SCE 594: Special Topics in Intelligent Automation & Robotics

Lecture 14: Dynamics of Fixed-base Manipulators II

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Outline

- Recap last lecture
- Analytic expression of $\dot{\mathcal{V}}_c^{c,0}$
- Two-link manipulator dynamics
- General n-link manipulator dynamics

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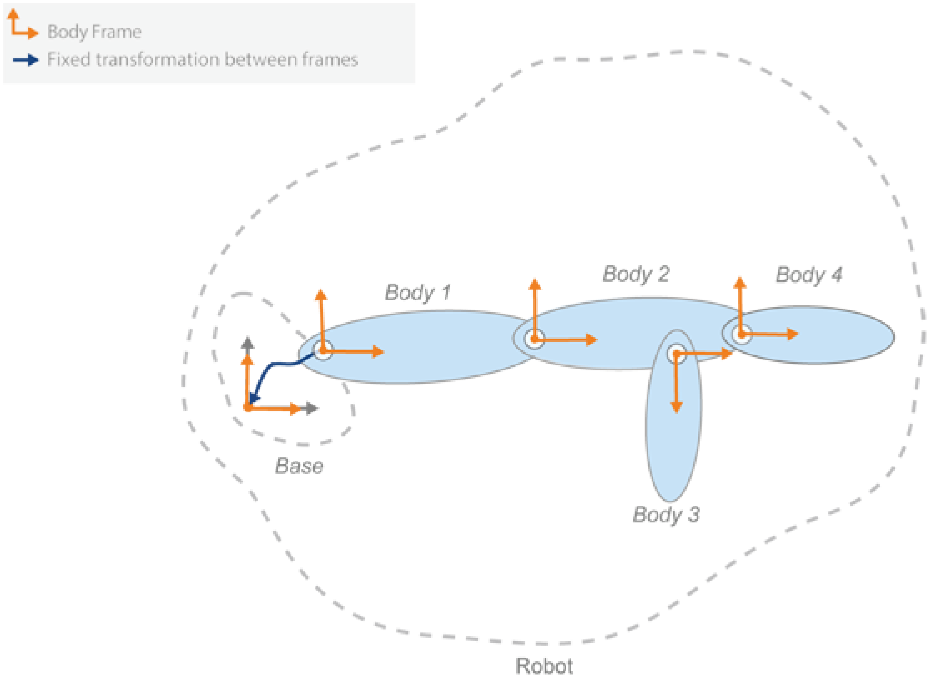
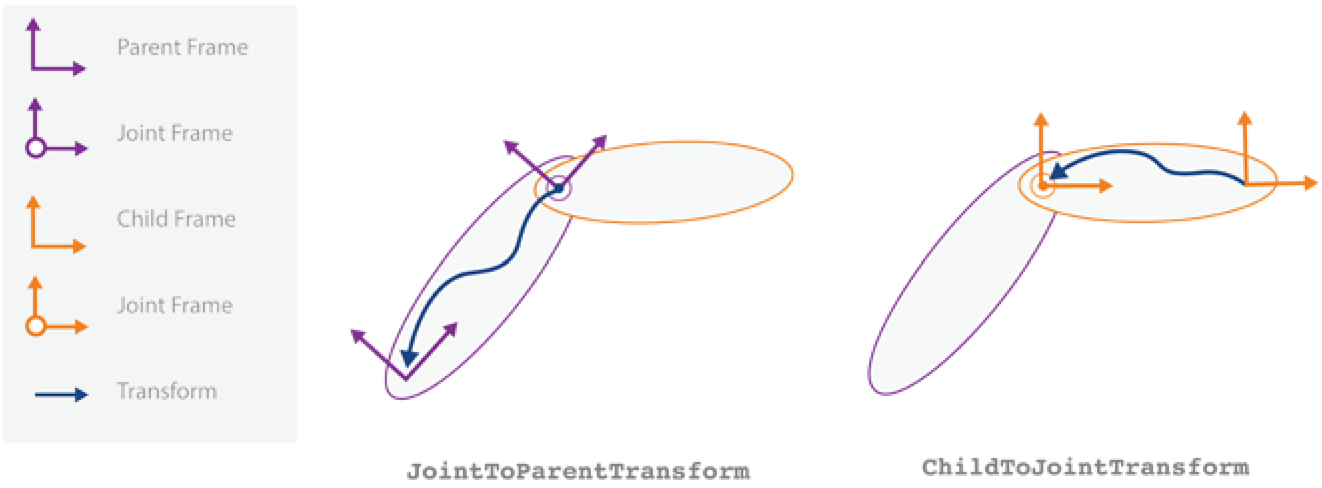
Recap: Parent-Child Dynamic Model

- In this lecture, we are developing a systematic modeling procedure for a generic fixed-base manipulator.
- A fixed-base manipulator in general consists of a set of rigid bodies linked together via joints that are usually activated by (electric) motors.



