1) H_b^s , $\mathcal{V}_b^{b,s}$

$\Rightarrow H_b^s \in \underbrace{\mathcal{H}_b}_{dim=b} \subseteq \underbrace{SE(3)}_{dim=6}$

$\Rightarrow \mathcal{V}_b^{b,s} := \imath_0(\dot{V})$

$\imath_0 : \mathbb{R}^b \longrightarrow \mathbb{R}^6$

* Examples:

i) 1 DoF joint: $b=1$

$H_i^j(\theta) \in \underbrace{\mathcal{H}_b}_{dim=1} \subset SE(3)$

$V = \dot{\theta}_i$

$$\underset{\mathbb{R}^6}{\underset{\uparrow}{\mathcal{V}_i^{i,j}}} = \underbrace{S_i^{i,j}}_{\imath_0} \underset{\mathbb{R}}{\underset{\uparrow}{\dot{\theta}_i}}$$

ii) 6 DOF floating joint: $b = 6$

$$\boxed{\dot{H}^{s}_{b} = H^{s}_{b}\,\tilde{\mathcal{V}}^{b,s}_{b}}$$

$$H^{s}_{b} \in \mathcal{H}_{b} \equiv SE(3)$$

$$\underset{\mathbb{R}^6}{\mathcal{V}^{b,s}_{b}} := z_0(\vec{V}) = I_6 \, \underset{\mathbb{R}^6}{(\mathcal{V}^{b,s}_{b})}$$

iii) Planar joint:

$$H^{s}_{b} = \begin{pmatrix} c_\theta & -s_\theta & 0 & \xi_x \\ s_\theta & c_\theta & 0 & \xi_y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \in \underbrace{\mathcal{H}_b}_{b=3} \subseteq \underset{6}{SE(3)}$$

$SE(2) = \mathcal{H}_b$

$$\mathbb{R}^3 \ni V = \begin{pmatrix} \omega_z \\ v_x \\ v_y \end{pmatrix} \xrightarrow{z_0} \mathcal{V}^{b,s}_{b} \in \mathbb{R}^6$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \qquad \mathcal{V}^{b,s}_{b} = \begin{pmatrix} 0 \\ 0 \\ \omega_z \\ v_x \\ v_y \\ 0 \end{pmatrix}$$

$\mathbb{R}^3 \to \mathbb{R}^6$

* Configuration Space:

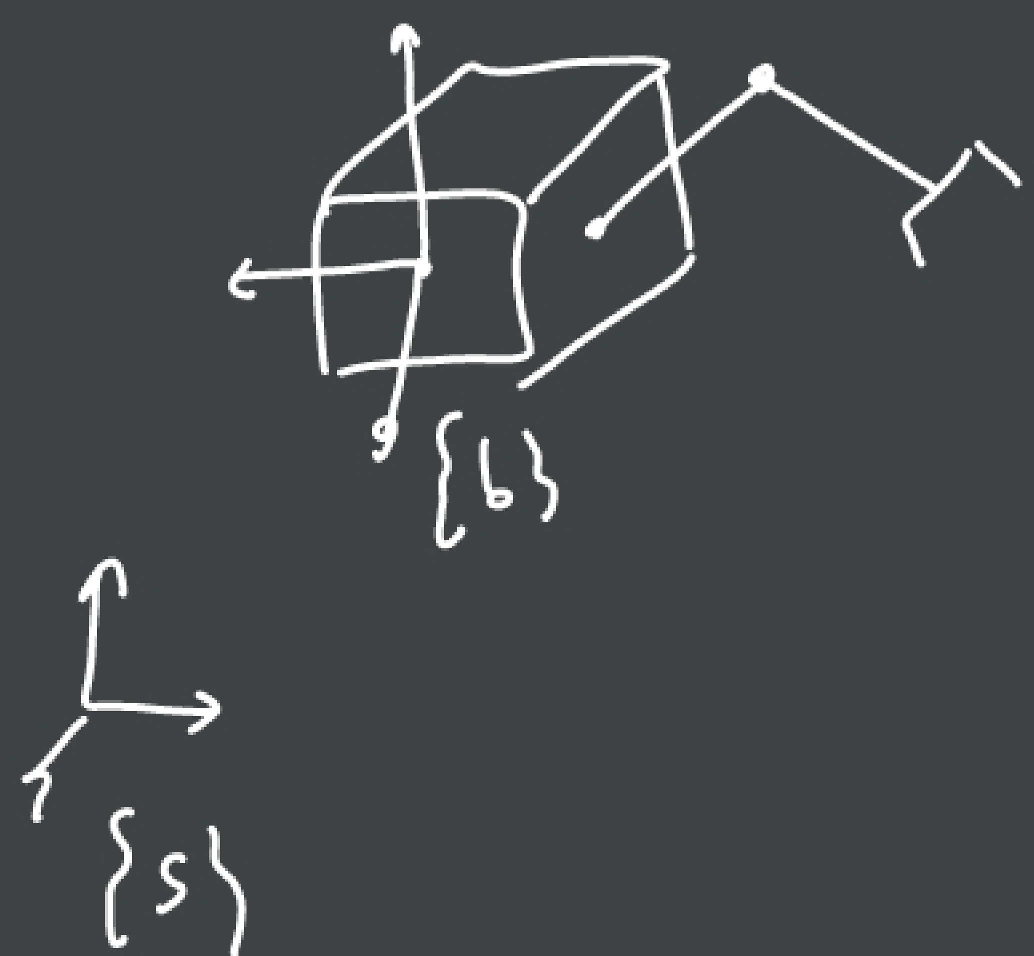
$\Rightarrow$ Unconstrained $\Rightarrow Q_{free} = [SE(3)]^{n+1}$

