SCE 594: Special Topics in Intelligent Automation & Robotics

Lecture 24: Impedance Control I

جامعة الملك فهد للبترول والمعادن
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Outline

- Recap last lecture
- Feedback Linearization Control
- From Motion Control to Impedance Control
- Impedance Control of a Point Mass
- Impedance Control of a n-link manipulator

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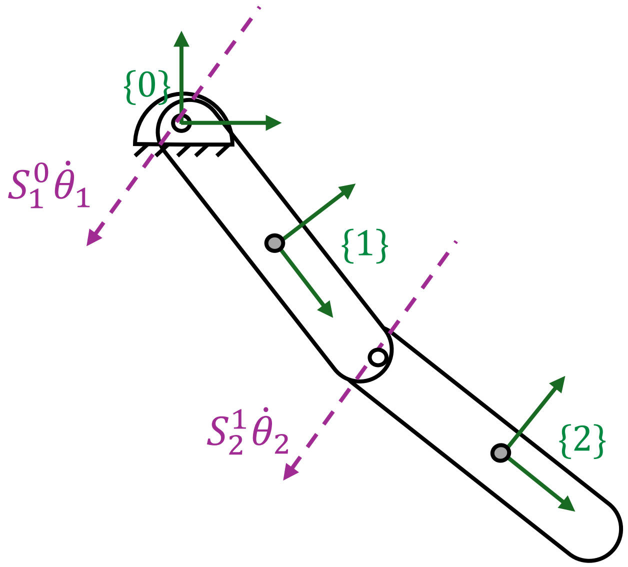
Recap: State Space Dynamics

- The governing equations of an n-link manipulator* with control torques τ are:

$$M(\theta)\ddot{\theta} + C\left(\theta, \dot{\theta}\right)\dot{\theta} + B\left(\dot{\theta}\right)\dot{\theta} + g(\theta) = \tau$$

where $\theta \in Q = \mathbb{S}^1 \times \cdots \times \mathbb{S}^1 \cong \mathbb{T}^n$.

- We can cast it into state space form
  - $x = (x_1, x_2) = \left(\theta, \dot{\theta}\right) \in TQ$
  - $\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ -M^{-1}(x_1)\,[c(x) + b(x_2) + g(x_1)] \end{pmatrix} + \begin{pmatrix} 0 \\ M^{-1}(x_1) \end{pmatrix} \tau$



$c(x) \coloneqq C(x_1, x_2)x_2 \in \mathbb{R}^n, \qquad b(x_2) \coloneqq B(x_2)x_2 \in \mathbb{R}^n, \qquad g(x_1) \in \mathbb{R}^n$

Recap: Stabilization Control

- The control law

$$\boxed{\begin{aligned}\tau &= \tau_p + \tau_d \\ &= -\nabla\Psi(x_1) - K_d x_2 = g(x_1) - K_p\,(x_1 - \theta_d) - K_d x_2\end{aligned}}$$

makes the closed loop system

$$\begin{pmatrix} \dot{x}_1 \\ M(x_1)\dot{x}_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ -c(x) - b(x_2) - K_p\,(x_1 - \theta_d) - K_d x_2 \end{pmatrix}$$

have a globally asymptotically equilibrium point at $x_d = (\theta_d, 0_2)$.

Lyapunov function: $V_{CL}(x_1, x_2) = V(x_1, x_2) + \Psi(x_1)$

$V(x_1, x_2) = \frac{1}{2} x_2^\top M(x_1) x_2 + E_{pot}(x_1)$, $\quad \Psi(x_1) = \frac{1}{2}\,(x_1 - \theta_d)^\top K_p\,(x_1 - \theta_d) - E_{pot}(x_1)$

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Feedback Linearization

- Feedback linearization control aims to cancel out the known nonlinearities by applying a torque command τ that directly “inverts” the dynamics.
- The result (in an ideal, no-uncertainty scenario) is a closed-loop system that behaves like a simple linear system.

Feedback Linearization

- For the state space model

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ -M^{-1}(x_1)\left[c(x) + b(x_2) + g(x_1) - \tau\right] \end{pmatrix}$$

- The feedback linearization control law is given by

$$\tau = c(x) + b(x_2) + g(x_1) - M(x_1)\left[K_p\,(x_1 - \theta_d) + K_d x_2\right]$$

yields the closed loop system

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ -K_p\,(x_1 - \theta_d) - K_d x_2 \end{pmatrix}$$

which is globally asymptotically stable at $x_d = (\theta_d, 0_2)$.

