SCE 594: Special Topics in Intelligent Automation & Robotics

Lecture 25: Impedance Control II

جامعة الملك فهد للبترول والمعادن
King Fahd University of Petroleum & Minerals

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Outline

- Recap last lecture
- Impedance Control of Manipulators
- Admittance Control of Manipulators

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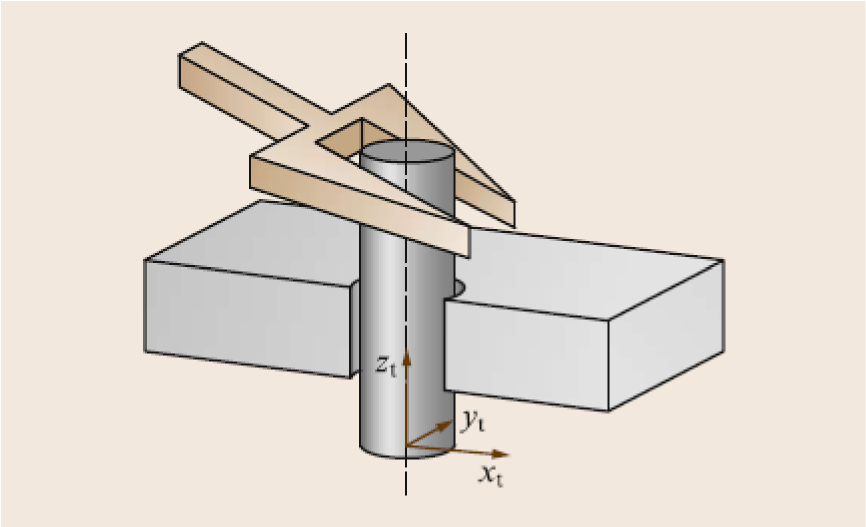
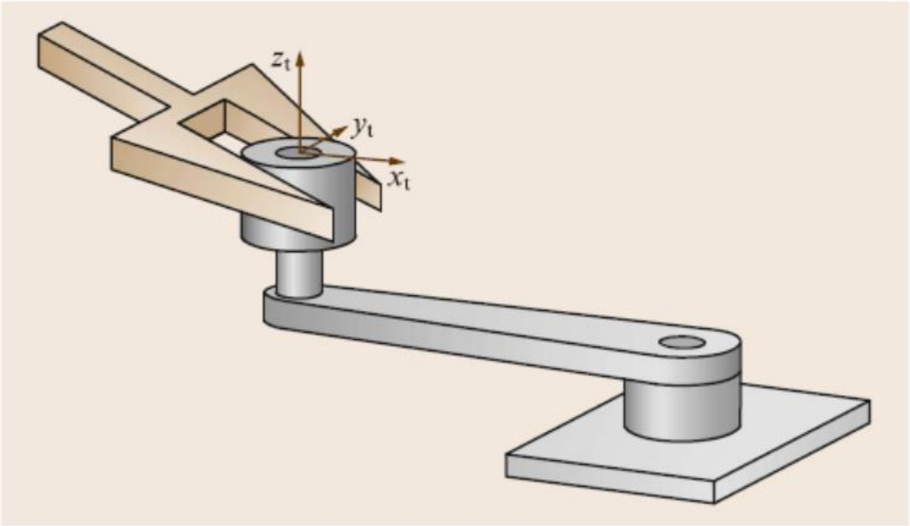
Recap: Interaction Control

- Controlling either the pose or wrench requires perfect knowledge of the task environment and when contact occurs or not, which is clearly not possible in practice.
- Instead, the behavior (relation between the wrench and pose) of the controlled robot could be modified independent of the environment.



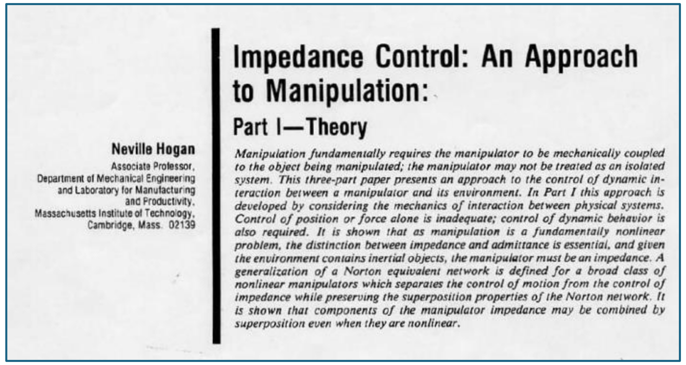
Insertion of a cylindrical peg into a hole



Turning a crank with an idle handle

Recap: Impedance Control

- Neville Hogan's idea of impedance control is that instead of commanding exact positions or forces, the robot is controlled to behave like a mechanical impedance:
  - It responds to external forces with a desired relationship between force, position, and velocity, like a spring-damper-mass system.
- The key is shaping the robot's dynamic behavior — its *stiffness*, *damping*, and *inertia* — to match the interaction task needs.



