SCE 594: Special Topics in Intelligent Automation & Robotics

Lecture 26: Floating-base Manipulators

جامعة الملك فهد للبترول والمعادن
King Fahd University of Petroleum & Minerals

Ramy Rashad
Assistant Professor
CIE Department

Outline

- Overview Floating-Base Manipulators
- Recap: Dynamics of Fixed-Base Manipulators
- Dynamics of Floating-Base Manipulators

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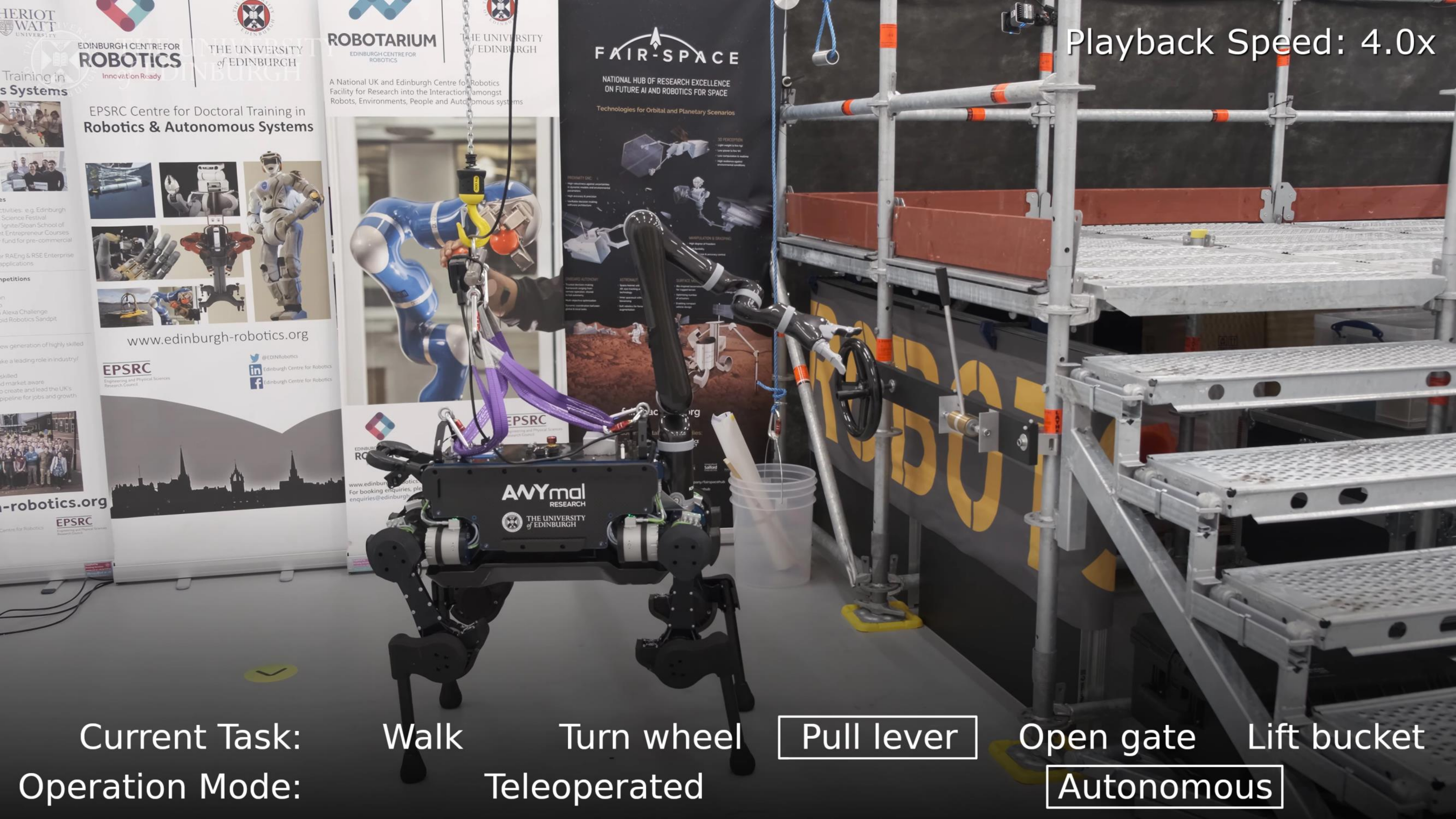
ENGINE COVER
Fully Autonomous


