* $d\Psi(R) \in T_R^{*} so(3)$

* $e_R \in (\mathbb{R}^3)^*$

$$
\begin{tikzcd}
\mathbb{R}^3 \arrow[r, "S"] \arrow[rr, bend left, "\beta_R"] & so(3) \arrow[r, "L_{R,*}"] & T_R SO(3)
\end{tikzcd}
$$

$$
\begin{tikzcd}
(\mathbb{R}^3)^* & so^*(3) \arrow[l, "S^*"'] & T_R^* SO(3) \arrow[l, "L_R^*"'] \arrow[ll, bend left, "\beta_R^*"]
\end{tikzcd}
$$

$$\langle e_R \,|\, \delta\omega \rangle_{\mathbb{R}^3} = \langle \tilde{e}_R \,|\, \widetilde{\delta\omega} \rangle_{so(3)} = \langle d\Psi(R) \,|\, \delta R \rangle_{T_R SO(3)},$$

$$e_R^T \delta\omega = \frac{1}{2} \operatorname{tr}\left(\tilde{e}_R^T \widetilde{\delta\omega} \right) = \operatorname{tr}\left(d\Psi(R)\, \delta R \right)$$

$$e_R^T \delta\omega = \frac{1}{2} tr(\tilde{e}_R^T \delta\tilde{\omega}) = tr(d\Psi(R)\,\delta R)$$

$$\downarrow$$

$$= \frac{1}{2} tr(\underbrace{\tilde{e}_R^T}_{\circledast} \delta\tilde{\omega}) = \frac{1}{2} tr(\underbrace{-R_d^T R} \delta\tilde{\omega})$$

$$= -\frac{1}{2} tr(sym(R_d^T R)\delta\tilde{\omega} + sk(R_d^T R)\delta\tilde{\omega})$$

$$= \underbrace{-\frac{1}{2} tr(sym(R_d^T R)\delta\tilde{\omega})}_{=0} + \frac{1}{2} tr(\underbrace{-sk(R_d^T R)}_{\circledast}\delta\tilde{\omega})$$

- For any $A \in \mathbb{R}^{3\times 3}$, we can write it as:

$$A = sym(A) + sk(A)$$

$$sym(A) := \frac{A+A^T}{2}, \quad sk(A) := \frac{A-A^T}{2}$$

- For any $A = A^T$, $B = -B^T$

$$tr(AB) = tr(BA) = 0$$

$\delta\omega \in \mathbb{R}^3$

$$\delta\tilde{\omega} = \begin{pmatrix} 0 & & \\ & 0 & \\ & & 0 \end{pmatrix}$$

$$\delta R = R\,\delta\tilde{\omega}$$

$$d\Psi(R) = -\frac{1}{2} R_d^T$$

$$\therefore \quad \tilde{e}_R^{\,T} = -sk(R_d^T R)$$

$$\Rightarrow \quad \tilde{e}_R = sk(R_d^T R)$$

$$\Rightarrow \quad e_R = S^{-1}\left(sk(R_d^T R) \right)$$

$$= \frac{1}{2} S^{-1}\left(R_d^T R - R^T R_d \right) \in \mathbb{R}^3$$

$$\Rightarrow \quad \boxed{\tau_p = -K_p \, e_R}$$

S

$\wedge : \mathbb{R}^3 \longrightarrow so(3)$

$\vee : so(3) \longrightarrow \mathbb{R}^3$

vee

S^{-1}

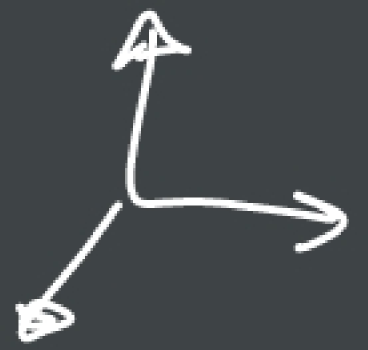
$V(R,p) = H(p) + \Psi(R)$

$$R_e = R_e^T$$

$$M = \{(R_{e,1},0),(R_{e,2},0),(R_{e,3},0),(R_{e,4},0)\}$$

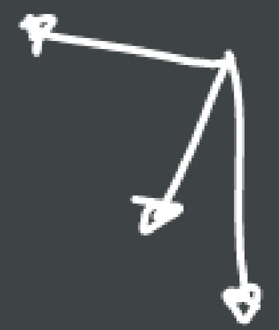
$\Psi(R_{e,1}) = 0$ $\quad \Psi(R_{e,2}) = \Psi(R_{e,3}) = \Psi(R_{e,4}) = 2$