Known in the literature as the inverse dynamics or computed torque control method

Comparison

- **Computed Torque Method (CTM):**
  - Full dynamic compensation: Cancels out all nonlinearities (inertia coupling, Coriolis/centrifugal forces, and gravity) to transform the system into decoupled linear systems.
  - Performance: Can achieve very fast, accurate trajectory tracking if the model is good.
  - Model dependence: Highly sensitive to model errors; requires accurate knowledge of inertial, Coriolis, and gravity terms.
- **PD with Gravity Compensation:**
  - Partial dynamic compensation: Only cancels gravity effects.
  - Performance: Good for slow or moderate speed motions; tracking degrades at high speeds due to unmodeled dynamics
  - Model dependence: Less sensitive to errors compared to CTM; only needs a reasonable estimate of the gravity vector.

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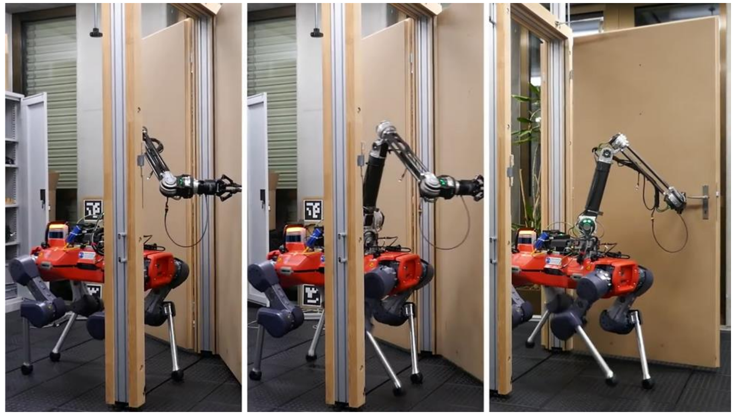
Paradigm Shift: Motion Control vs. Interaction Control

- **Motion Control**
- Main objective is to achieve stability and reject external disturbances to maximize performance.

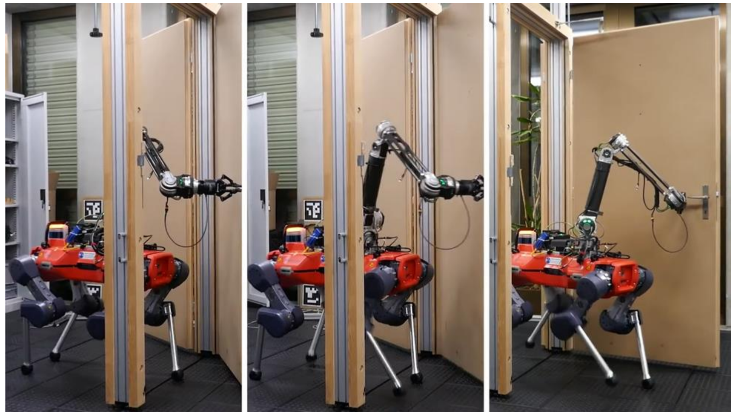
- **Interaction Control**
- Main objective is to be able to interact with an unknown environment in a stable & safe manner.



Paradigm Shift: Motion Control vs. Interaction Control

- **Motion Control**
- Closed dynamical system.
- Stability analysis requires closed loop model.

- **Interaction Control**
- Open dynamical system.
- Unknown environment needs to be incorporated in closed loop stability analysis.



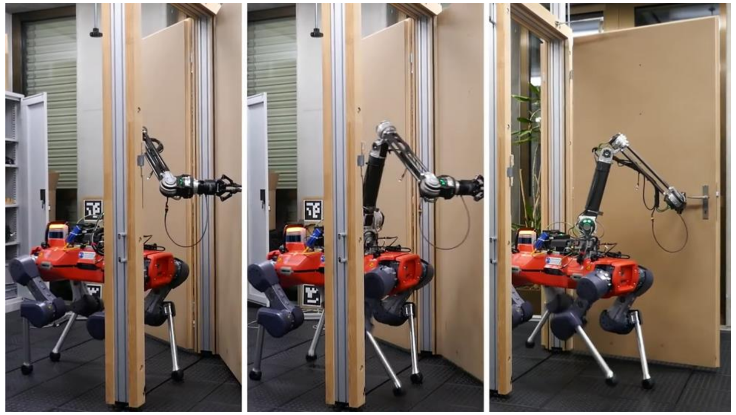
Paradigm Shift: Motion Control vs. Interaction Control