Recap: Parent-Child Dynamic Model

- Last lecture, we describe the dynamic model of two rigid bodies connected by a 1 degree-of-freedom revolute joint.
- This basic building block will be used to model a complete fixed-base manipulator.

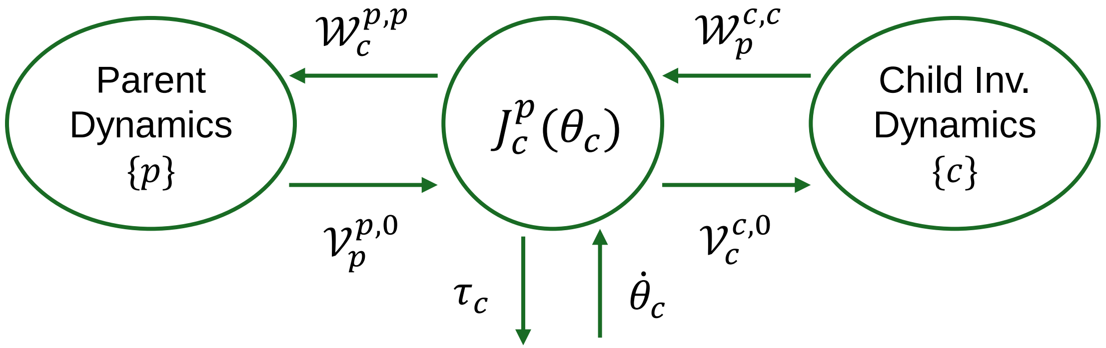


Recap: Parent-Child Dynamic Model

- Relative pose $$H_c^p(\theta_c) = H_c^p(0)e^{\tilde{S}_c^{c,p}\theta_c}$$

$$\boxed{\begin{pmatrix} \mathcal{W}_c^{p,p} \\ \mathcal{V}_c^{c,0} \\ \tau_c \end{pmatrix} = J_c^p(\theta_c)\begin{pmatrix} \mathcal{V}_p^{p,0} \\ \mathcal{W}_p^{c,c} \\ \dot{\theta}_c \end{pmatrix}}$$

- Joint Transformation $$J_c^p(\theta_c) = \begin{pmatrix} 0 & -\mathrm{Ad}^{\mathsf{T}}_{H_p^c(\theta_c)} & 0 \\ \mathrm{Ad}_{H_p^c(\theta_c)} & 0 & S_c^{c,p} \\ 0 & \left(S_c^{c,p}\right)^{\mathsf{T}} & 0 \end{pmatrix}$$



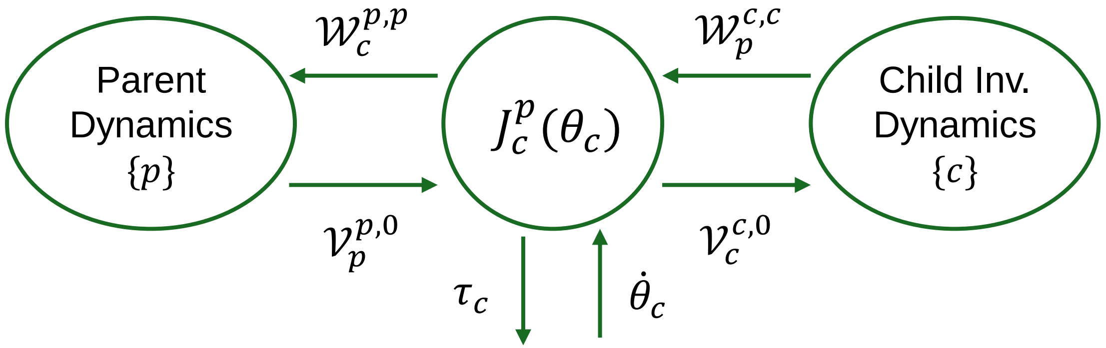
Recap: Parent-Child Dynamic Model

- Parent Dynamics

$$\mathfrak{I}^{p,p}\dot{\mathcal{V}}_p^{p,0} = \mathrm{ad}^{\top}_{\mathcal{V}_p^{p,0}}\left(\mathfrak{I}^{p,p}\mathcal{V}_p^{p,0}\right) + \mathcal{W}_c^{p,p}$$