Playback Speed: 4.0x
Current Task:
Walk
Turn wheel
Pull lever
Open gate
Lift bucket
Operation Mode:
Teleoperated
Autonomous
EDINBURGH CENTRE FOR ROBOTICS
Innovation Ready
THE UNIVERSITY of EDINBURGH
EPSRC Centre for Doctoral Training in Robotics & Autonomous Systems
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ROBOTARIUM
EDINBURGH CENTRE FOR ROBOTICS
A National UK and Edinburgh Centre for Robotics Facility for Research into the Interaction amongst Robots, Environments, People and Autonomous systems
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NATIONAL HUB OF RESEARCH EXCELLENCE ON FUTURE AI AND ROBOTICS FOR SPACE
Technologies for Orbital and Planetary Scenarios
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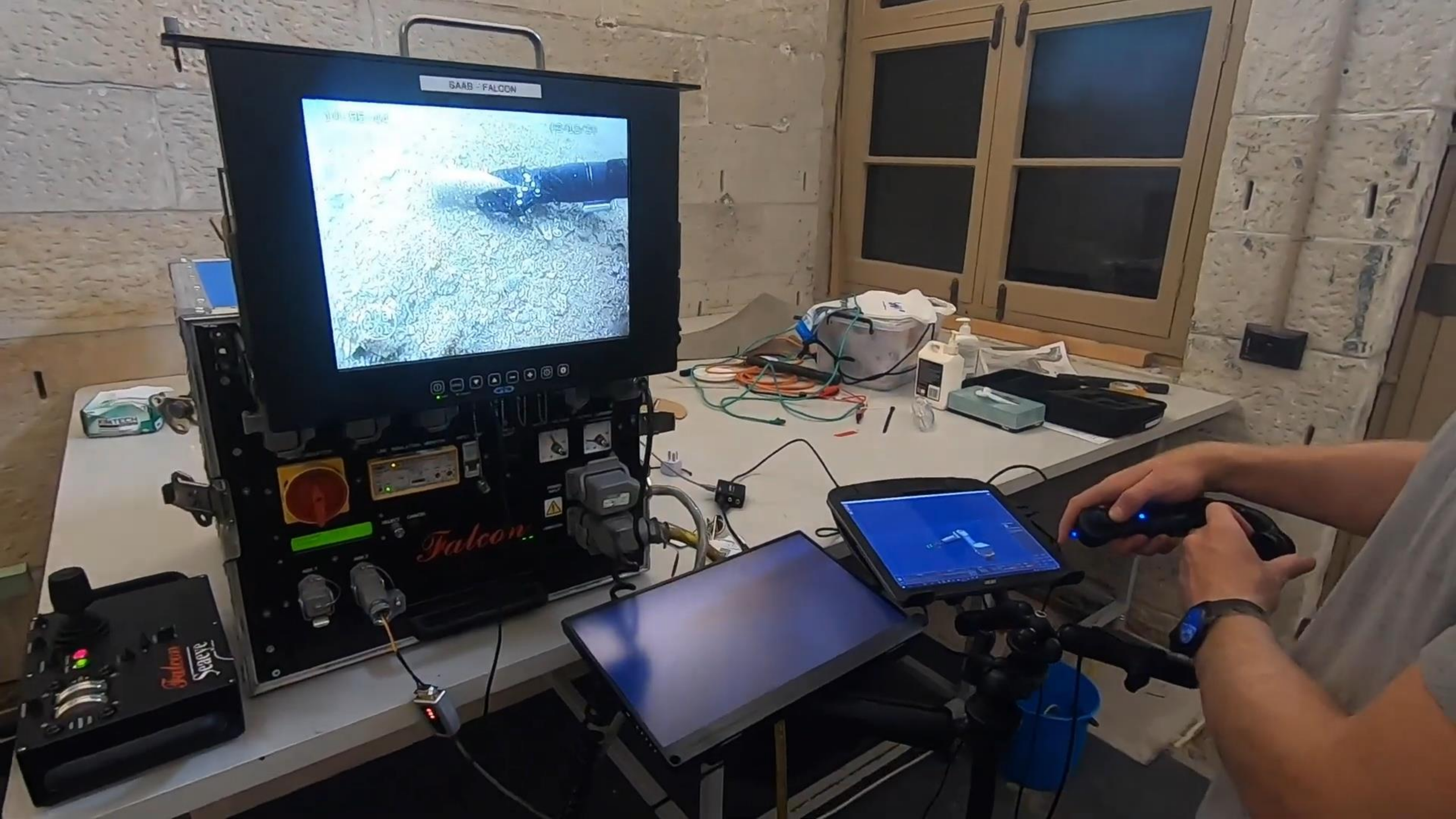
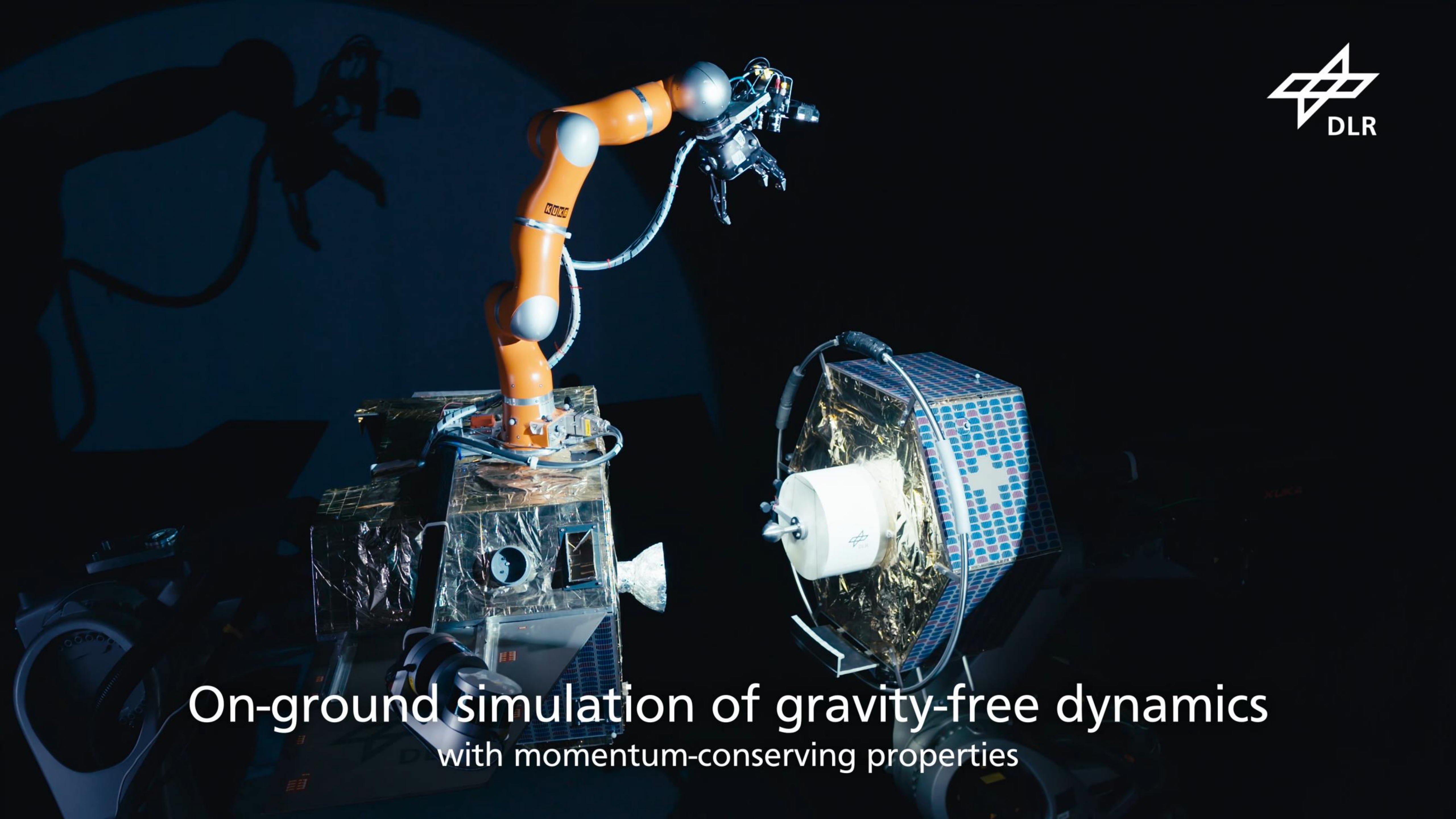