- **Motion Control**
- Based on unilateral signals.

e.g., control of position or velocity

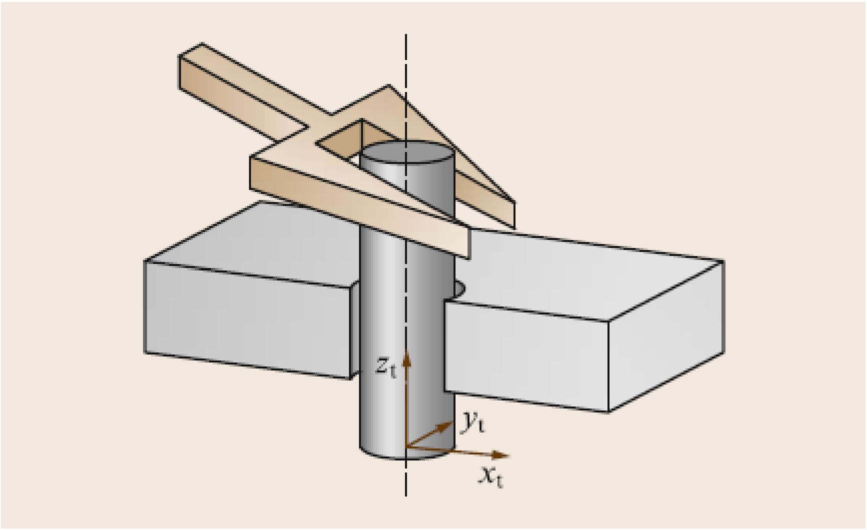
- **Interaction Control**
- Based on bilateral signals

e.g., control of relation between velocity and force.

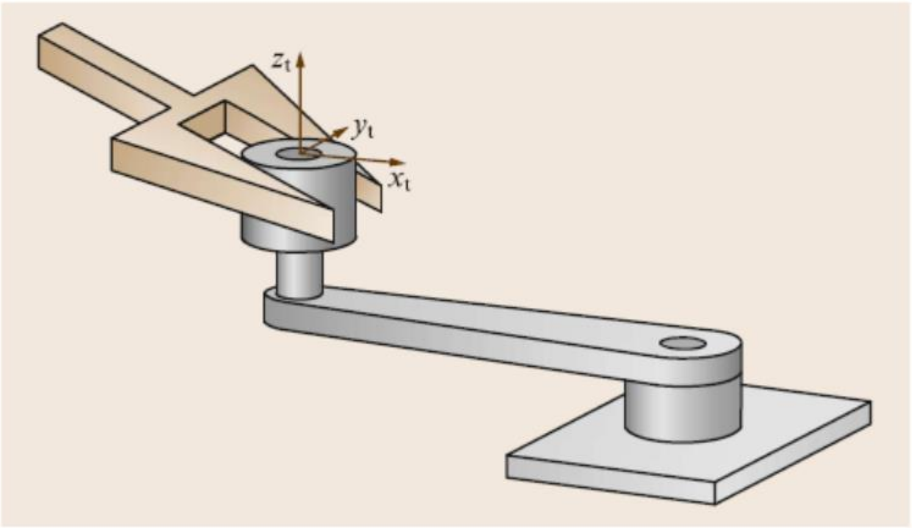


Interaction Control

- Controlling either the pose or wrench requires perfect knowledge of the task environment and when contact occurs or not, which is clearly not possible in practice.
- Instead, the behavior (relation between the wrench and pose) of the controlled robot could be modified independent of the environment.