Impedance Control: An Approach to Manipulation:

Part I—Theory

Neville Hogan
Associate Professor,
Department of Mechanical Engineering
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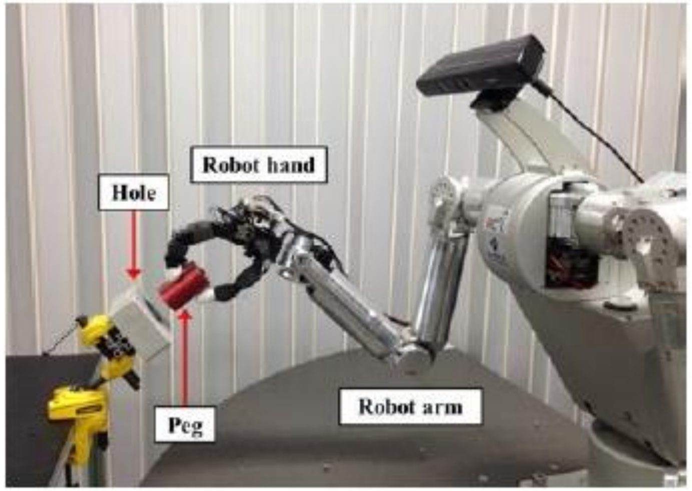
and Productivity,
Massachusetts Institute of Technology,
Cambridge, Mass. 02139

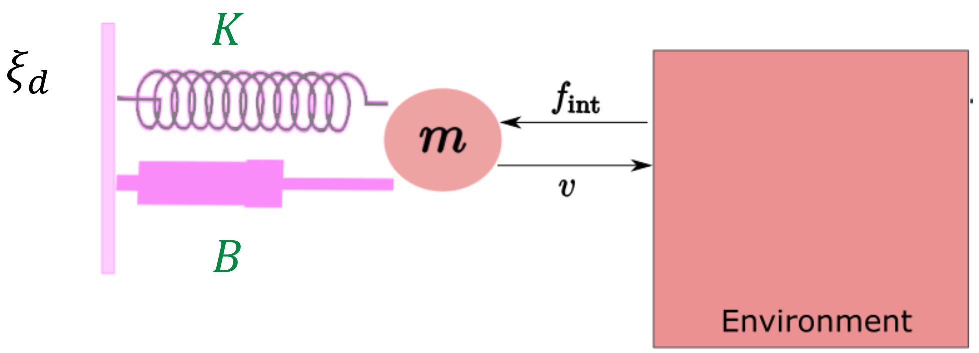
Manipulation fundamentally requires the manipulator to be mechanically coupled to the object being manipulated; the manipulator may not be treated as an isolated system. This three-part paper presents an approach to the control of dynamic interaction between a manipulator and its environment. In Part I this approach is developed by considering the mechanics of interaction between physical systems. Control of position or force alone is inadequate; control of dynamic behavior is also required. It is shown that as manipulation is a fundamentally nonlinear problem, the distinction between impedance and admittance is essential, and given the environment contains inertial objects, the manipulator must be an impedance. A generalization of a Norton equivalent network is defined for a broad class of nonlinear manipulators which separates the control of motion from the control of impedance while preserving the superposition properties of the Norton network. It is shown that components of the manipulator impedance may be combined by superposition even when they are nonlinear.



Recap: Impedance Control of a Point Mass

- To provide some intuition, let's start in a simple Euclidean space $\mathbb{R}^n$.
- The governing equations of a point mass (with no gravity) are:
  - $\dot{\xi} = v, \qquad m\dot{v} = f_{\text{con}} + f_{\text{int}}$
- The goal of impedance control is to implement the task-space behavior

$$\boxed{m\,\ddot{q} + B\dot{q} + Kq = f_{\text{int}}\,,} \qquad q \coloneqq \xi - \xi_d,$$



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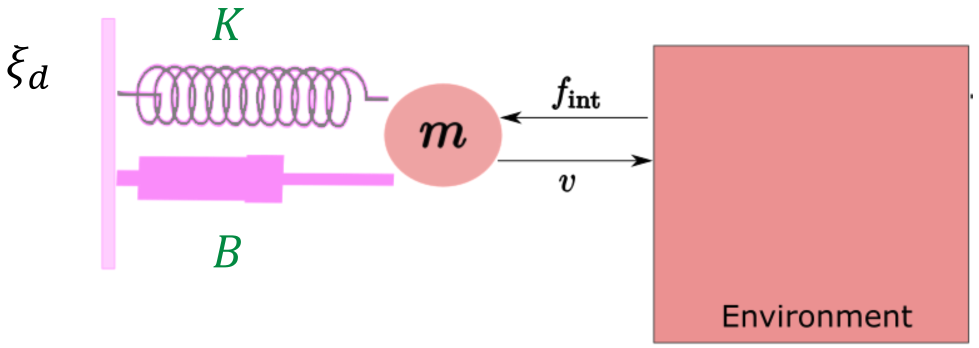
$$\boxed{m\,\ddot{q} + B\dot{q} + Kq = f_{\text{int}}\ ,} \qquad q \coloneqq \xi - \xi_d,$$

- The impedance control law that yields this impedance behavior is

$$\boxed{f_{\text{con}} = m\ddot{\xi}_d - B\left(v - \dot{\xi}_d\right) - K(\xi - \xi_d)}$$



