$$x = (x_1, x_2) = (\theta, \dot{\theta})$$

$$\dot{V}(x) = -\frac{1}{2} c_2 x_2^2$$

$eig = (0, c_2)$

$c_1, c_2 > 0$

$$= -\frac{1}{2} \begin{pmatrix} x_1 & x_2 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & c_2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$= -\frac{1}{2} x^T C x$$

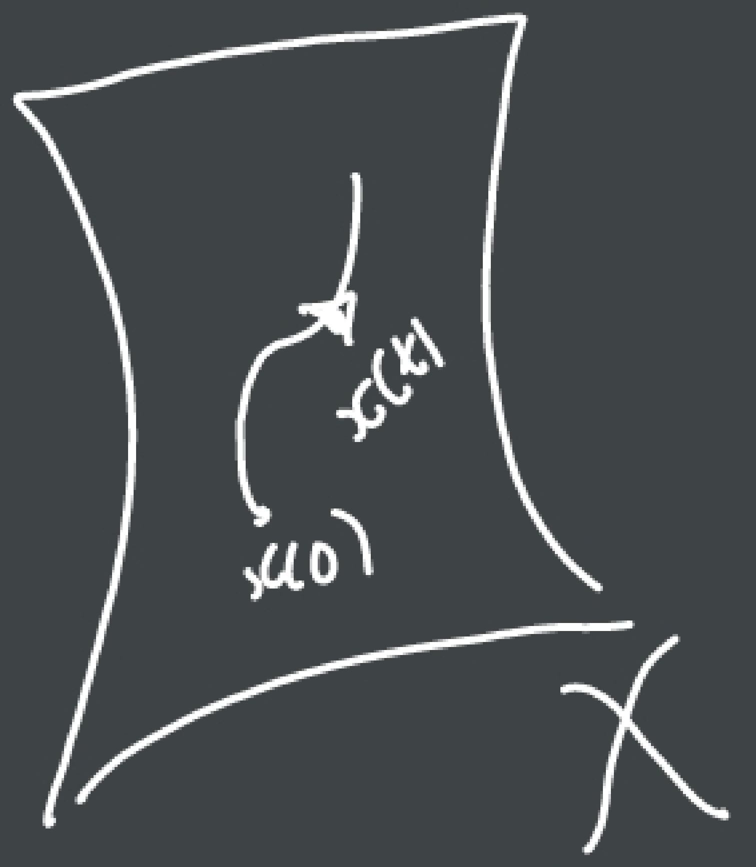
$$\boxed{C \succeq 0}$$

* Geometric Interpretation of $\dot{V}(x)$:

$$V : X \longrightarrow \mathbb{R}$$

$$\dot{V} = \left(\frac{\partial V}{\partial x}\right)^T \dot{x}$$

$$\dot{x}(t) = f(x(t))$$



$$\frac{d}{dt} V(x(t)) = \underbrace{dV(x(t))}_{\mathbb{R}^{1\times n}} \; \underset{\mathbb{R}^{n\times 1}}{\dot{x}(t)} = \Big\langle \underbrace{dV(x(t))}_{\in T^*_{x(t)}X} \,\Big|\, \underbrace{\dot{x}(t)}_{\in T_{x(t)}X} \Big\rangle$$

σ_f

if $x \in \mathbb{R}^n$

$$= dV(x)\,(f(x))$$

$$= \mathcal{L}_{\sigma_f} V(x)$$

$$\dot{x}(t) \in T_{x(t)}X$$

$v \in V$, $\alpha \in V^* : V \to \mathbb{R}$

$$\langle \alpha | v \rangle_V := \alpha(v)$$

$$f : \mathbb{R}^n \to \mathbb{R}$$

$$df(x) = \left(\frac{\partial f(x)}{\partial x_1}, \ldots, \frac{\partial f(x)}{\partial x_n}\right)$$

$$\boxed{V : \mathcal{X} \to \mathbb{R}}$$

$$\boxed{\dot{V} := \mathcal{L}_{\mathfrak{F}} V}$$

$$dV(R) \in T_R^* SO(3)$$

$$\dot{R} \in T_R SO(3)$$

$$\mathcal{X} \cong SO(3)$$

$$V : SO(3) \to \mathbb{R}$$

$$\cancel{\dot{V} = \left(\frac{\partial V}{\partial R}\right)^T \dot{R}}$$

Assuming

$$SO(3) = \mathbb{R}^3$$

* Lyapunov Stability of Satellite:

* $V(\underline{0}) = 0$

- $V(\omega) = \frac{1}{2}\omega^T J \omega$, $\omega \in \mathbb{R}^3$

$J = J^T$

- $\dot{V}(\omega) = dV(\omega)\ \dot{\omega}$

$$= \left(\frac{\partial V}{\partial \omega_1}(\omega), \frac{\partial V}{\partial \omega_2}(\omega), \frac{\partial V}{\partial \omega_3}(\omega) \right) \dot{\omega}$$

$$= (J\omega)^T \dot{\omega}$$

$$= \omega^T J \dot{\omega} = \omega^T (J\omega)^{\sim} \omega + \omega^T \tau$$

$$= \omega^T \tau = -\omega^T K \omega < 0$$

$$\tau = \gamma(\omega) = -\underset{\mathbb{R}^{3\times 3}}{K}\,\omega$$

Damping Controller Stabilization

$$\dot{\omega} = \begin{pmatrix} c_1\ \omega_2\ \omega_3 \\ c_2\ \omega_1\ \omega_3 \\ c_3\ \omega_1\ \omega_2 \end{pmatrix} + \begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{pmatrix}$$

$$\boxed{\omega_{*,1} = (0,0,0)}$$

$$J\dot{\omega} = -\omega \wedge (J\omega) + \tau$$

$$= (J\omega)^{\sim}\omega + \tau$$

$\omega_{*,1} = 0$ is a stable eq point

$x \in \mathbb{R}^n$

$$x^T \underbrace{J(x)}_{= -J^T(x)} x = 0$$