SCE 594: Special Topics in Intelligent Automation & Robotics

Lecture 8: Twists

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Outline

- Recap Last Lectures
- Lie algebra $se(3)$ of $SE(3)$
- Properties of Angular velocities & Twists

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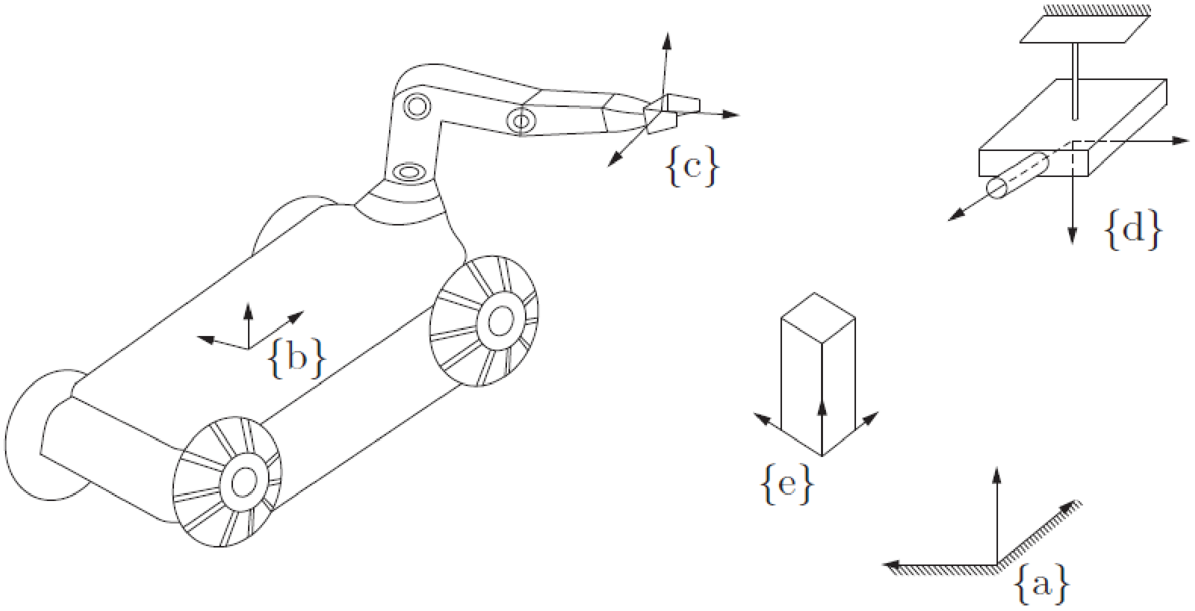
Recap: Kinematic Modeling Notation

- A frame will be denoted by Ψ_i or $\{i\}$.
- The relative pose of $\{i\}$ with respect to $\{k\}$ is described by

$$H_i^k = \begin{pmatrix} R_i^k & \xi_i^k \\ 0 & 1 \end{pmatrix} \in SE(3), \qquad R_i^k \in SO(3), \qquad \xi_i^k \in \mathbb{R}^3$$