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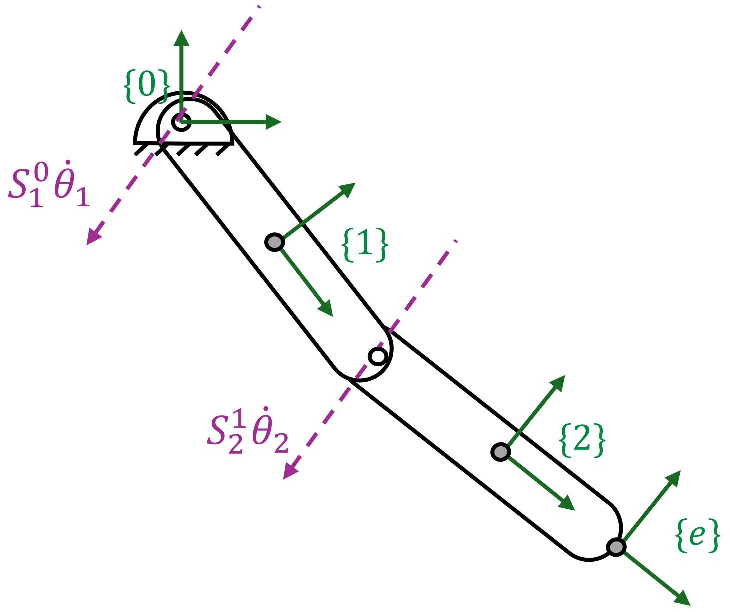
Impedance Control of n-link Manipulator

- Now we consider the impedance control problem of a n-link manipulator governed by

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + B(\dot{\theta})\dot{\theta} + g(\theta) = \tau_{\text{con}} + \tau_{\text{int}}$$

- where $\tau_{\text{con}} \in \mathbb{R}^n$ are joint control torques and $\tau_{\text{int}} \in \mathbb{R}^n$ are torques due to interaction with the environment.



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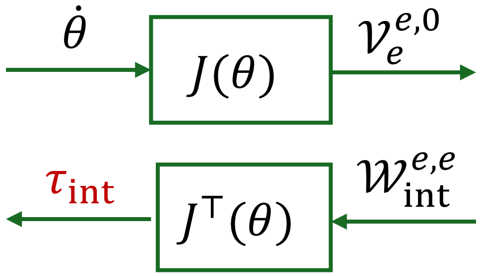
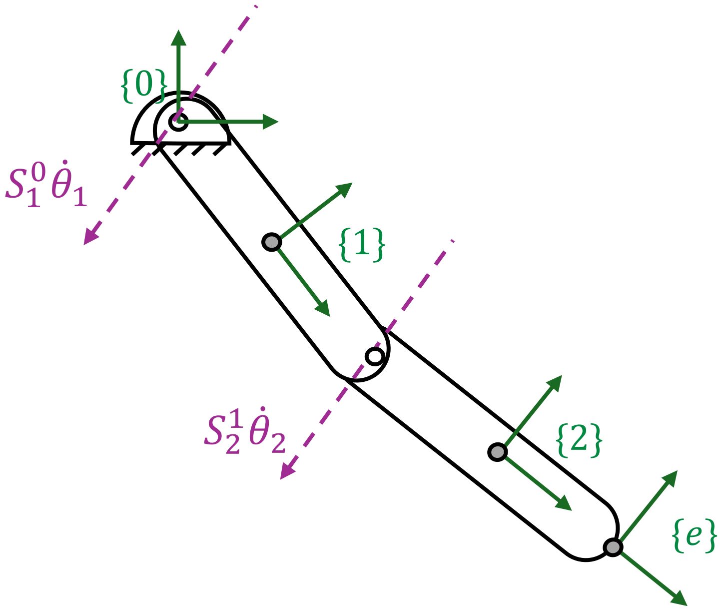
- Recall that:





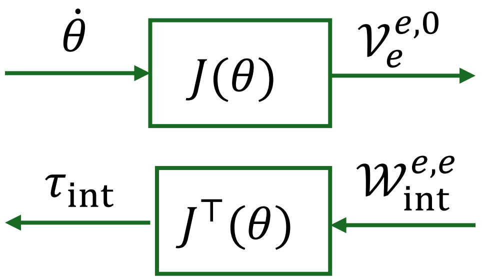
where $J(\theta)$ is the geometric Jacobian.

Geometric Jacobian Revisited

- The geometric Jacobian is a matrix that linearly maps the joint velocities to the end-effector’s twist

$$\mathcal{V}_e^{e,0} = \begin{pmatrix} \omega_e^{e,0} \\ v_e^{e,0} \end{pmatrix} = J(\theta)\dot{\theta}$$

- It provides an instantaneous kinematic relation between the joint space and the task space.