- Child Inverse Dynamics

$$\mathcal{W}_p^{c,c} = \mathfrak{I}^{c,c}\dot{\mathcal{V}}_c^{c,0} - \mathrm{ad}^{\top}_{\mathcal{V}_c^{c,0}}\left(\mathfrak{I}^{c,c}\mathcal{V}_c^{c,0}\right)$$



$\dot{\mathcal{V}}_c^{c,0}$ can be computed analytically

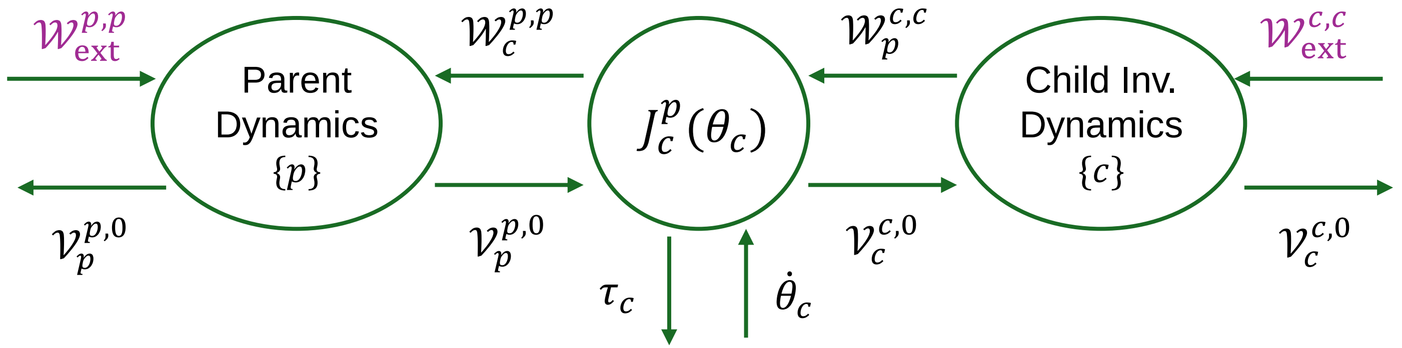
Recap: Parent-Child Dynamic Model + External wrench

- Parent Dynamics

$$\mathfrak{I}^{p,p}\dot{\mathcal{V}}_p^{p,0} = \mathrm{ad}^{\mathsf{T}}_{\mathcal{V}_p^{p,0}}\left(\mathfrak{I}^{p,p}\mathcal{V}_p^{p,0}\right) + \mathcal{W}_c^{p,p} + \mathcal{W}_{\mathrm{ext}}^{p,p}$$

- Child Inverse Dynamics

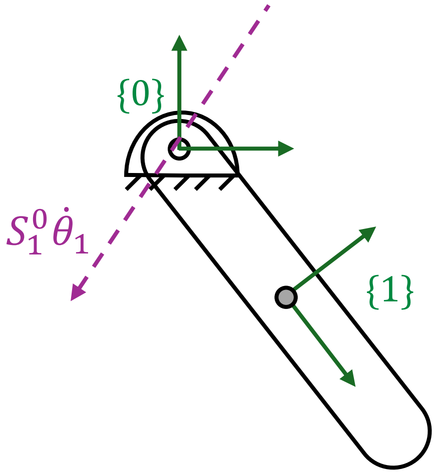
$$\mathcal{W}_p^{c,c} = \mathfrak{I}^{c,c}\dot{\mathcal{V}}_c^{c,0} - \mathrm{ad}^{\mathsf{T}}_{\mathcal{V}_c^{c,0}}\left(\mathfrak{I}^{c,c}\mathcal{V}_c^{c,0}\right) - \mathcal{W}_{\mathrm{ext}}^{c,c}$$