SAAB - FALCON
Falcon
Falcon
Seaeye


Change the joint configuration in midair


DLR
On-ground simulation of gravity-free dynamics
with momentum-conserving properties

Complexity of Floating-base Manipulators

- **Fixed-base:** All degrees of freedom (DOFs) are typically actuated.
- **Floating-base:** The base is unactuated (or indirectly actuated), leading to under-actuation.

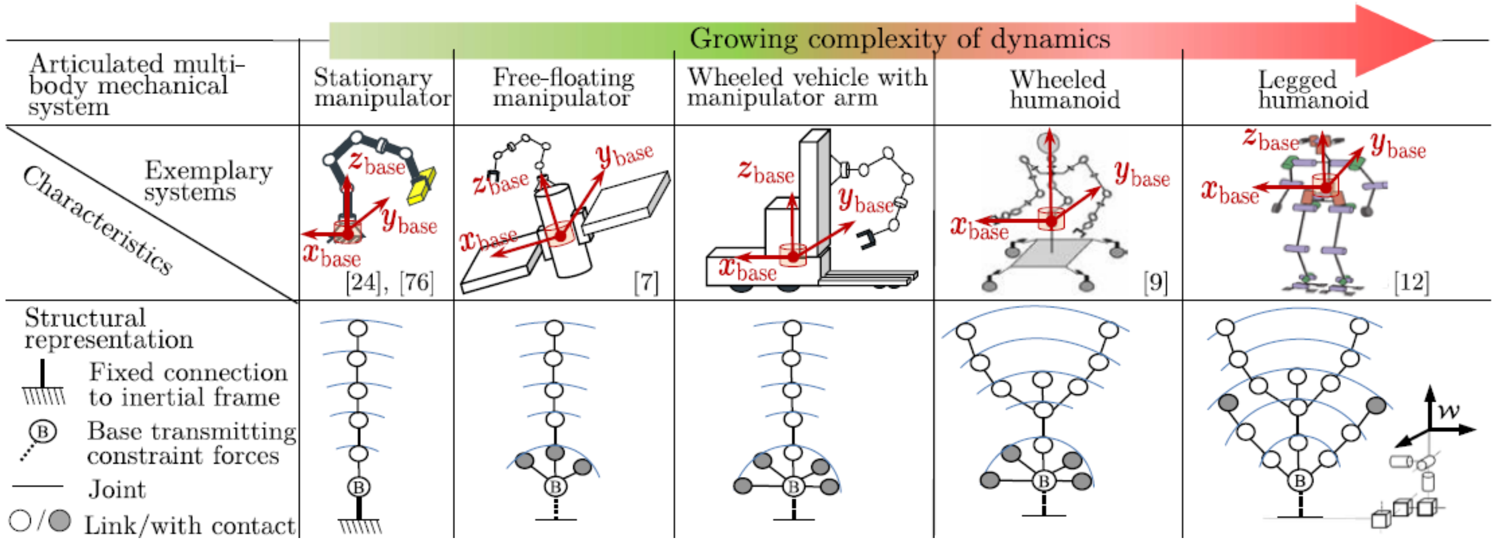


Figure from Mazin Hamad et al. IEEE TRO 2023

Complexity of Floating-base Manipulators

- **Fixed-base:** The base is stationary; dynamics are primarily local to the manipulator.
- **Floating-base:** Base and arm(s) dynamics are tightly coupled, and the base has holonomic or non-holonomic constraints.