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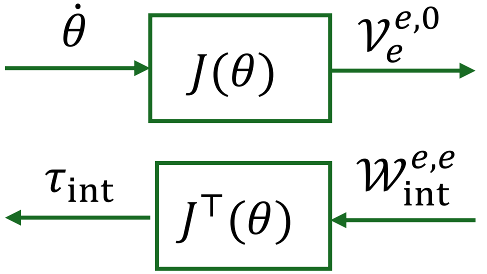
- It provides an instantaneous kinematic relation between the joint space and the task space.
- Each column of $J(\theta)$ is a joint screw axis represented in $\{e\}$ frame.

$$J(\theta) \coloneqq \left(Ad_{H_0^e(\theta)}\mathcal{S}_1^{0,0}\ , Ad_{H_1^e(\theta)}\mathcal{S}_2^{1,1}, \cdots, Ad_{H_{n-1}^e(\theta)}\mathcal{S}_n^{n-1,n-1}\right)$$

$J(\theta) \in \mathbb{R}^{6\times n}$



Geometric Jacobian Revisited

- The dual (or simply transpose) of the geometric Jacobian linearly maps any wrench at the end effector to joint torques

$$\tau_{\text{int}} = J^{\top}(\theta)\, \mathcal{W}_{\text{int}}^{e,e} = J^{\top}(\theta) \begin{pmatrix} \tau_{\text{int}}^{e,e} \\ f_{\text{int}}^{e,e} \end{pmatrix}$$

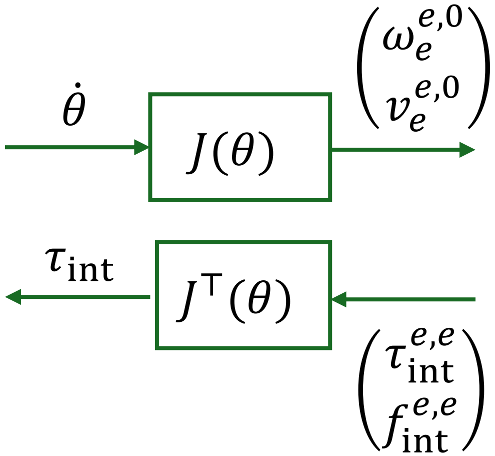
- Essential to utilize in interaction control, human-robot collaboration, and teleoperation.
- Analyzing properties of this matrix is essential for identifying kinematic singularities of the robot.

$$\dot{\theta} \longrightarrow \boxed{J(\theta)} \longrightarrow \mathcal{V}_e^{e,0}$$

$$\tau_{\text{int}} \longleftarrow \boxed{J^{\top}(\theta)} \longleftarrow \mathcal{W}_{\text{int}}^{e,e}$$

$J^{\top}(\theta) \in \mathbb{R}^{n\times 6}$

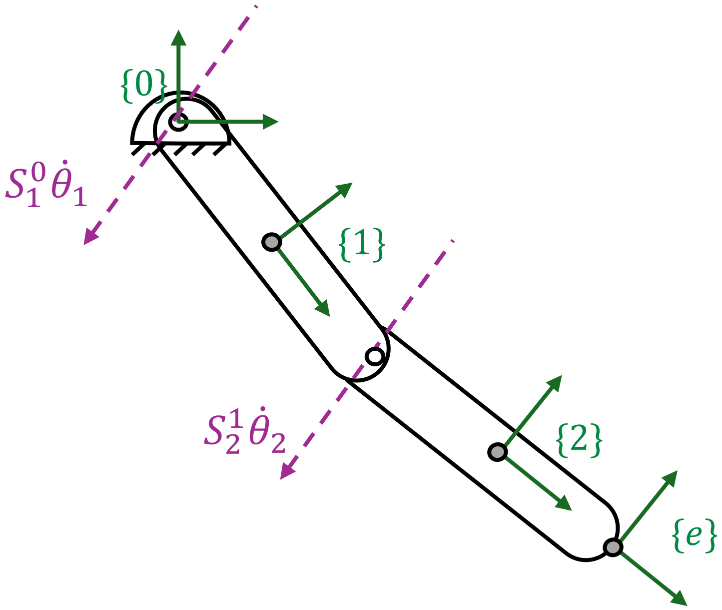
Example: 2-Link Manipulator



- Recall our two-link manipulator example.
- The geometric Jacobian takes the form

$$J(\theta) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ j_1(\theta) & 0 \\ j_2(\theta) & l_2 \\ 0 & 0 \end{pmatrix}$$

where j_1, j_2 are scalar functions of θ.



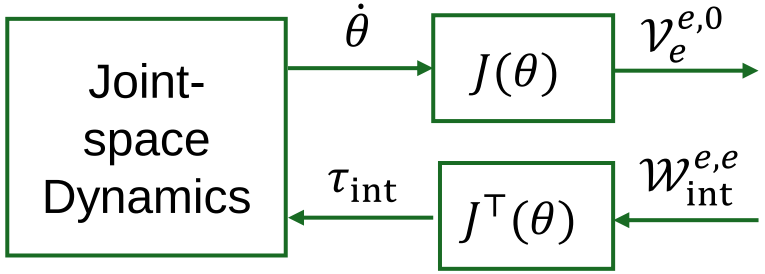
$$J(\theta) := \left(Ad_{H_0^e(\theta)}\mathcal{S}_1^{0,0}\ , Ad_{H_1^e(\theta)}\mathcal{S}_2^{1,1}\right) \in \mathbb{R}^{6\times 2}$$

Manipulator Dynamics in the Task Space

- If we do not apply any actuator torques ($\tau_{\text{con}} = 0$), the dynamics in the joint-space are given by

$$M(\theta)\ddot{\theta} + c(\theta, \dot{\theta}) + b(\dot{\theta}) + g(\theta) = J^\top(\theta)\, \mathcal{W}_{\text{int}}^{e,e}$$

- An interesting question is what does the dynamics “feel like” seen from the robot’s end-effector perspective?