$\dot{\mathcal{V}}_c^{c,0}$ can be computed analytically

Recap: Pendulum dynamics

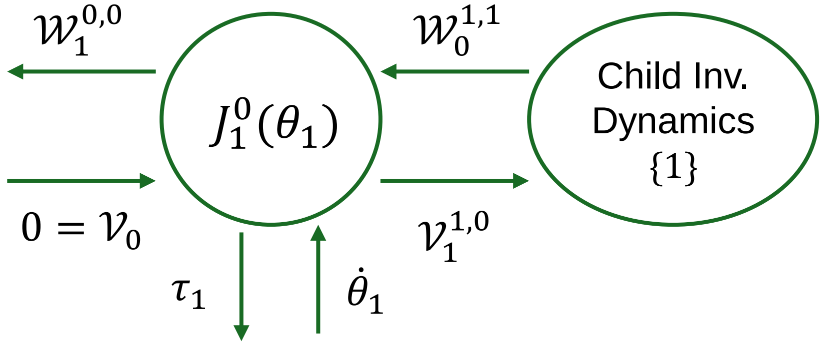
- Dynamics
  - $\mathcal{V}_1^{1,0} = S_1^{1,0}\dot{\theta}_1$
  - $\dot{\mathcal{V}}_1^{1,0} = S_1^{1,0}\ddot{\theta}_1$
  - $\mathcal{W}_0^{1,1} = \mathfrak{T}^{1,1}\dot{\mathcal{V}}_1^{1,0} - \mathrm{ad}_{\mathcal{V}_1^{1,0}}^{\mathsf{T}}\left(\mathfrak{T}^{1,1}\mathcal{V}_1^{1,0}\right)$
  - $\tau_1 = \left(S_1^{1,0}\right)^{\mathsf{T}}\mathcal{W}_0^{1,1}$



- Reduced Dynamics

$$\boxed{\tau_1 = M\ddot{\theta}_1}$$

with $M := \left(S_1^{1,0}\right)^{\mathsf{T}}\mathfrak{T}^{1,1}S_1^{1,0}$



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Analytic expression of $\dot{\mathcal{V}}_c^{c,0}$

- Recall that

$$\mathcal{V}_c^{c,0} = \mathcal{V}_p^{c,0} + \mathcal{V}_c^{c,p} = \mathrm{Ad}_{H_p^c(\theta_c)} \mathcal{V}_p^{p,0} + S_c^{c,p} \dot{\theta}_c$$

- Then

$$\dot{\mathcal{V}}_c^{c,0} = \mathrm{Ad}_{H_p^c(\theta_c)} \dot{\mathcal{V}}_p^{p,0} + \frac{d}{dt}\left(\mathrm{Ad}_{H_p^c(\theta_c)}\right) \mathcal{V}_p^{p,0} + S_c^{c,p} \ddot{\theta}_c$$

Analytic expression of $\dot{\mathcal{V}}_c^{c,0}$

- Recall that

$$\mathcal{V}_c^{c,0} = \mathcal{V}_p^{c,0} + \mathcal{V}_c^{c,p} = \mathrm{Ad}_{H_p^c(\theta_c)} \mathcal{V}_p^{p,0} + S_c^{c,p} \dot{\theta}_c$$

- Then

$$\dot{\mathcal{V}}_c^{c,0} = \mathrm{Ad}_{H_p^c(\theta_c)} \dot{\mathcal{V}}_p^{p,0} + \frac{d}{dt}\left(\mathrm{Ad}_{H_p^c(\theta_c)}\right) \mathcal{V}_p^{p,0} + S_c^{c,p} \ddot{\theta}_c$$

- Which can be written as

$$\dot{\mathcal{V}}_c^{c,0} = \mathrm{Ad}_{H_p^c(\theta_c)} \dot{\mathcal{V}}_p^{p,0} + \mathrm{ad}_{\mathcal{V}_c^{c,0}} S_c^{c,p} \dot{\theta}_c + S_c^{c,p} \ddot{\theta}_c$$

Homework problem

or

$$\dot{\mathcal{V}}_c^{c,0} = \mathrm{Ad}_{H_p^c(\theta_c)} \dot{\mathcal{V}}_p^{p,0} - \mathrm{ad}_{S_c^{c,p} \dot{\theta}_c} \mathcal{V}_c^{c,0} + S_c^{c,p} \ddot{\theta}_c$$

or

$$\dot{\mathcal{V}}_c^{c,0} = \mathrm{Ad}_{H_p^c(\theta_c)} \dot{\mathcal{V}}_p^{p,0} - \mathrm{ad}_{S_c^{c,p} \dot{\theta}_c} \left(\mathrm{Ad}_{H_p^c(\theta_c)} \mathcal{V}_p^{p,0}\right) + S_c^{c,p} \ddot{\theta}_c$$

