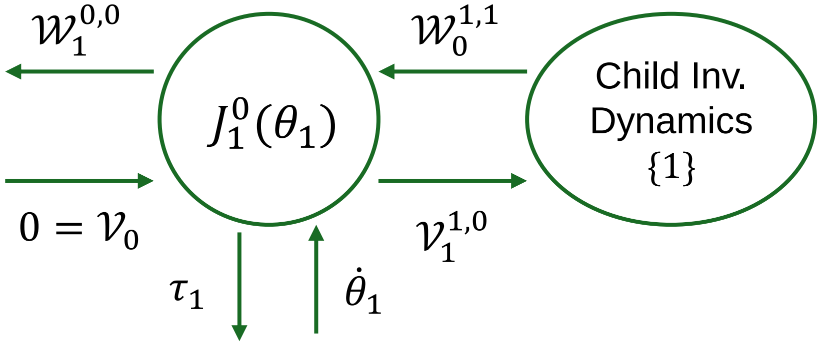
- Dynamics
  - $\mathcal{V}_1^{1,0} = S_1^{1,0}\dot{\theta}_1$
  - $\dot{\mathcal{V}}_1^{1,0} = S_1^{1,0}\ddot{\theta}_1$
  - $\mathcal{W}_0^{1,1} = \mathfrak{I}^{1,1}\dot{\mathcal{V}}_1^{1,0} - \mathrm{ad}^{\mathsf{T}}_{\mathcal{V}_1^{1,0}}\left(\mathfrak{I}^{1,1}\mathcal{V}_1^{1,0}\right)$
  - $\tau_1 = \left(S_1^{1,0}\right)^{\mathsf{T}}\mathcal{W}_0^{1,1}$



- Reduced Dynamics

$$\boxed{\tau_1 = M\ddot{\theta}_1}$$

with $M := \left(S_1^{1,0}\right)^{\mathsf{T}}\mathfrak{I}^{1,1}S_1^{1,0}$



Pendulum dynamics

- $S_1^{1,0} = \left(0,0,1,0,\frac{L}{2},0\right)$
- $\mathfrak{T}^{1,1} = \begin{pmatrix} J^{1,1} & 0 \\ 0 & m\, I_3 \end{pmatrix}$
- $J^{1,1} = \begin{pmatrix} J_{xx} & J_{xy} & J_{xz} \\ J_{xy} & J_{yy} & J_{yz} \\ J_{xz} & J_{yz} & J_{zz} \end{pmatrix}$
- $M = (S_1^{1,0})^\top \mathfrak{T}^{1,1} S_1^{1,0} = J_{zz} + m\left(\frac{L}{2}\right)^2$