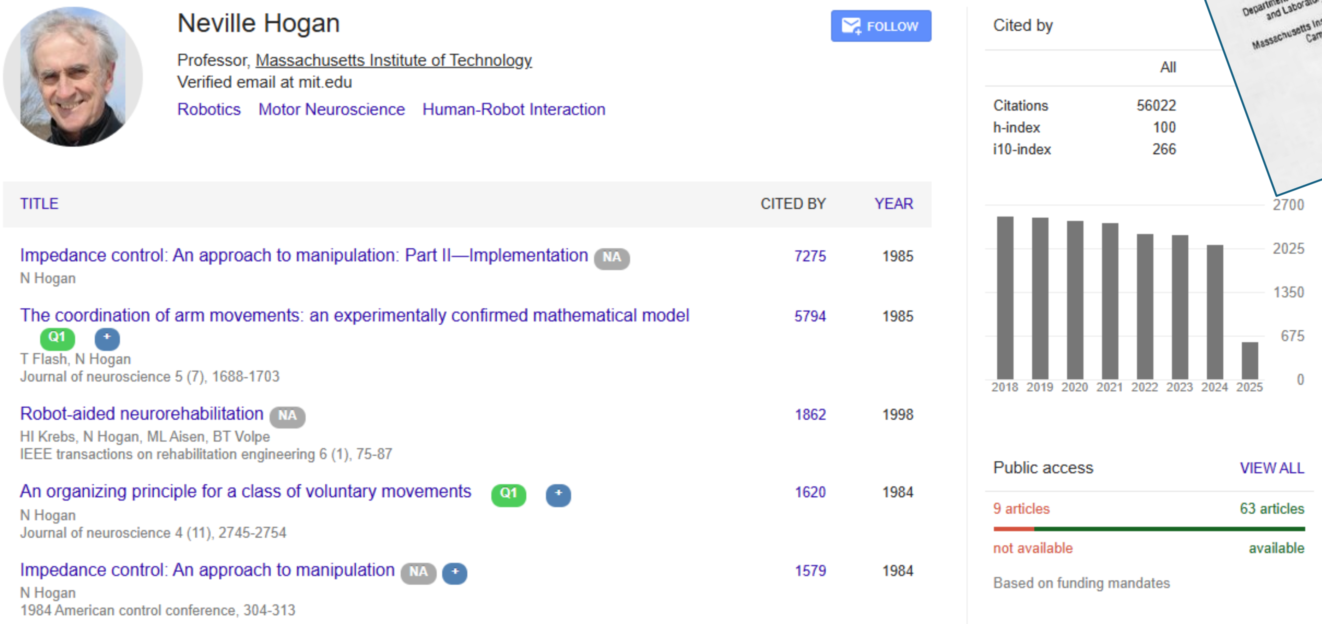
Insertion of a cylindrical peg into a hole



Turning a crank with an idle handle

Impedance Control

- The first one to address this issue in the field of robotics was Neville Hogan in his seminal trilogy.



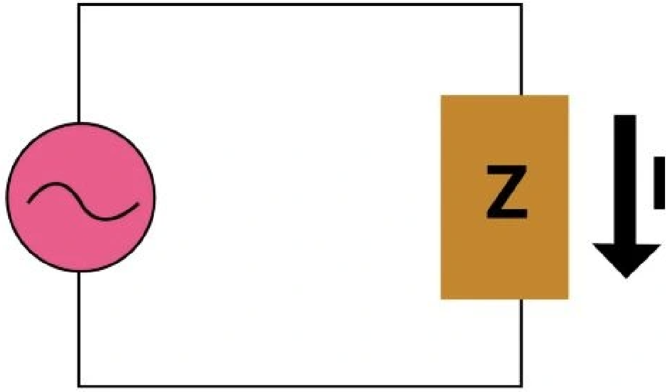
Electrical Impedance and Admittance

- In electrical engineering, impedance (Z) is the total opposition a circuit offers to the flow of alternating current.

$$\vec{Z} = \frac{\vec{V}}{\vec{I}} = R + j\,X$$

- Admittance (Y) is the reciprocal of impedance and is a measure of how easily a circuit or device will allow a current to flow.

$$\vec{Y} = \frac{\vec{I}}{\vec{V}} = G + j\,B$$



V: Voltage [V]
I: Current [A]

Phasors (frequency dependent):
$\vec{V} = |V|e^{j(\omega t+\phi_V)}$, $\vec{I} = |I|e^{j(\omega t+\phi_I)}$

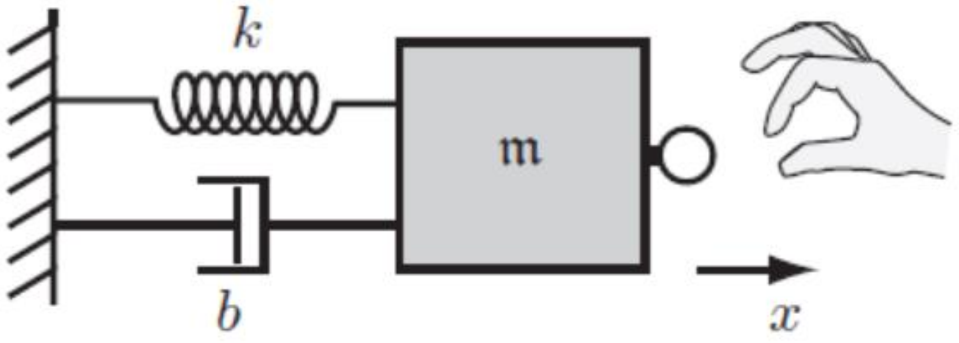
Mechanical Impedance and Admittance

- In mechanical engineering, impedance is a measure of how much a structure resists motion when subjected to a harmonic force.