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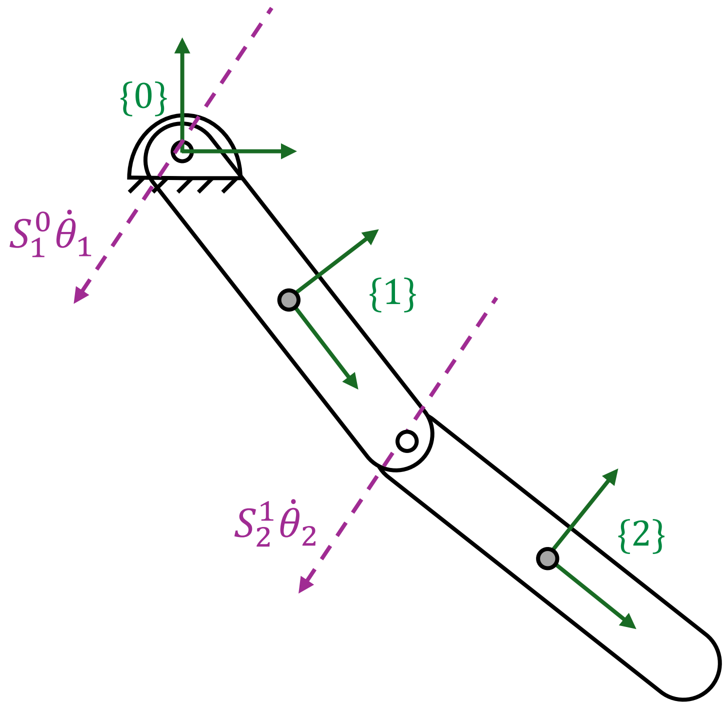
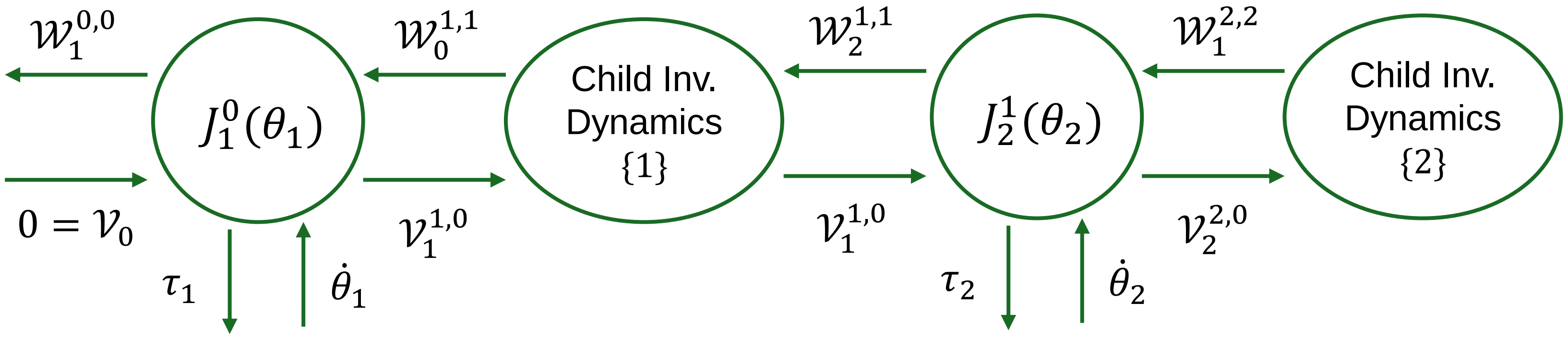
Two-link manipulator