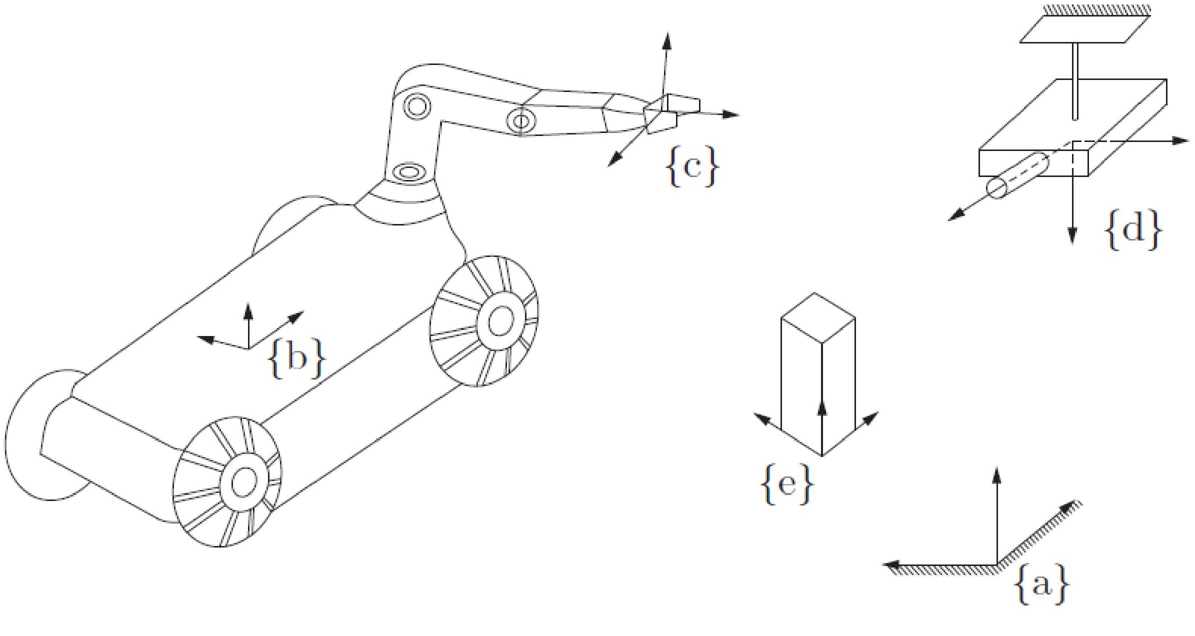
Recap: Kinematic Modeling Notation

- Given the relative pose of $\{i\}$ with respect to $\{j\}$ and the relative pose of $\{j\}$ with respect to $\{k\}$, we have that

$$H_i^k = H_j^k H_i^j \in SE(3)$$

$$H_j^k := \left(H_k^j\right)^{-1} = \begin{pmatrix} R_j^k & -R_j^k \xi_k^j \\ 0 & 1 \end{pmatrix} \in SE(3)$$

$$R_j^k := \left(R_k^j\right)^\top \in SO(3)$$



Recap: Kinematic Relations

1. Point mass translation :

- Configuration:

$$\xi_b^s \in \mathbb{R}^3$$

- Rate-of-change of configuration:

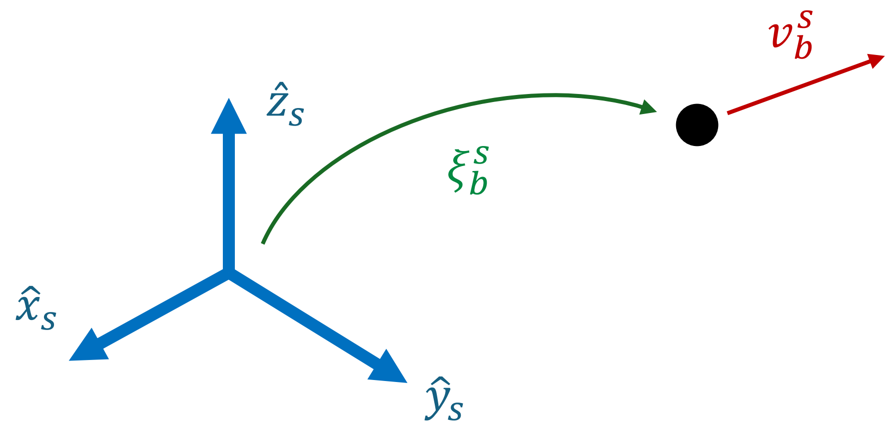
$$\dot{\xi}_b^s \in \mathbb{R}^3$$

- Velocity expressed in $\{s\}$:

$$v_b^{s,s} \in \mathbb{R}^3$$

- Kinematic relation:

$$\boxed{\dot{\xi}_b^s = v_b^{s,s}}$$



Recap: Kinematic Relations

2. Rigid body rotation :

- Configuration:

$$R_b^s \in SO(3)$$

- Rate-of-change of configuration:

$$\dot{R}_b^s \in T_{R_b^s} SO(3)$$

- Angular velocity expressed in $\{*\}$:

$$\tilde{\omega}_b^{*,s} \in T_I SO(3) =: so(3)$$

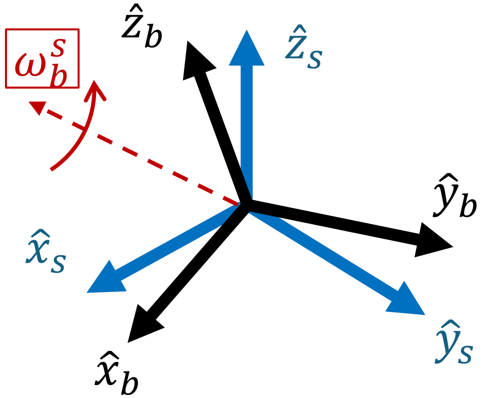
- Kinematic relation:

$$\boxed{\dot{R}_b^s = R_b^s \tilde{\omega}_b^{b,s}}$$

$$\boxed{\dot{R}_b^s = \tilde{\omega}_b^{s,s} R_b^s}$$

$$\boxed{\begin{aligned} S: \mathbb{R}^3 &\to so(3) \\ \omega &\mapsto \tilde{\omega} \end{aligned}}$$

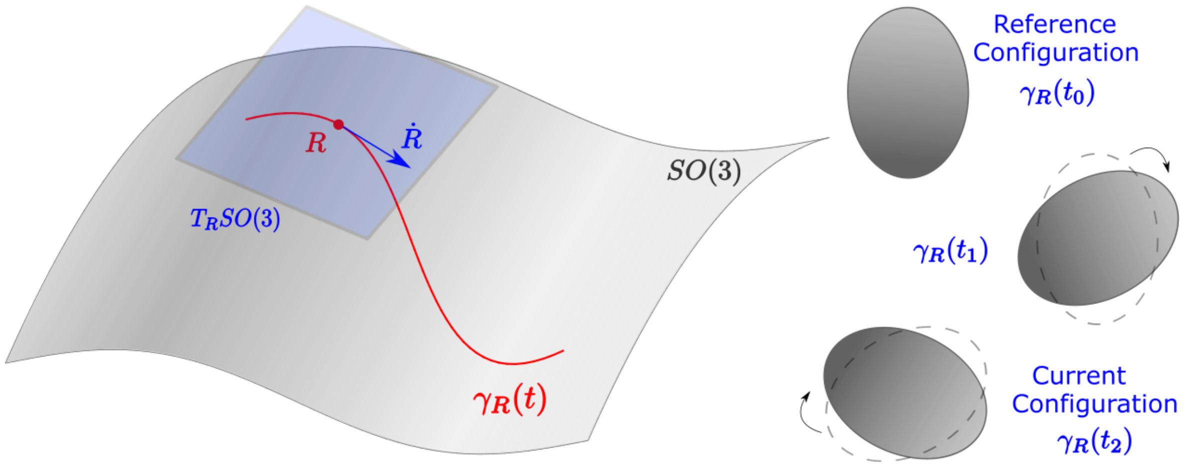
$$\boxed{\begin{aligned} S^{-1}: so(3) &\to \mathbb{R}^3 \\ \tilde{\omega} &\mapsto \omega \end{aligned}}$$

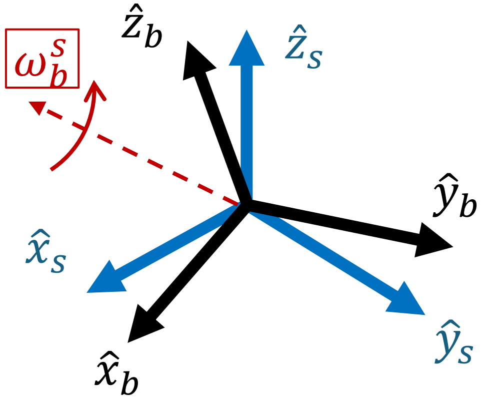


Recap: Lie group structure of SO(3)

- A rigid body rotational motion is represented mathematically by a curve

$$\gamma_R : I \subset \mathbb{R} \to SO(3)$$
$$t \mapsto \gamma_R(t) =: R_b^s(t)$$