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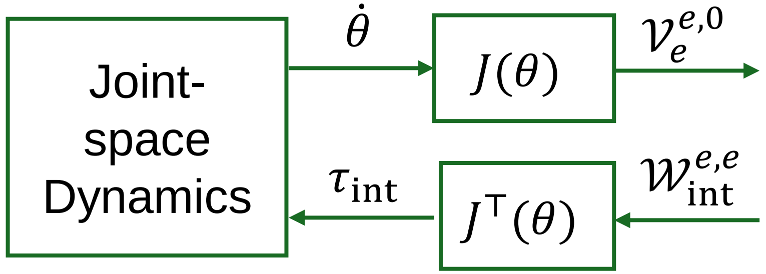
- An interesting question is what does the dynamics “feel like” seen from the robot’s end-effector perspective?
- In other words, someone is pushing/pulling the robot’s end-effector, will feel the dynamics in the task-space given by

$$\Lambda(\theta)\dot{\mathcal{V}}_e^{e,0} + \eta\big(\theta, \mathcal{V}_e^{e,0}\big)\mathcal{V}_e^{e,0} + \gamma(\theta) = \mathcal{W}_{\mathrm{int}}^{e,e}$$



Supplementary material will be provided for reading !

Manipulator Dynamics in the Task Space

- Therefore, a human interacting with an-unactuated robot arm* will experience the behavior given by

$$\underbrace{\Lambda(\theta)}_{\text{Apparent inertia}} \dot{\mathcal{V}}_e^{e,0} + \underbrace{\eta\left(\theta, \mathcal{V}_e^{e,0}\right)}_{\text{Apparent nonlinear friction}} \mathcal{V}_e^{e,0} + \underbrace{\gamma(\theta)}_{\text{Apparent gravitational forces}} = \mathcal{W}_{\mathrm{int}}^{e,e}$$

* This is only possible in so called back-drivable robots

Manipulator Dynamics in the Task Space

- Therefore, a human interacting with an-unactuated robot arm* will experience the behavior given by

$$\Lambda(\theta)\dot{\mathcal{V}}_e^{e,0} + \eta\left(\theta, \mathcal{V}_e^{e,0}\right)\mathcal{V}_e^{e,0} + \gamma(\theta) = \mathcal{W}_{\text{int}}^{e,e}$$

- With an impedance controller,

$$\Lambda(\theta)\dot{\mathcal{V}}_e^{e,0} + \eta\left(\theta, \mathcal{V}_e^{e,0}\right)\mathcal{V}_e^{e,0} + \gamma(\theta) = J^{-\top}(\theta)\tau_{\text{con}} + \mathcal{W}_{\text{int}}^{e,e}$$

we can now shape the task-space dynamics to be our desired behavior

$$\mathfrak{M}\,\ddot{q} + \mathfrak{B}\dot{q} + \mathfrak{K}q = \mathcal{W}_{\text{int}}^{e,e}$$

* This is only possible in so called back-drivable robots

Impedance Control of n-link manipulator

- Therefore, the control law that will yields the desired impedance behavior at the end effector

$$\mathfrak{M}\,\ddot{q} + \mathfrak{B}\dot{q} + \mathfrak{K}q = \mathcal{W}_{\text{int}}^{e,e}$$

will be given by

$$\boxed{\tau_{\text{con}} = J^{\top}(\theta)\left[\Lambda(\theta)\dot{\mathcal{V}}_e^{e,0} + \eta\left(\theta, \mathcal{V}_e^{e,0}\right)\mathcal{V}_e^{e,0} + \gamma(\theta) - (\mathfrak{M}\,\ddot{q} + \mathfrak{B}\dot{q} + \mathfrak{K}q)\right]}$$

Impedance Control of n-link manipulator

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Alternative formulations 1

- Impedance Control law

$$\tau_{\text{con}} = J^{\top}(\theta)\left[\underbrace{\Lambda(\theta)\dot{\mathcal{V}}_e^{e,0} + \eta\left(\theta, \mathcal{V}_e^{e,0}\right)\mathcal{V}_e^{e,0} + \gamma(\theta)}_{\text{task−dynamics compensation}} - \underbrace{\left(\mathfrak{M}\,\ddot{q} + \mathfrak{B}\dot{q}\; + \mathfrak{K}q\right)}_{\text{desired behavior}}\right]$$

- One can also rewrite the impedance control law as

$$\tau_{\text{con}} = \underbrace{M(\theta)\ddot{\theta} + c\left(\theta, \dot{\theta}\right) + b\left(\dot{\theta}\right) + g(\theta)}_{\text{joint−dynamics compensation}} - J^{\top}(\theta)\underbrace{\left(\mathfrak{M}\,\ddot{q} + \mathfrak{B}\dot{q}\; + \mathfrak{K}q\right)}_{\text{desired behavior}}$$