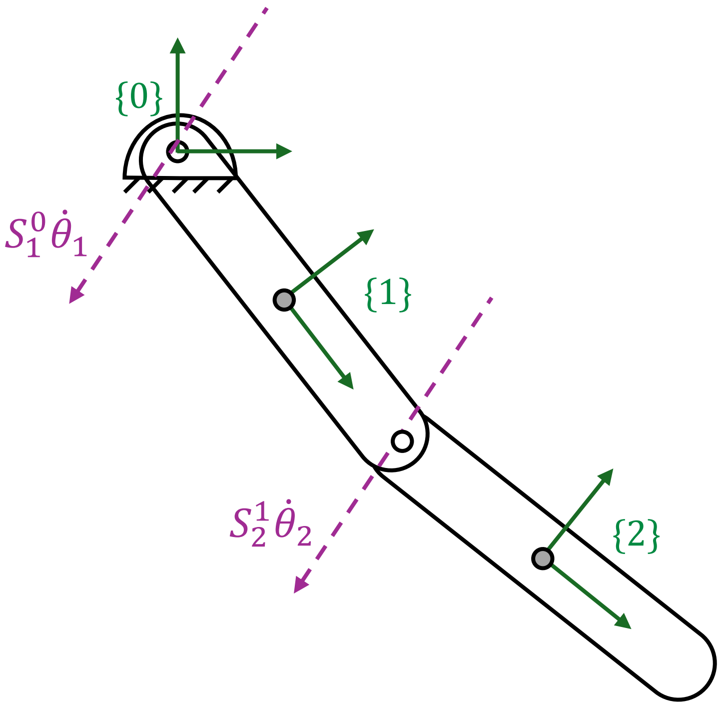
- Newton-Euler Inverse Dynamics Algorithm
  - Forward iterations
    - $H_i^{i-1}(\theta_i) = H_i^{i-1}(0)e^{\tilde{S}_i^{i,i-1}\theta_i}$
    - $\mathcal{V}_i^{i,0} = \mathrm{Ad}_{H_{i-1}^{i}(\theta_i)}\mathcal{V}_{i-1}^{i-1,0} + S_i^{i,i-1}\dot{\theta}_i$
    - $\dot{\mathcal{V}}_i^{i,0} = \mathrm{Ad}_{H_{i-1}^{i}(\theta_i)}\mathcal{V}_{i-1}^{i-1,0} - \mathrm{ad}_{S_i^{i,i-1}\dot{\theta}_i}\left(\mathrm{Ad}_{H_{i-1}^{i}(\theta_i)}\mathcal{V}_{i-1}^{i-1,0}\right) + S_i^{i,i-1}\ddot{\theta}_i$

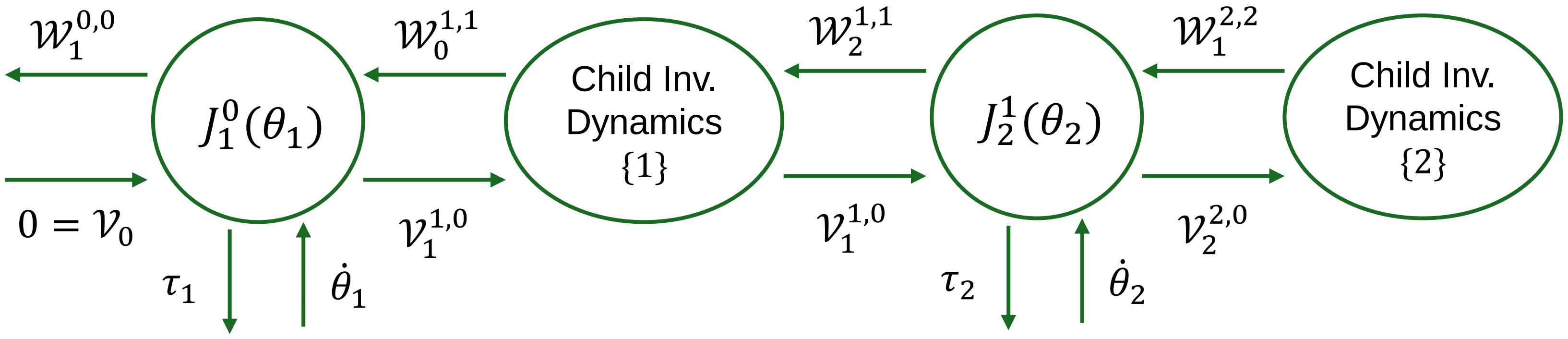




Two-link manipulator

- Newton-Euler Inverse Dynamics Algorithm
  - Backward iterations
    - $\mathcal{W}_{i-1}^{i,i} = \mathfrak{I}^{i,i}\dot{\mathcal{V}}_i^{i,0} - \mathrm{ad}_{\mathcal{V}_i^{i,0}}^{\top}\left(\mathfrak{I}^{i,i}\mathcal{V}_i^{i,0}\right) + \mathrm{Ad}_{H_i^{i+1}(\theta_{i+1})}^{\top}\mathcal{W}_i^{i+1,i+1}$
    - $\tau_i = \left(S_i^{i,i-1}\right)^{\top}\mathcal{W}_{i-1}^{i,i}$