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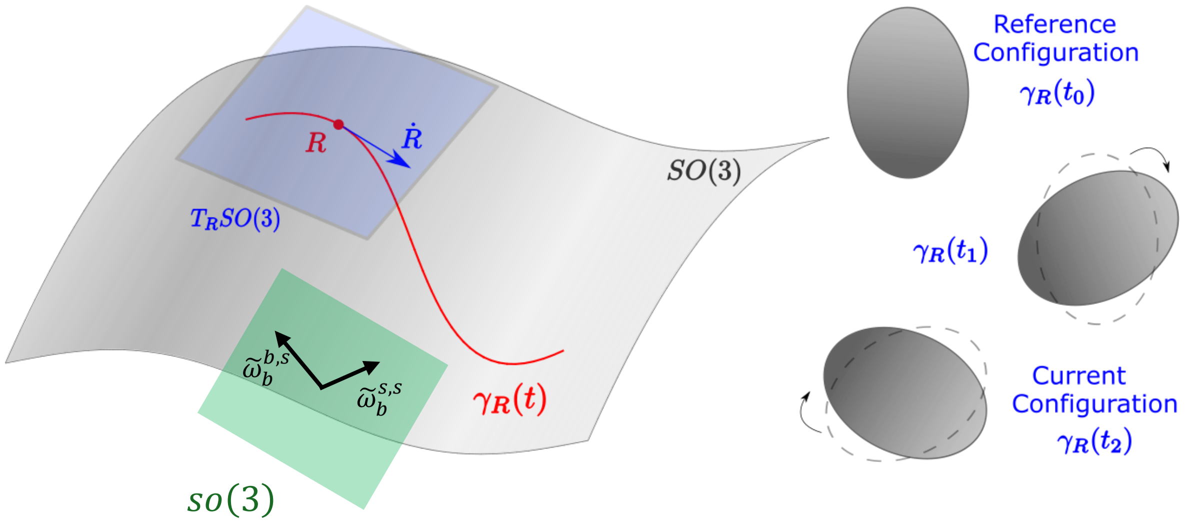
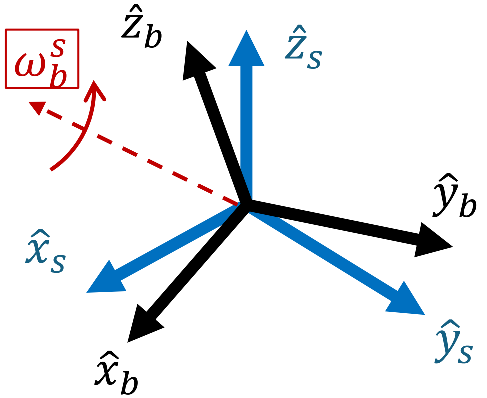




Recap: Kinematic Relations

3. Rigid body motion :

$$\tilde{S}: \mathbb{R}^6 \to se(3)$$
$$\mathcal{V} \mapsto \tilde{\mathcal{V}}$$

$$\tilde{S}^{-1}: se(3) \to \mathbb{R}^6$$
$$\tilde{\mathcal{V}} \mapsto \mathcal{V}$$

- Configuration:

$$H_b^s \in SE(3)$$

- Rate-of-change of configuration:

$$\dot{H}_b^s \in T_{H_b^s} SE(3)$$

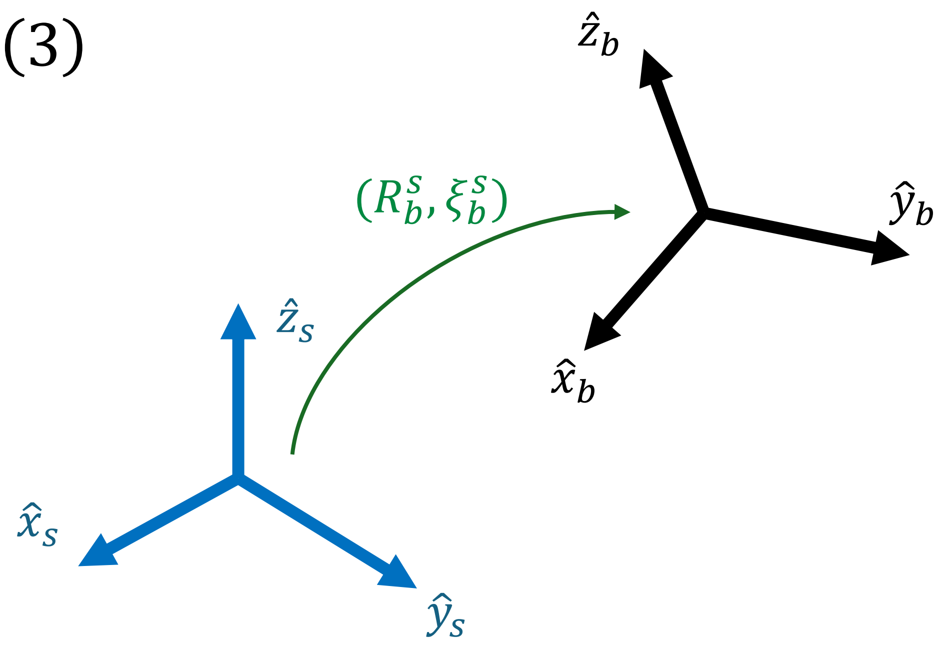
- Twist expressed in $\{*\}$:

$$\tilde{\mathcal{V}}_b^{*,s} \in T_I SE(3) =: se(3)$$

- Kinematic relation:

$$\boxed{\dot{H}_b^s = H_b^s \tilde{\mathcal{V}}_b^{b,s}}$$

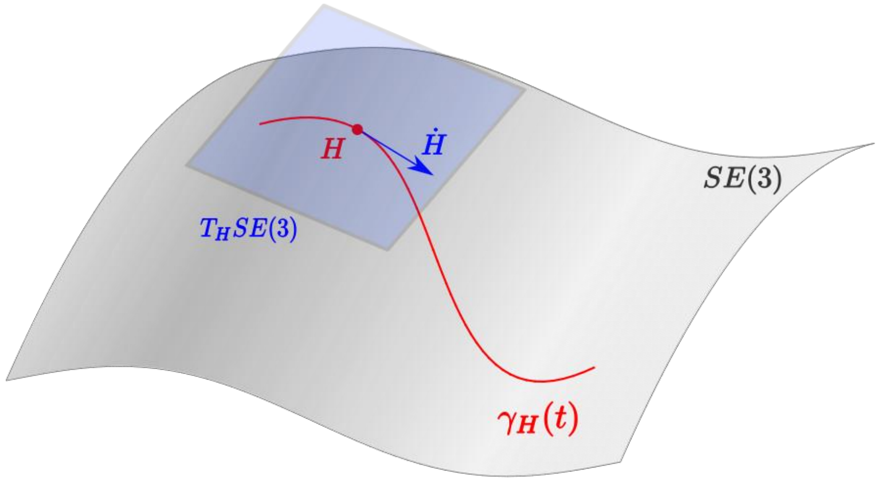
$$\boxed{\dot{H}_b^s = \tilde{\mathcal{V}}_b^{s,s} H_b^s}$$

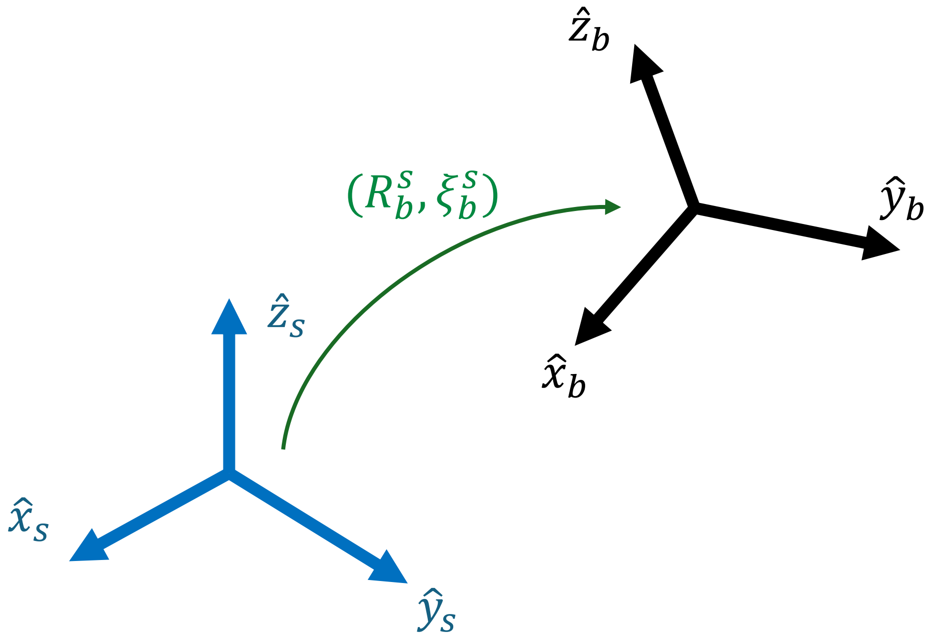


Recap: Lie group structure of SE(3)