Alternative formulations 2

- Impedance Control law

$$\tau_{\text{con}} = J^{\top}(\theta)\left[\Lambda(\theta)\dot{\mathcal{V}}_e^{e,0} + \eta\left(\theta, \mathcal{V}_e^{e,0}\right)\mathcal{V}_e^{e,0} + \gamma(\theta) - (\mathfrak{M}\,\ddot{q} + \mathfrak{B}\dot{q} + \mathfrak{K}q)\right]$$

- The above controller requires measurement of accelerations of the end effector $\left(\dot{\mathcal{V}}_e^{e,0}, \ddot{q}\right)$ which might be noisy in practice.
- A common alternative is

$$\tau_{\text{con}} = J^{\top}(\theta)\left[\eta\left(\theta, \mathcal{V}_e^{e,0}\right)\mathcal{V}_e^{e,0} + \gamma(\theta) - (\mathfrak{B}\dot{q} + \mathfrak{K}q)\right]$$

- However, in this case the actual mass of the manipulator will be apparent to the user at the end-effector (Unless the robot is very lightweight).

Alternative formulations 3

- Impedance Control law

$$\boxed{\tau_{\text{con}} = J^{\top}(\theta)\left[\Lambda(\theta)\dot{\mathcal{V}}_e^{e,0} + \eta\left(\theta, \mathcal{V}_e^{e,0}\right)\mathcal{V}_e^{e,0} + \gamma(\theta) - (\mathfrak{M}\,\ddot{q} + \mathfrak{B}\dot{q} + \mathfrak{K}q)\right]}$$

- Another common alternative, that is useful in slow interaction tasks does not compensate the full robot nonlinearities but only the gravity terms:

$$\boxed{\begin{aligned}\tau_{\text{con}} &= J^{\top}(\theta)[\gamma(\theta) - (\mathfrak{B}\dot{q} + \mathfrak{K}q)] \\ &= g(\theta) - J^{\top}(\theta)(\mathfrak{B}\dot{q} + \mathfrak{K}q)\end{aligned}}$$

Similar to PD controller
+ gravity compensation !

Desired Impedance in 6D !

- Let's take a closer look at the desired impedance behavior

$$\mathfrak{M}\,\ddot{q} + \mathfrak{B}\dot{q} + \mathfrak{K}q = \mathcal{W}_{\mathrm{int}}^{e,e}$$

- Caution must be considered in defining this impedance in a geometrically consistent manner.

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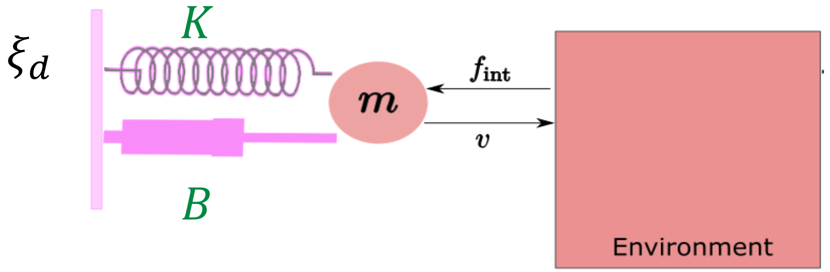
- Caution must be considered in defining this impedance in a geometrically consistent manner.
- In the linear case, we had that $q \coloneqq \xi - \xi_d \in \mathbb{R}^3$, and the impedance behavior:

$$\boxed{m\,\ddot{q} + B\dot{q} + Kq = f_{\mathrm{int}}\,,}$$



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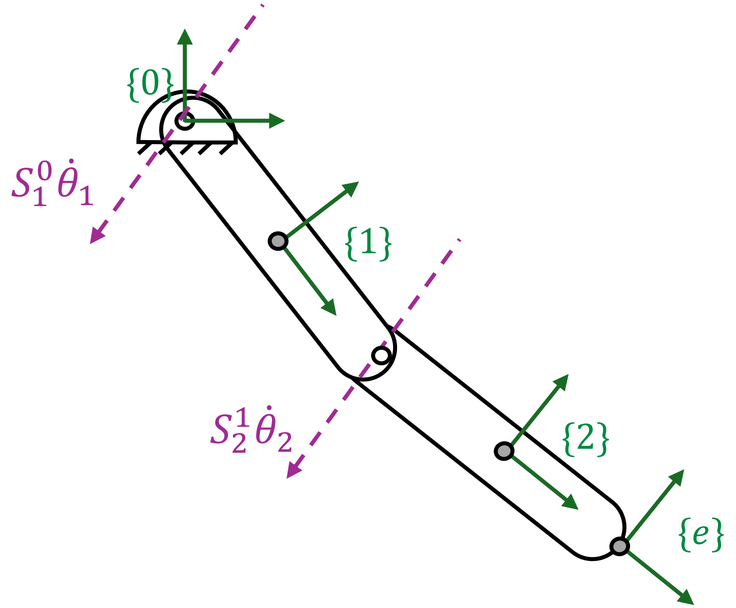
$$\mathfrak{M}\,\ddot{q} + \mathfrak{B}\dot{q} + \mathfrak{K}q = \mathcal{W}_{\mathrm{int}}^{e,e}$$

- Caution must be considered in defining this impedance in a geometrically consistent manner.
- However, for the end-effector as a rigid body, we cannot simply define $q \in \mathbb{R}^6$ as $q = H_e^0 - H_d^0 \notin SE(3)$.