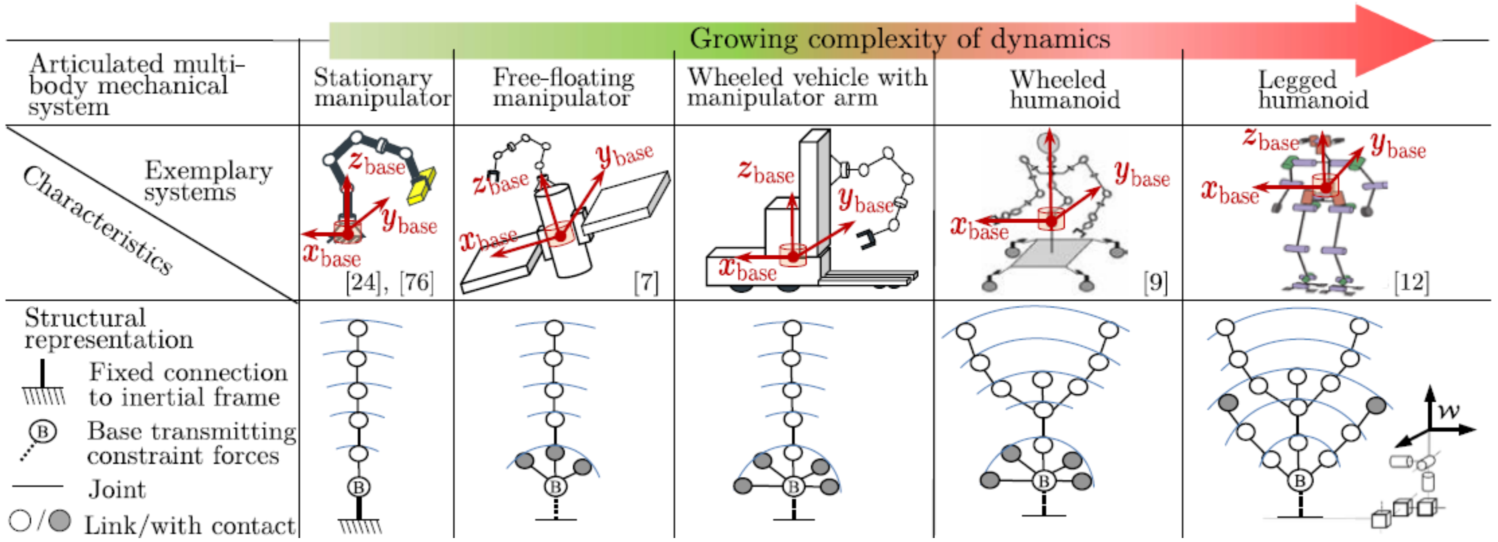


Figure from Mazin Hamad et al. IEEE TRO 2023

Complexity of Floating-base Manipulators

- **Fixed-base:** Stability is not a concern as the base is immobile.
- **Floating-base:** Especially in ground or legged platforms, stability is dynamic and must be actively maintained.