- A rigid body general motion is represented mathematically by a curve

$$\begin{aligned} \gamma_H : I \subset \mathbb{R} &\to SE(3) \\ t &\mapsto \gamma_H(t) =: H_b^s(t) \end{aligned}$$





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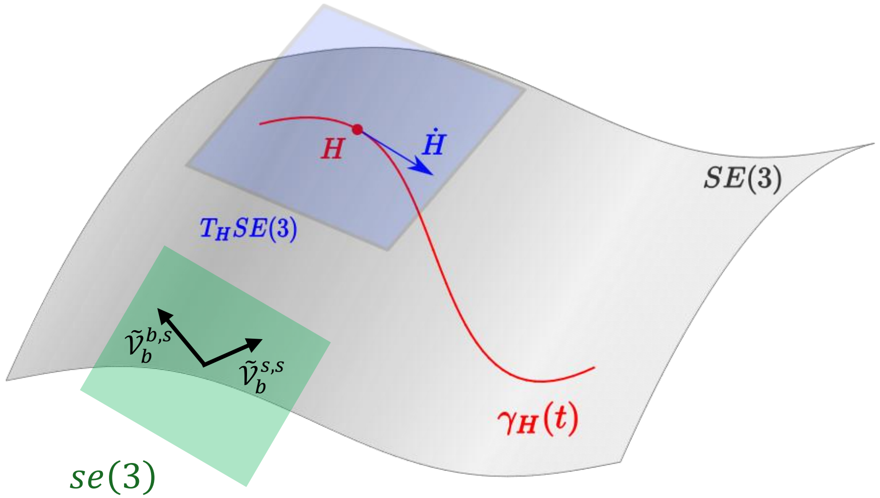

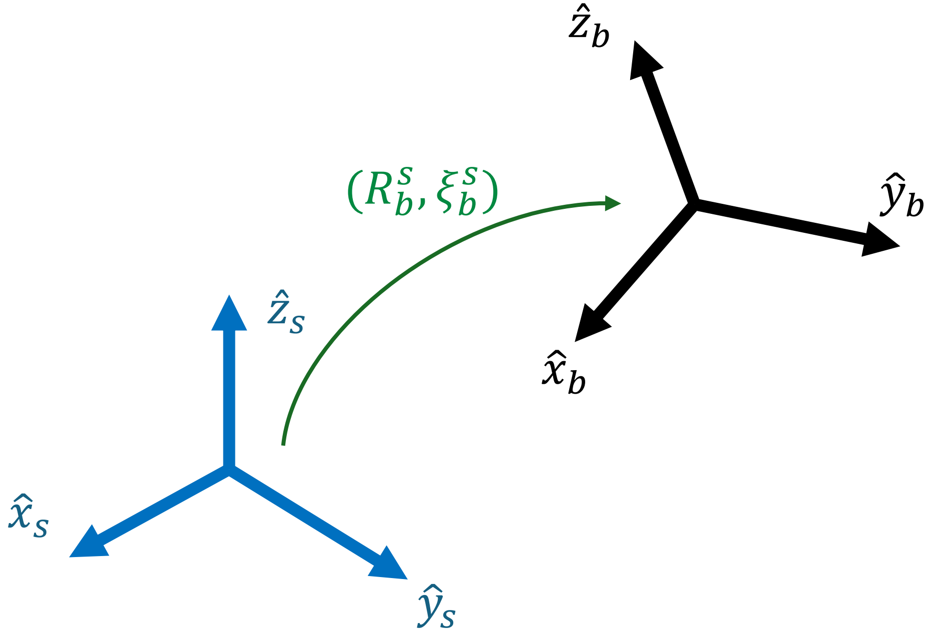

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