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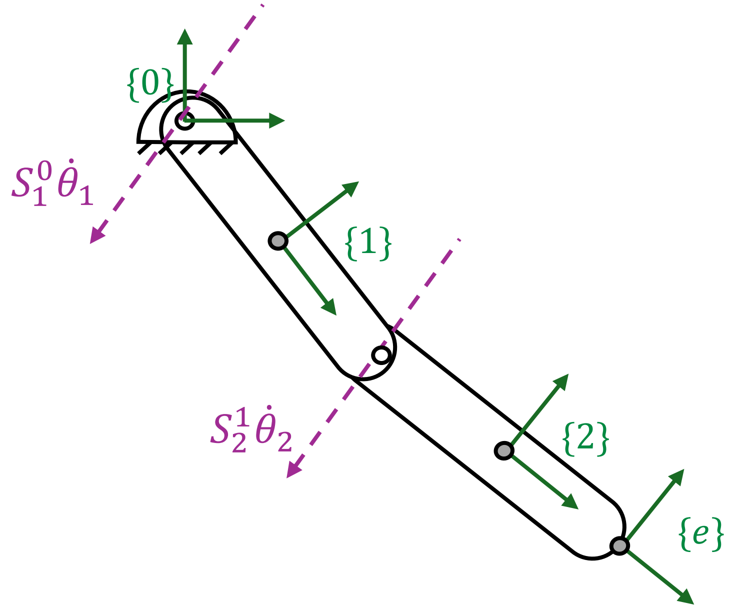

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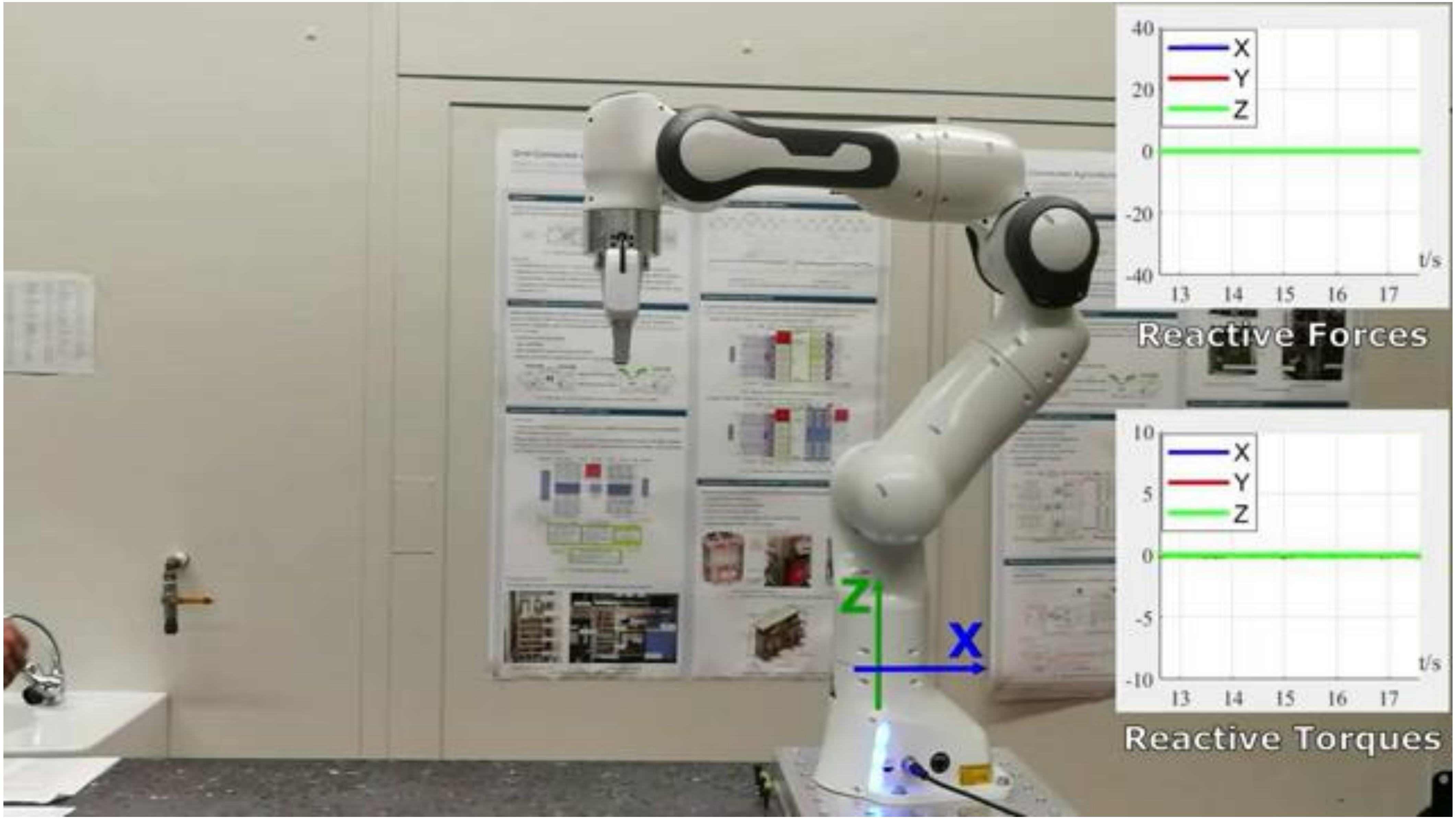
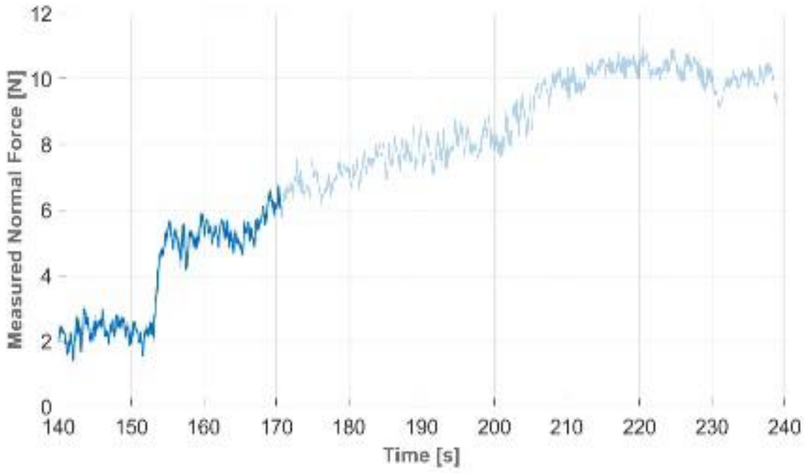
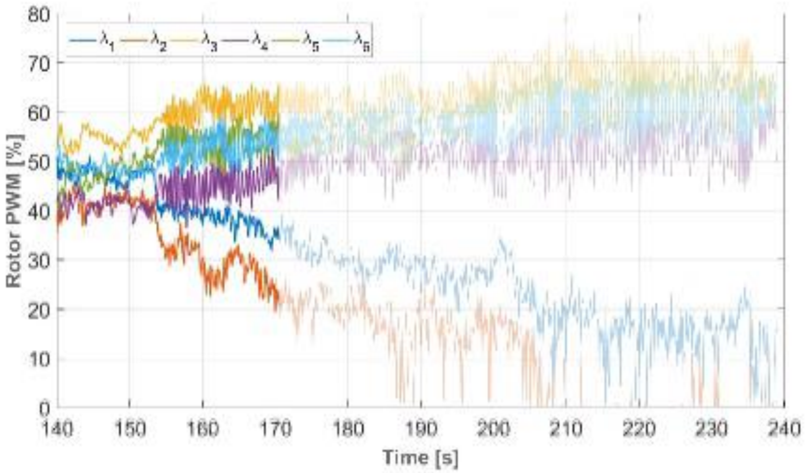
Impedance Control of Franka Panda



