SCE594: Special Topics in Intelligent Automation & Robotics

Homework 1

Question 1:

Let $A := \{1,5,31,56,101\}$, $B := \{22,56,5,103,87\}$, and $C := \{41,13,7,101,48\}$

Write the sets resulting from:

- $A \cap B$
- $C \cup A$

Question 2:

List all the subsets of:

- $\{1,2,3\}$
- $\{\varphi, \lambda\}$

Question 3:

Let $A := \{1,3,5\}$, and $B := \{a, b\}$.

Then

1. Find $A \times B$ and $B \times A$
2. Are $A \times B$ and $B \times A$ isomorphic (as sets) to each other? If yes, can you define a bijective map between them.

Question 4:

Rewrite each set-theoretic expression or mapping notation below as a complete sentence in words.

For example:

- $A \coloneqq \{x \in \mathbb{Z} \mid x > 0\}$
  Answer: A is defined as the set of all integers x such that x is greater than zero.

- $f: \mathbb{R} \to \mathbb{R},\ f(x) = x^2$
  Answer: f is a map from the set of real numbers to itself, that maps a real number to its square value.

i. $B: = \{x \in \mathbb{N} \mid x \text{ is odd}\}$
ii. $S^1: = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$
iii. $f: \mathbb{Z} \to \mathbb{Z}, \quad f(n) = 2n + 1$
iv. $g: A \to B, \quad g(x) = x^3$
v. $P \subseteq Q$
vi. $A \cap B = \{3,5\}$
vii. $\mid A \mid = 7$

Question 5:

The function f is defined by $f: \mathbb{R} \to \mathbb{R},\ x \mapsto x^2 + 2$.

a) Give an example to show that f is not injective.

b) Give an example to show that f is not surjective.

Question 6:

Prove that the set of 2×2 diagonal matrices with real entries equipped with matrix addition forms an Abelian group.

Hint: For any two diagonal matrices A and B, check:

a) Closure under the group operation, i.e., whether A+B is still a diagonal matrix,
b) Associativity
c) Commutativity
d) Does an identity element of the group exist?
e) Does each matrix have an additive inverse?