$\Rightarrow \dim = 6(n+1)$

$\Rightarrow$ Reduced $\Rightarrow Q = \mathcal{H}_{base} \times Q_{mani}$

$\Rightarrow \dim Q = \dim \mathcal{H}_b + \dim Q_n$

$= b + n$



b=6

b=3

$$\mathcal{V}_i^{i,s} = J_i^{i,s}(q) \begin{pmatrix} V \\ \dot{q} \end{pmatrix}$$

$$\begin{pmatrix} \mathcal{V}_b^{b,s} \\ \mathcal{V}_1^{1,s} \\ \mathcal{V}_2^{2,s} \\ \vdots \\ \mathcal{V}_n^{n,s} \end{pmatrix}_{6(n+1)} = \underbrace{\begin{pmatrix} Z_0 & 0_6 & 0_6 & \cdots & 0_6 \\ Ad_{H_b^1} Z_0 & S_1^{1,0} & 0_6 & \cdots & 0_6 \\ Ad_{H_b^2} Z_0 & Ad_{H_1^2} S_1^{1,0} & S_2^{2,1} & \cdots & 0_6 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ Ad_{H_b^n} Z_0 & Ad_{H_1^n} S_1^{1,0} & Ad_{H_2^n} S_2^{2,1} & & S_n^{n,n-1} \end{pmatrix}}_{J(q)} \begin{pmatrix} V \\ \dot{q} \end{pmatrix}_{b+n}$$

$J_m(q)$

$q \in Q_m$

$q = \begin{pmatrix} q_1 \\ \vdots \\ q_n \end{pmatrix}$

$H_b^s \in \mathcal{H}_b$

$\cancel{\boxed{q_b \in \mathbb{R}^b}}$

$$J(q) = \begin{pmatrix} \iota_0 & 0 \\ h_{mb}(q)\iota_0 & J_m(q) \end{pmatrix}$$

$$\cancel{\boxed{\dot{q}_b = V \in \mathbb{R}^b}}$$

$$\boxed{M(q) \begin{pmatrix} \dot{V} \\ \ddot{q} \end{pmatrix} + C(q, V, \dot{q}) \begin{pmatrix} V \\ \dot{q} \end{pmatrix} = \begin{pmatrix} W \\ \tilde{T} \end{pmatrix}}$$

$$M(q) = \begin{pmatrix} M_b(q) & M_{bm}(q) \\ M_{bm}^T(q) & M_m(q) \end{pmatrix} = J^T(q) \underset{\substack{\parallel \\ \mathrm{diag}\left(I^{b,b}, \underbrace{I^{1,1}, \dots, I^{n,n}}_{I_m}\right)}}{\underset{\sim}{I}} J(q)$$

$$\boxed{M_b(q) := \iota_0^T \left(I^{b,b} + h_{mb}^T(q)\, I_m\, h_{mb}(q)\right) \iota_0}$$