$$R_{e_1} = I$$

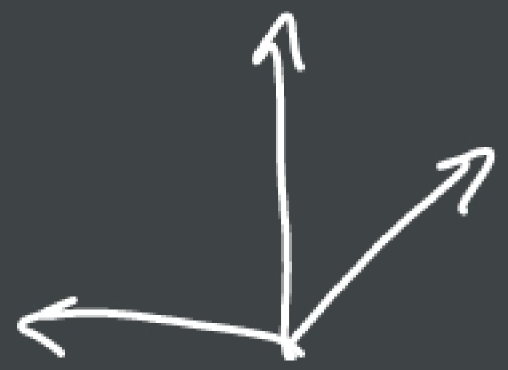


$$R_{e_3} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

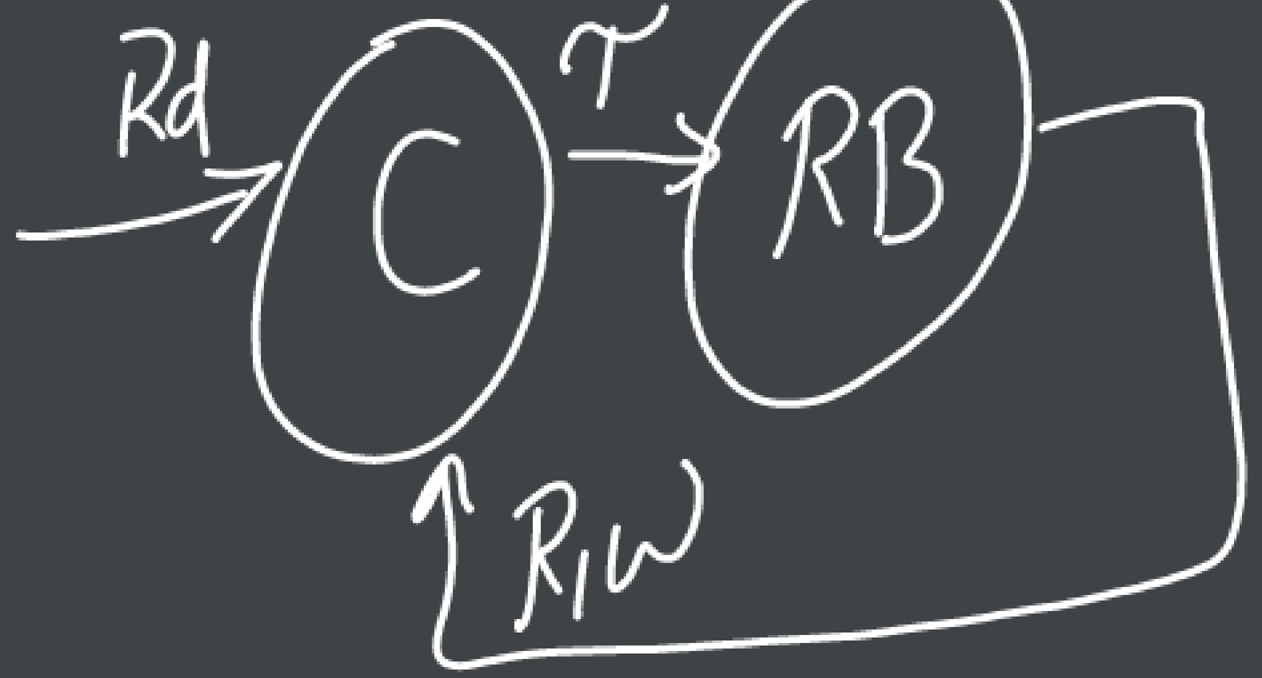
$$R_{e_2} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$



$$R_{e_4} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



Almost G.A.S $(R = R_d, 0)$



$SO(3)$

$$\dot{R} = R\tilde{\omega}$$

$$\dot{p} = \tilde{p}\omega + \tau_p + \tau_d$$

$$\tau_p = -K_p e_R$$

$$e_R = \frac{1}{2} S^{-1}(R_d^T R - R^T R_d)$$

$$\tau_d = -K_d \omega$$

$(R, p) = (R_d, 0) \Longrightarrow$ Almost G.A.S.

$$\dot{\xi} = v$$

$$\dot{p} = u_p + u_d$$

$$u_p = -K_p(\xi - \xi_d)$$

$$u_d = -K_d v$$

$\mathbb{R}^3$

$$\Psi(\xi) = \frac{1}{2}(\xi - \xi_d)^T$$

$$K_p(\xi - \xi_d)$$

*Alternative:-

$$\Phi : SO(3) \longrightarrow \mathbb{R}$$

$$\Phi(R) := \frac{1}{2} \operatorname{tr}\left(G_p \left(I - R_d^T R \right) \right)$$

$$\tau_p = - \beta_R^*\left(d\Phi(R) \right) = - \operatorname{Sk}\left(G_p R_d^T R \right)$$

$$\boxed{\begin{aligned} G_p &= \frac{1}{2} \operatorname{tr}(K_p) I - K_p \\ K_p &= \end{aligned}}$$