$$\vec{Z} = \frac{\vec{F}}{\vec{v}}$$

- Mechanical admittance is the reciprocal of impedance.

$$\vec{Y} = \frac{\vec{v}}{\vec{F}}$$

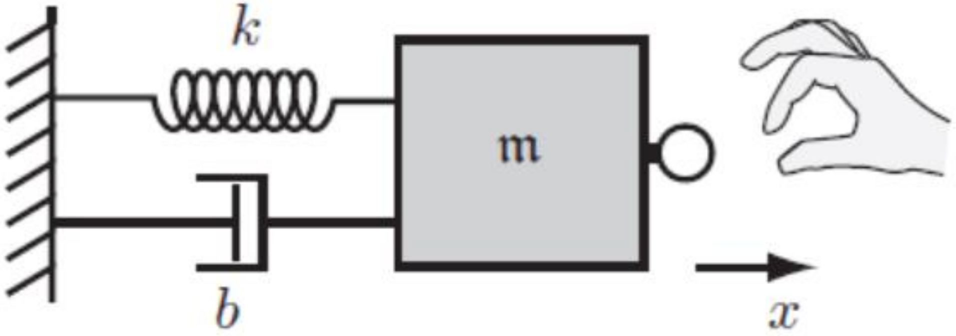


Impedance vs. Admittance

- **Impedance**
- $F(s) = Z(s)X(s)$
- $Z(s) = m\,s^2 + b\,s + k$
- Motion input, Force output

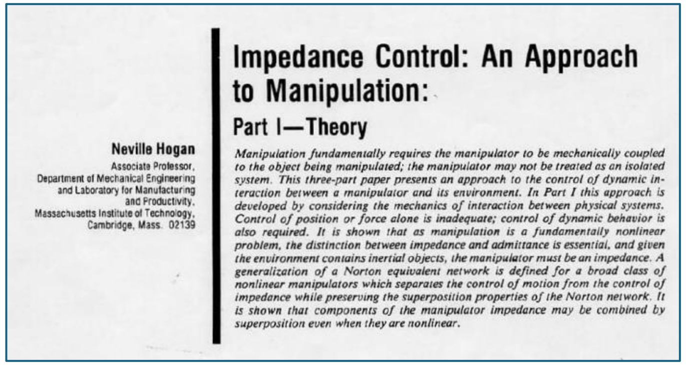
- **Admittance**
- $X(s) = Y(s)F(s)$
- $Y(s) = \frac{1}{m\,s^2 + b\,s + k}$
- Force input, Motion output

$$\boxed{m\,\ddot{x} + b\,\dot{x} + k\,x = F(t)}$$



Impedance Control

- Neville Hogan's idea of impedance control is that instead of commanding exact positions or forces, the robot is controlled to behave like a mechanical impedance:
  - It responds to external forces with a desired relationship between force, position, and velocity, like a spring-damper-mass system.
- The key is shaping the robot's dynamic behavior — its *stiffness*, *damping*, and *inertia* — to match the interaction task needs.



Impedance Control: An Approach to Manipulation:

Part I—Theory

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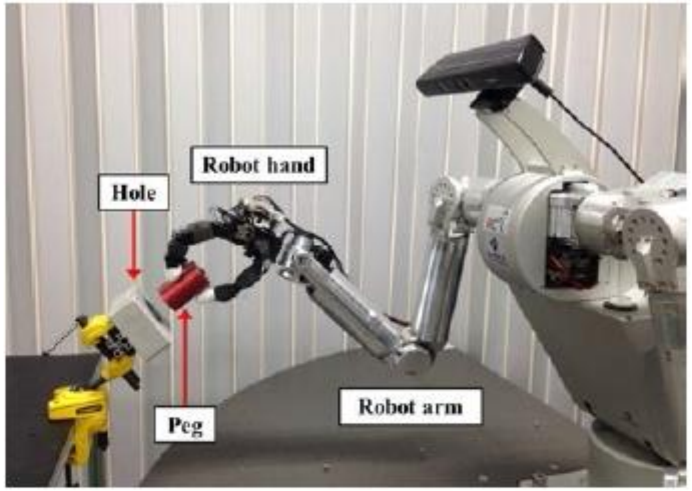
Manipulation fundamentally requires the manipulator to be mechanically coupled to the object being manipulated; the manipulator may not be treated as an isolated system. This three-part paper presents an approach to the control of dynamic interaction between a manipulator and its environment. In Part I this approach is developed by considering the mechanics of interaction between physical systems. Control of position or force alone is inadequate; control of dynamic behavior is also required. It is shown that as manipulation is a fundamentally nonlinear problem, the distinction between impedance and admittance is essential, and given the environment contains inertial objects, the manipulator must be an impedance. A generalization of a Norton equivalent network is defined for a broad class of nonlinear manipulators which separates the control of motion from the control of impedance while preserving the superposition properties of the Norton network. It is shown that components of the manipulator impedance may be combined by superposition even when they are nonlinear.



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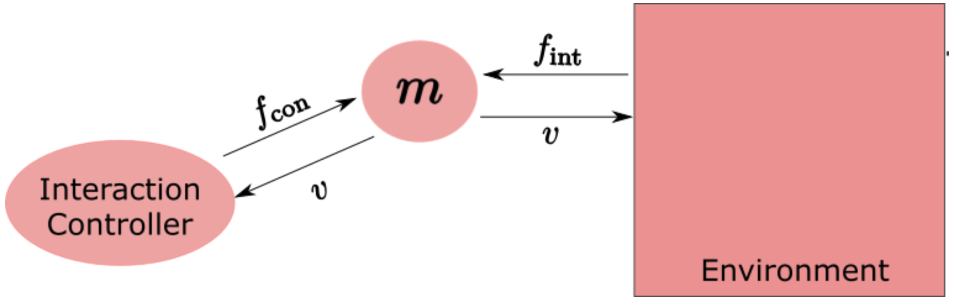
Point Mass Dynamics

- To provide some intuition, let's start in a simple Euclidean space $\mathbb{R}^n$.
- The governing equations of a point mass (with no gravity) are:
  - $\dot{\xi} = v, \qquad m\dot{v} = f_{\text{con}} + f_{\text{int}}$



Impedance Control of a Point Mass

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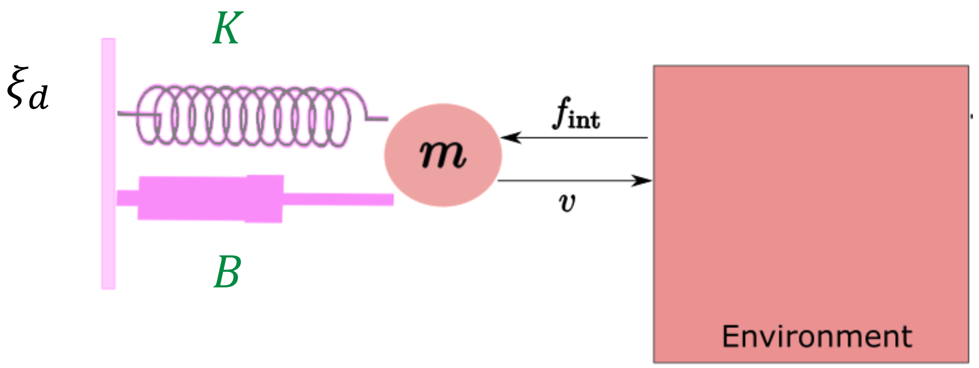
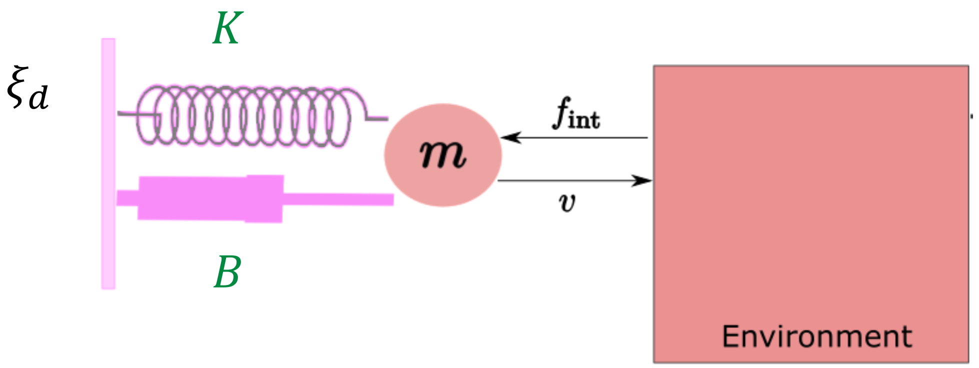
- The governing equations of a point mass (with no gravity) are:
  - $\dot{\xi} = v, \qquad m\dot{v} = f_{\text{con}} + f_{\text{int}}$
- The goal of impedance control is to implement the task-space behavior
  - $m\,\ddot{q} + B\dot{q} + Kq = f_{\text{int}}\,, \qquad q \coloneqq \xi - \xi_d,$