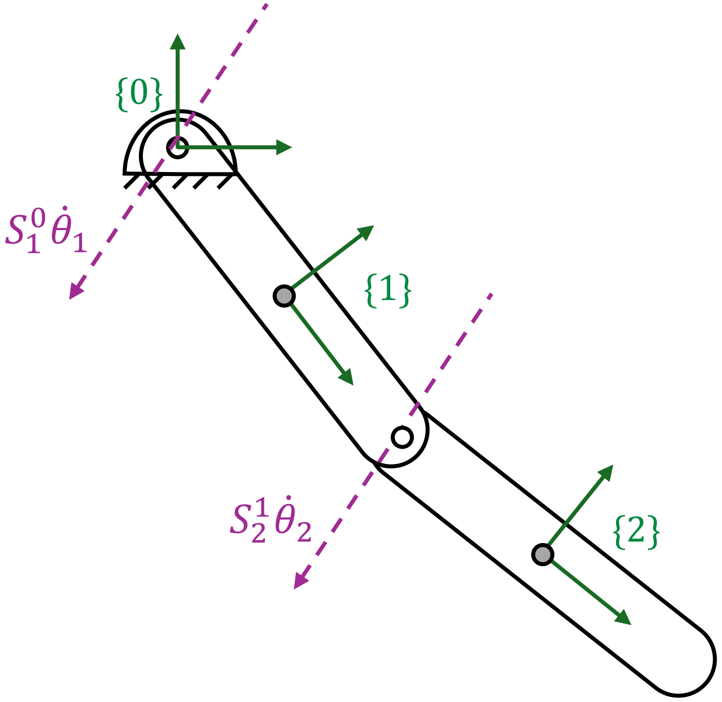


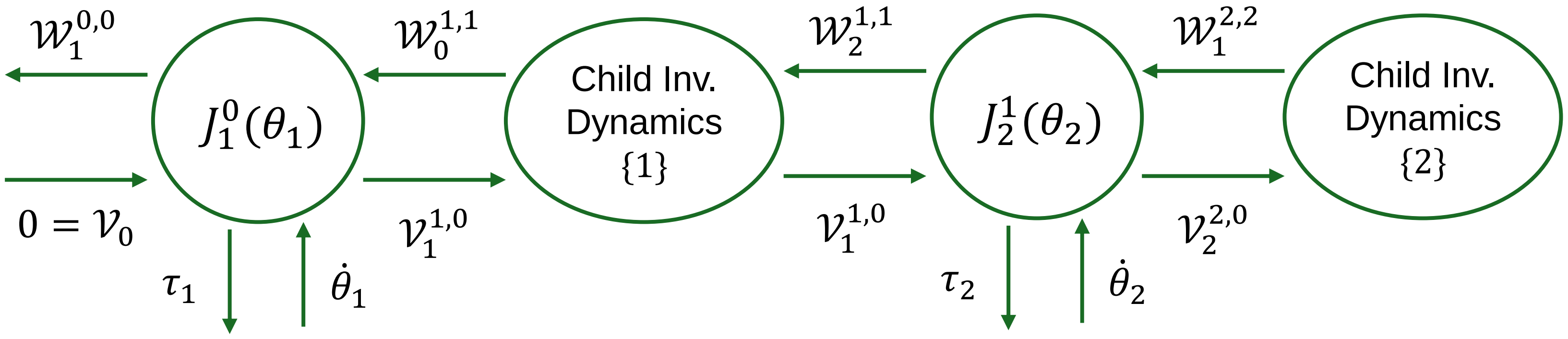


Two-link manipulator

- Reduced Manipulator Dynamics

$$\boxed{\begin{pmatrix}\tau_1\\ \tau_2\end{pmatrix} = M(\theta)\begin{pmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{pmatrix} + C(\theta,\dot{\theta})\begin{pmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{pmatrix}}$$





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