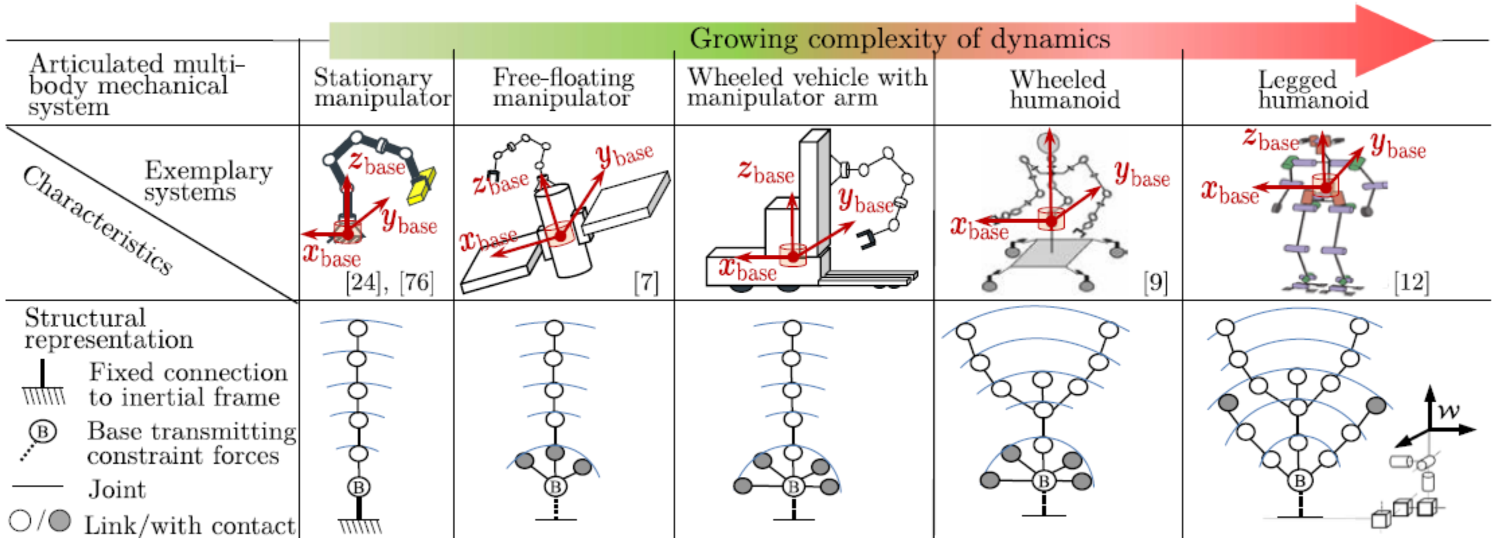


Figure from Mazin Hamad et al. IEEE TRO 2023

Complexity of Floating-base Manipulators

- **Fixed-base:** External reaction forces are naturally transferred to the environment.
- **Floating-base:** Internal forces and torques cannot rely on ground reactions (linear and angular momentum must be managed by control carefully)

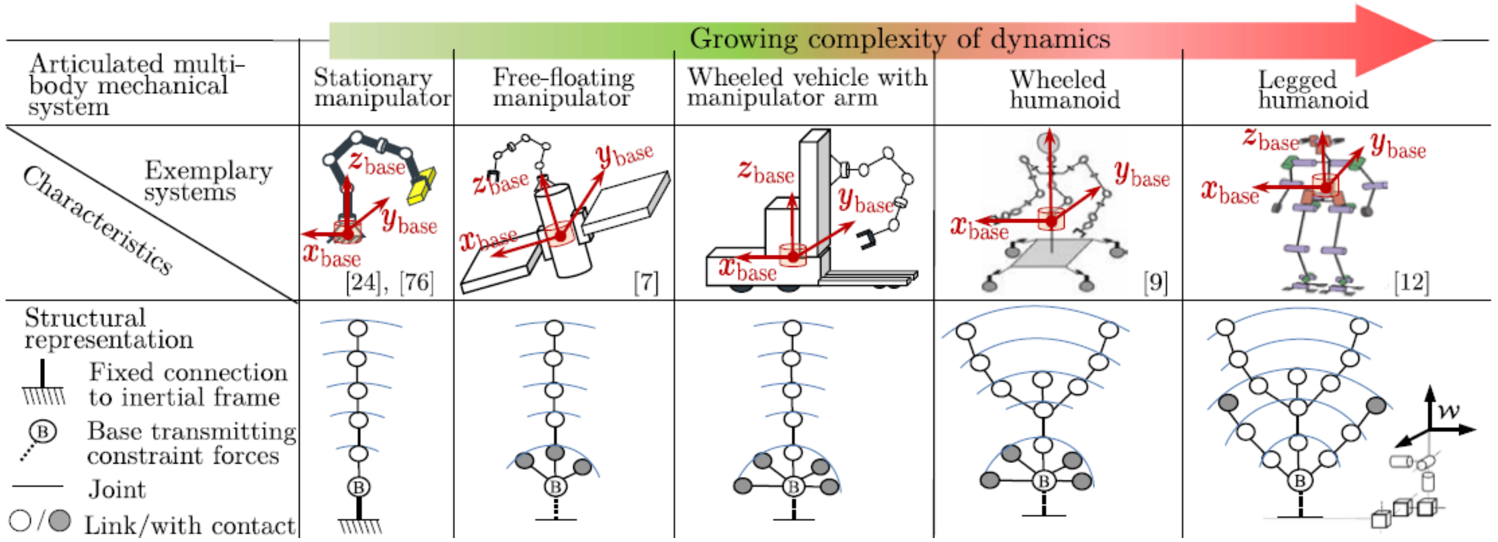


Figure from Mazin Hamad et al. IEEE TRO 2023

Complexity of Floating-base Manipulators

- **Fixed-base:** Simpler joint-space or task-space control suffices in many cases.
- **Floating-base:** Requires whole-body control, prioritization, and constraint handling.

