SCE594: Special Topics in Intelligent Automation & Robotics

Homework 3

Question 1:

Let $x = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \in \mathbb{R}^3$ and $y = \begin{bmatrix} d \\ e \\ f \end{bmatrix} \in \mathbb{R}^3$

- **a)** Compute the cross product $x \wedge y \in \mathbb{R}^3$ of x and y
- **b)** Compute the skew-symmetric representation $\tilde{x} \in so(3)$ and $\tilde{y} \in so(3)$ of the vectors x and y, respectively.
- **c)** Show that the following identity is true $\tilde{x}y = x \wedge y$
- **d)** Show that the following identity is true $\tilde{y}x = -x \wedge y$

Question 2:
Given the identities

$$\dot{H}_b^s = H_b^s \widetilde{\boldsymbol{\mathcal{V}}}_b^{b,s} = \widetilde{\boldsymbol{\mathcal{V}}}_b^{s,s} H_b^s,$$

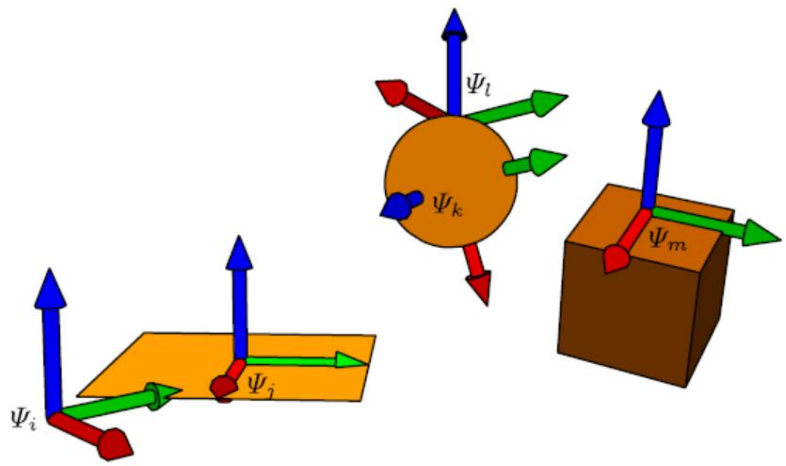
Derive the expressions for $\omega_b^{b,s}, v_b^{b,s}, \omega_b^{s,s}, v_b^{s,s} \in \mathbb{R}^3$ in terms of R_b^s, ξ_b^s and $\dot{R}_b^s, \dot{\xi}_b^s$.

Question 3:

Using the identity $S(R\,\omega) = \widetilde{R\,\omega} = R\,\tilde{\omega}\,R^{\top}$, show that $\tilde{\mathcal{V}}_b^{s,s} = H_b^s \tilde{\mathcal{V}}_b^{b,s} H_s^b$ is equivalent to

$$\mathcal{V}_b^{s,s} = \mathrm{Ad}_{H_b^s}\,\mathcal{V}_b^{b,s} \qquad \text{with } \mathrm{Ad}_H := \begin{pmatrix} R & 0_{3\times 3} \\ \tilde{\xi} R & R \end{pmatrix}.$$

Question 4:
Consider the following setup showing three rigid bodies with several attached coordinate frames. We have that Ψ_i and Ψ_j rigidly attached
to one rigid body (the plane), Ψ_k and Ψ_l rigidly attached to another rigid body (the sphere), and Ψ_m attached to the final rigid body (the cube).



Prove that the following identities hold:

a) $\mathcal{V}_j^{i,i} = 0$
b) $\mathcal{V}_m^{j,j} = \mathcal{V}_k^{j,j} + \mathcal{V}_m^{j,k}$
c) $\mathcal{V}_k^{j,j} = -\mathcal{V}_j^{j,k}$