Impedance Control of a Point Mass

- To provide some intuition, let's start in a simple Euclidean space $\mathbb{R}^n$.
- The governing equations of a point mass (with no gravity) are:
  - $\dot{\xi} = v, \qquad m\dot{v} = f_{\text{con}} + f_{\text{int}}$
- The goal of impedance control is to implement the task-space behavior
  - $m\,\ddot{q} + B\dot{q} + Kq = f_{\text{int}}\,, \qquad q := \xi - \xi_d,$
- The impedance control law then takes the form

$$\boxed{f_{\text{con}} = m\ddot{\xi}_d - B(v - \dot{\xi}_d) - K(\xi - \xi_d)}$$



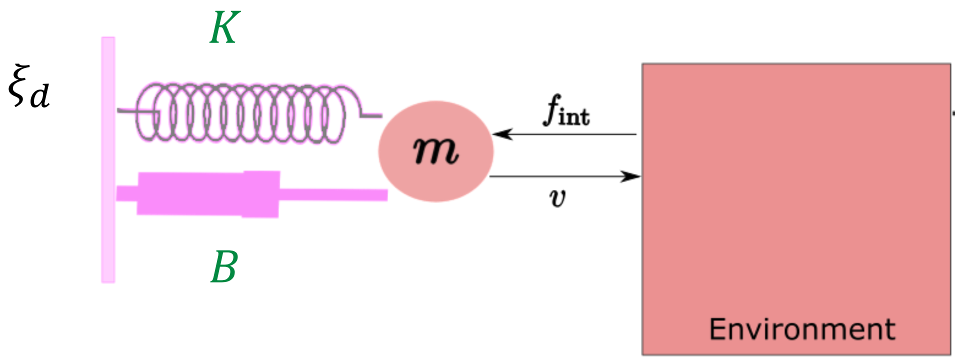
Impedance Control of a Point Mass

- During static interaction ($\ddot{q} = \dot{q} = 0$), the impedance behavior then becomes

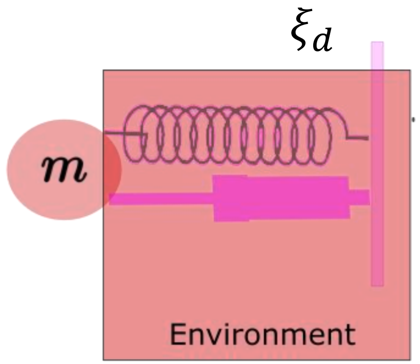
$$f_{\mathrm{int}} = K(\xi - \xi_d)$$

Motion input,
Force Output !

- Therefore, K plays the role of an active stiffness.
- The force applied to the environment depends on K and the virtual setpoint ξ_d.



Before Interaction



During Interaction

Impedance Control vs. PD Control

Aspect	PD Control	Impedance Control
Primary goal	Track a desired position (no interaction considered)	Shape the dynamic interaction (force vs motion)
External forces	Seen as disturbances to reject	Part of the behavior to regulate
If environment pushes	Robot tries hard to return to position (may push back aggressively)	Robot "complies" according to set stiffness/damping
Output behavior	Stiff behavior unless you manually tune gains	Adjustable compliance via virtual mass-spring-damper model

$$\boxed{f_{\mathrm{pd}} = -B(v) - K(\xi - \xi_d)}$$

$$\boxed{f_{\mathrm{imp}} = m\ddot{\xi}_d - B\left(v - \dot{\xi}_d\right) - K(\xi - \xi_d)}$$