Impedance Control of Fully-actuated Hexarotor





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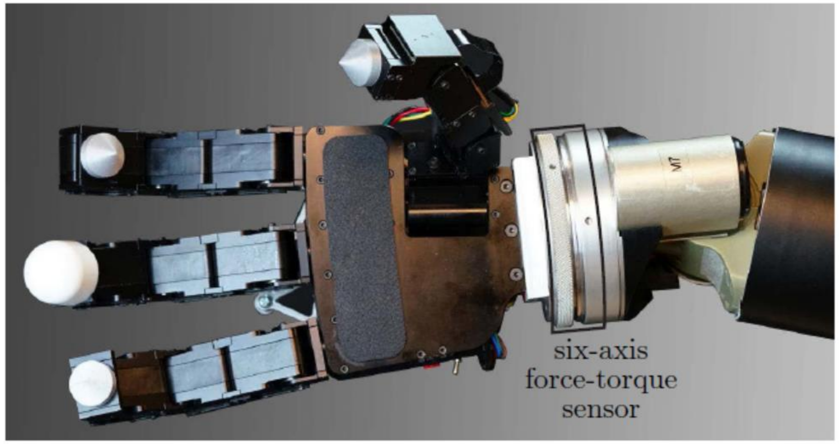
Admittance Control

- In an admittance-control algorithm, the wrench applied by the user at the end-effector is sensed using a force/torque sensor.
- The robot then should respond with the desired end-effector acceleration $\ddot{q}_d$ satisfying

$$\ddot{q}_d = \mathfrak{M}^{-1}\left(-\mathfrak{B}\dot{q} \ - \mathfrak{K}q + \mathcal{W}_{\text{int}}^{e,e}\right)$$

- Using the inverse kinematics, one can transform this to desired joint accelerations $\ddot{\theta}_d$ and then use the inverse dynamics to calculate the control torques.



Impedance: Motion input, Force output
Admittance: Force input, Motion output

Admittance Control of UR5e

Admittance Control of Quadrotor