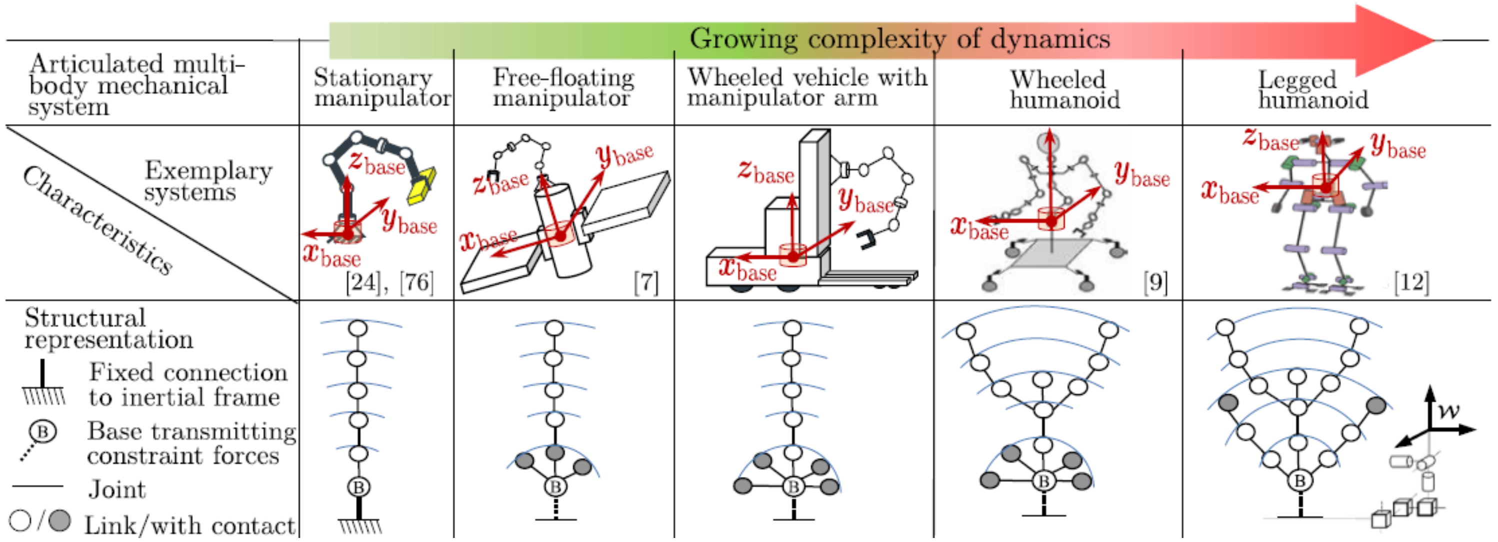


Figure from Mazin Hamad et al. IEEE TRO 2023

Outline

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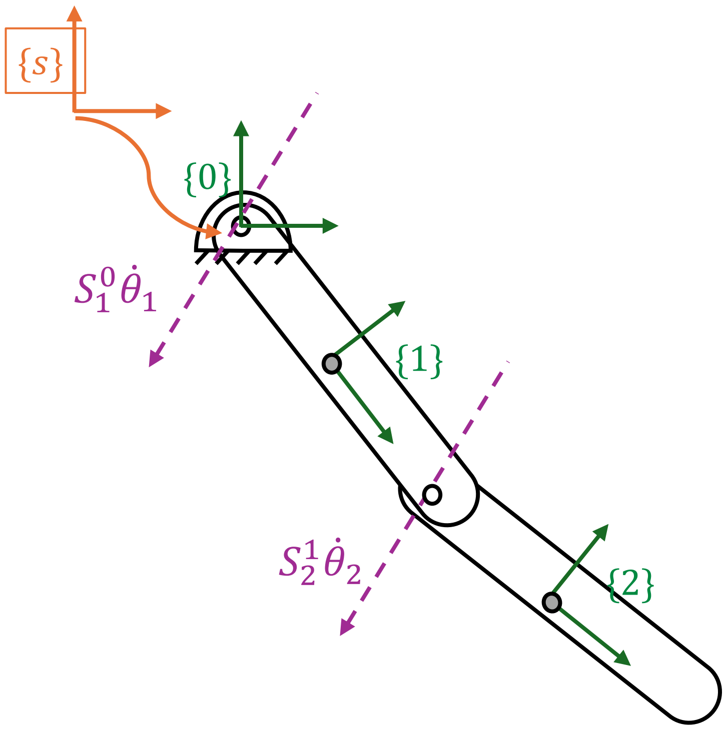
- Recap: Dynamics of Fixed-Base Manipulators
- Dynamics of Floating-Base Manipulators

Recap: Fixed-base Dynamics

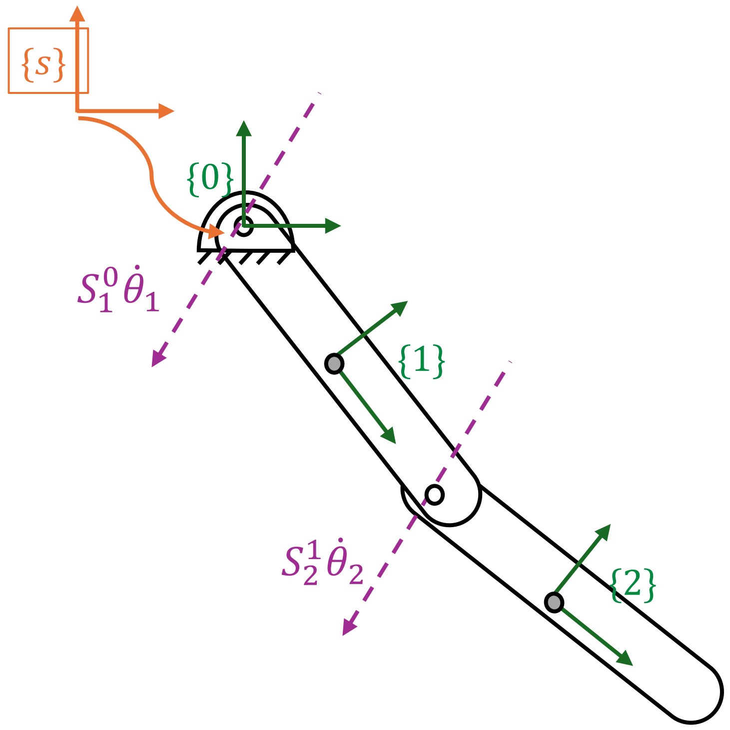
- Structure:
  - $n \times$ Rigid Bodies
  - $n_{\mathrm{r}} \times$ 1-DOF Revolute Joints
  - $n_{\mathrm{p}} \times$ 1-DOF Prismatic Joints
  - Frame $\{i\}$ is attached to i-th body, $\quad i \in \{1, \cdots, n\}$
  - Frame $\{0\} \equiv \{s\}$ is stationary.
- Configuration Space:
  - Unconstrained: $Q_{free} \coloneqq \left(SE(3)\right)^n$
  - Constrained/Reduced: $Q = \underbrace{\mathbb{S}^1 \times \cdots \times \mathbb{S}^1}_{n_{\mathrm{r}}} \times \underbrace{\mathbb{R} \times \cdots \times \mathbb{R}}_{n_{\mathrm{p}}}$
  - $\dim Q_{free} = 6n \quad , \qquad \dim Q = n$