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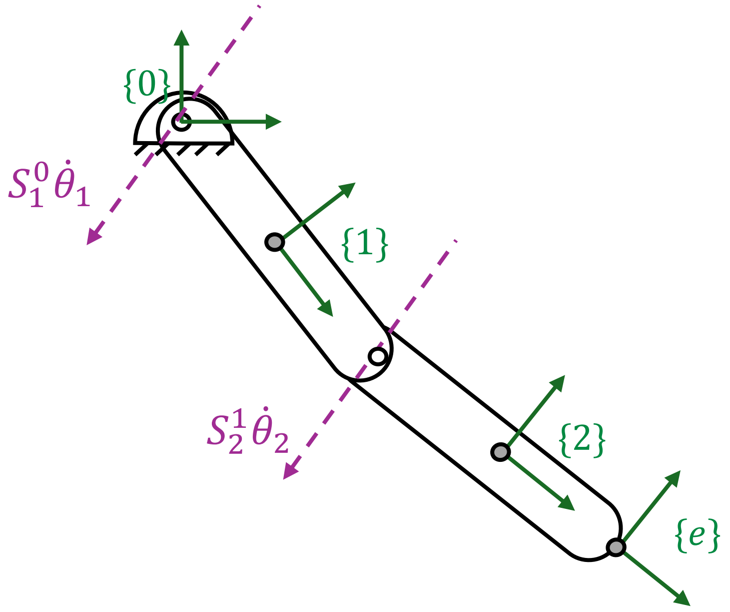
Impedance Control of n-link Manipulator

- Now we consider the impedance control problem of a n-link manipulator governed by

$$M(\theta)\ddot{\theta} + C\left(\theta, \dot{\theta}\right)\dot{\theta} + B\left(\dot{\theta}\right)\dot{\theta} + g(\theta) = \tau_{\mathrm{con}} + \tau_{\mathrm{int}}$$

- where $\tau_{\mathrm{con}} \in \mathbb{R}^n$ are joint control torques and $\tau_{\mathrm{int}} \in \mathbb{R}^n$ are torques due to interaction with the environment.



Impedance Control of n-link Manipulator

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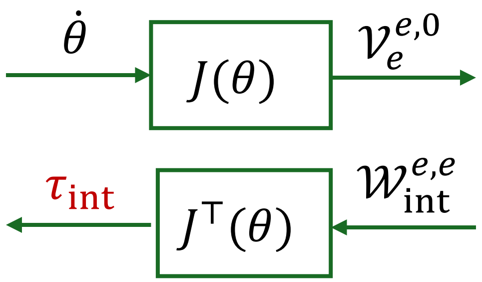
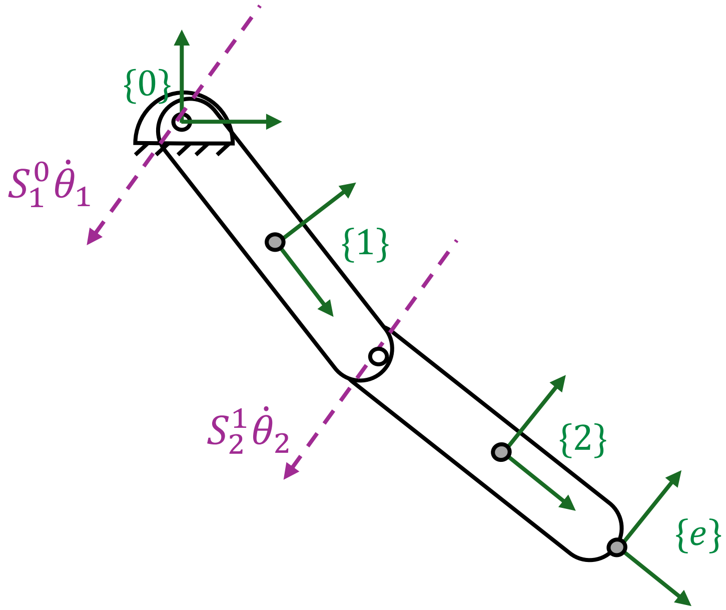
$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + B(\dot{\theta})\dot{\theta} + g(\theta) = \tau_{\text{con}} + \tau_{\text{int}}$$

- where $\tau_{\text{con}} \in \mathbb{R}^n$ are joint control torques and $\tau_{\text{int}} \in \mathbb{R}^n$ are torques due to interaction with the environment.
- Recall that:





where $J(\theta)$ is the geometric Jacobian.