$n_{\mathrm{r}} + n_{\mathrm{p}} = n$

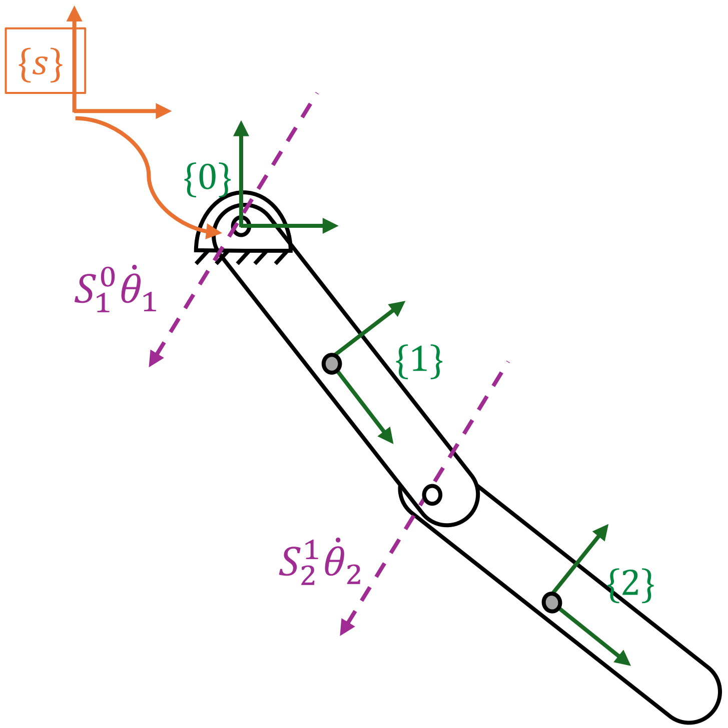
Recap: Fixed-base Dynamics

- 1-DOF Joints:
  - Joint configuration: $q_i \in \mathbb{S}^1$ or $q_i \in \mathbb{R}$
  - Joint velocity: $\dot{q}_i \in \mathbb{R}$
  - Relative Pose: $H_i^{i-1}(q_i) = H_i^{i-1}(0)\, e^{\tilde{S}_i^{i,i-1} q_i} \in SE(3)$
  - Relative Twist: $\mathcal{V}_i^{i,i-1} = S_i^{i,i-1} \dot{q}_i \in \mathbb{R}^6$



Recap: Fixed-base Dynamics

- Recursive Newton-Euler Inverse Dynamics
  - $\mathcal{V}_i^{i,s} = \mathrm{Ad}_{H_{i-1}^{i}} \mathcal{V}_{i-1}^{i-1,s} + S_i^{i,i-1} \dot{q}_i$
  - $\dot{\mathcal{V}}_i^{i,s} = \mathrm{Ad}_{H_{i-1}^{i}} \dot{\mathcal{V}}_{i-1}^{i-1,s} - \mathrm{ad}_{S_i^{i,i-1}\dot{q}_i}\left(\mathrm{Ad}_{H_{i-1}^{i}} \mathcal{V}_{i-1}^{i-1,s}\right) + S_i^{i,i-1} \ddot{q}_i$
  - $\mathcal{W}_{i-1}^{i,i} = \mathfrak{I}^{i,i}\dot{\mathcal{V}}_i^{i,s} - \mathrm{ad}^{\top}_{\mathcal{V}_i^{i,s}}\left(\mathfrak{I}^{i,i}\mathcal{V}_i^{i,s}\right) + \mathrm{Ad}^{\top}_{H_i^{i+1}} \mathcal{W}_i^{i+1,i+1}$
  - $\tau_i = \left(S_i^{i,i-1}\right)^{\top} \mathcal{W}_{i-1}^{i,i}$



For i = 1, $\mathcal{V}_0^{0,s} = 0$

Recap: Fixed-base Dynamics

- Compact Newton-Euler Inverse Dynamics
  - $\mathcal{V} = \mathcal{L}(q)\,\mathcal{S}\,\dot{q}$
  - $\dot{\mathcal{V}} = \mathcal{L}(q)\,\mathcal{S}\,\ddot{q} - \mathcal{L}(q)\,\mathrm{ad}_{\mathcal{S}\,\dot{q}}\,\mathcal{A}(q)\,\mathcal{V}$
  - $\mathcal{W} = \mathcal{L}^\top(q)\,\mathfrak{I}\,\dot{\mathcal{V}} - \mathcal{L}^\top(q)\,\mathrm{ad}_{\mathcal{V}}^\top\,\mathfrak{I}\,\mathcal{V}$
  - $\tau = \mathcal{S}^\top\mathcal{W}$

Unconstrained Variables

$$\mathcal{V} = \begin{pmatrix} \mathcal{V}_1^{1,s} \\ \vdots \\ \mathcal{V}_n^{n,s} \end{pmatrix} \in \mathbb{R}^{6n}, \qquad \mathcal{W} = \begin{pmatrix} \mathcal{W}_0^{1,1} \\ \vdots \\ \mathcal{W}_{n-1}^{n,n} \end{pmatrix} \in (\mathbb{R}^{6n})^*$$

Constrained/Reduced Variables

$$\dot{q} = \begin{pmatrix} \dot{q}_1 \\ \vdots \\ \dot{q}_n \end{pmatrix} \in \mathbb{R}^{n}, \qquad \tau = \begin{pmatrix} \tau_1 \\ \vdots \\ \tau_n \end{pmatrix} \in (\mathbb{R}^{n})^*$$

Recap: Fixed-base Dynamics

- Compact Newton-Euler Inverse Dynamics
  - $\mathcal{V} = \mathcal{J}(q)\dot{q}$
  - $\dot{\mathcal{V}} = \mathcal{J}(q)\ddot{q} - \mathcal{L}(q)\,\mathrm{ad}_{\mathcal{S}\,\dot{q}}\,\mathcal{A}(q)\,\mathcal{V}$
  - $\mathcal{W} = \mathcal{L}^{\top}(q)\,\mathfrak{I}\,\dot{\mathcal{V}} - \mathcal{L}^{\top}(q)\,\mathrm{ad}_{\mathcal{V}}^{\top}\,\mathfrak{I}\,\mathcal{V}$
  - $\tau = \mathcal{S}^{\top}\mathcal{W}$

Geometric Body Jacobian of all n-links

$$\mathcal{J}(q) = \begin{pmatrix} S_1^{1,0} & 0 & \cdots & \cdots & 0 \\ \mathrm{Ad}_{H_1^2(q)} S_1^{1,0} & S_2^{2,1} & \cdots & \cdots & 0 \\ \mathrm{Ad}_{H_1^3(q)} S_1^{1,0} & \mathrm{Ad}_{H_2^3(q)} S_2^{2,1} & S_3^{3,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \mathrm{Ad}_{H_1^n(q)} S_1^{1,0} & \mathrm{Ad}_{H_2^n(q)} S_2^{2,1} & \mathrm{Ad}_{H_3^n(q)} S_3^{3,2} & \cdots & S_n^{n,n-1} \end{pmatrix} = \begin{pmatrix} \mathcal{J}_1^{1,s} \\ \mathcal{J}_2^{2,s}(q) \\ \mathcal{J}_3^{3,s}(q) \\ \vdots \\ \mathcal{J}_n^{n,s}(q) \end{pmatrix}$$

Recap: Fixed-base Dynamics

- Reduced Dynamics

- $\mathcal{V} = \mathcal{J}(q)\dot{q}$
- $\dot{\mathcal{V}} = \mathcal{J}(q)\ddot{q} - \mathcal{L}(q)\,\mathrm{ad}_{\mathcal{S}\,\dot{q}}\,\mathcal{A}(q)\,\mathcal{V}$
- $\mathcal{W} = \mathcal{L}^{\top}(q)\,\mathfrak{T}\,\dot{\mathcal{V}} - \mathcal{L}^{\top}(q)\,\mathrm{ad}_{\mathcal{V}}^{\top}\,\mathfrak{T}\,\mathcal{V}$
- $\tau = \mathcal{S}^{\top}\mathcal{W}$

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} = \tau$$

$$M(q) \coloneqq \mathcal{J}^{\top}(q)\,\mathfrak{T}\,\mathcal{J}(q)$$
$$C(q,\dot{q}) \coloneqq \mathcal{J}^{\top}(q)\left[-\mathfrak{T}\mathcal{L}(q)\mathrm{ad}_{\mathcal{S}\,\dot{q}}\,\mathcal{A}(q) - \mathrm{ad}_{\mathcal{L}(q)\,\mathcal{S}\,\dot{q}}^{\top}\mathfrak{T}\right]\mathcal{J}(q)$$

Recap: Fixed-base Dynamics

- Reduced Dynamics with gravity and end-effector wrench

$$\boxed{\begin{aligned} &M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau + \mathcal{J}_{\mathrm{e}}^{\top}(q)\mathcal{W}_{\mathrm{int}}^{e,e} \\ &\mathcal{V}_e^{e,s} = \mathcal{J}_e(q)\dot{q} \end{aligned}}$$

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