



## SCE594: Special Topics in Intelligent Automation & Robotics

### Homework 4

#### Question 1:

- a) Let  $\text{Rot}(\hat{n}, \theta) := e^{\hat{n}\theta} \in SO(3)$ , where  $\tilde{n} := S(\hat{n}) \in so(3)$  and  $\hat{n} \in \mathbb{S}^2$  is a unit vector. Show using Rodrigues' formula that rotation operations about the coordinate-frame axes  $\hat{x} := (1,0,0), \hat{y} := (0,1,0), \hat{z} := (0,0,1)$  are given by

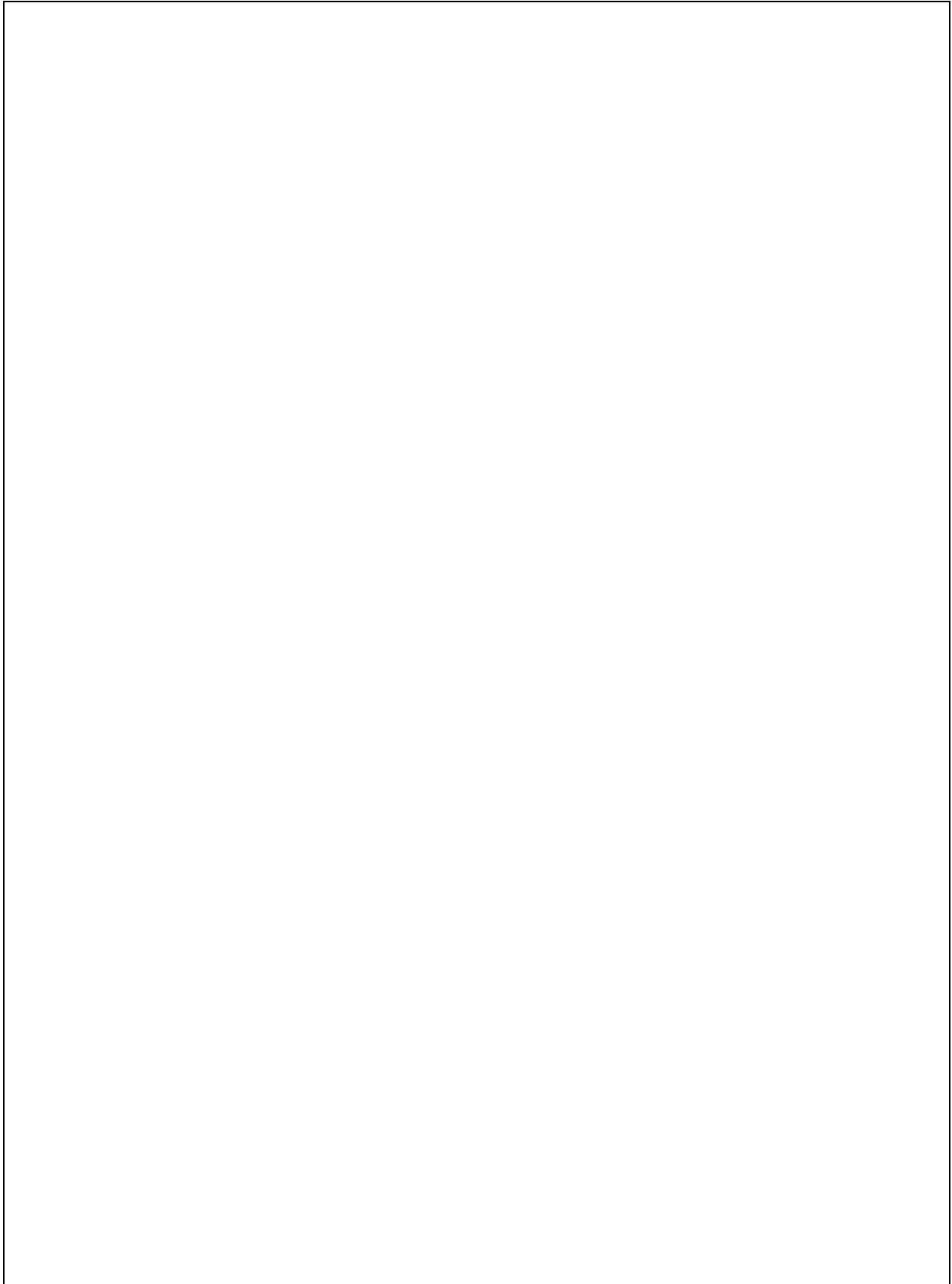
$$\text{Rot}(\hat{x}, \theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix}, \quad \text{Rot}(\hat{y}, \theta) = \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix},$$

$$\text{Rot}(\hat{z}, \theta) = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

- b) Show using Rodrigues' formula that the rotation operation for a general  $\hat{\omega} = (\hat{\omega}_1, \hat{\omega}_2, \hat{\omega}_3) \in \mathbb{S}^2$  is given by

$$\text{Rot}(\hat{\omega}, \theta) = \begin{bmatrix} c_\theta + \hat{\omega}_1^2(1 - c_\theta) & \hat{\omega}_1\hat{\omega}_2(1 - c_\theta) - \hat{\omega}_3s_\theta & \hat{\omega}_1\hat{\omega}_3(1 - c_\theta) + \hat{\omega}_2s_\theta \\ \hat{\omega}_1\hat{\omega}_2(1 - c_\theta) + \hat{\omega}_3s_\theta & c_\theta + \hat{\omega}_2^2(1 - c_\theta) & \hat{\omega}_2\hat{\omega}_3(1 - c_\theta) - \hat{\omega}_1s_\theta \\ \hat{\omega}_1\hat{\omega}_3(1 - c_\theta) - \hat{\omega}_2s_\theta & \hat{\omega}_2\hat{\omega}_3(1 - c_\theta) + \hat{\omega}_1s_\theta & c_\theta + \hat{\omega}_3^2(1 - c_\theta) \end{bmatrix},$$

**Tip: You can write a MATLAB script that computes for you symbolically.**





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**Question 2:**

Consider the following properties for  $\tilde{V} \in se(3)$  and  $H \in SE(3)$

- $e^{H_i^k} \tilde{v}_*^{i,\circ} H_k^i = H_i^k e^{\tilde{v}_*^{i,\circ}} H_k^i$ ,
- $H_i^k \tilde{V}_*^{i,\circ} H_k^i = \tilde{V}_*^{k,\circ}$
- $H_n^0 = H_1^0 H_2^1 \dots H_n^{n-1}$
- $H_i^{i-1}(\theta_i) = e^{\tilde{s}_i^{i-1,i-1} \theta_i} H_i^{i-1}(0)$

Using these properties, show that

$$H_1^0(\theta_1) H_2^1(\theta_2) H_3^2(\theta_3) = e^{\tilde{s}_1^{0,0} \theta_1} e^{\tilde{s}_2^{0,1} \theta_2} e^{\tilde{s}_3^{0,2} \theta_3} H_3^0(0)$$



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**Question 3:**

Consider the following properties for  $\tilde{V} \in se(3)$  and  $H \in SE(3)$

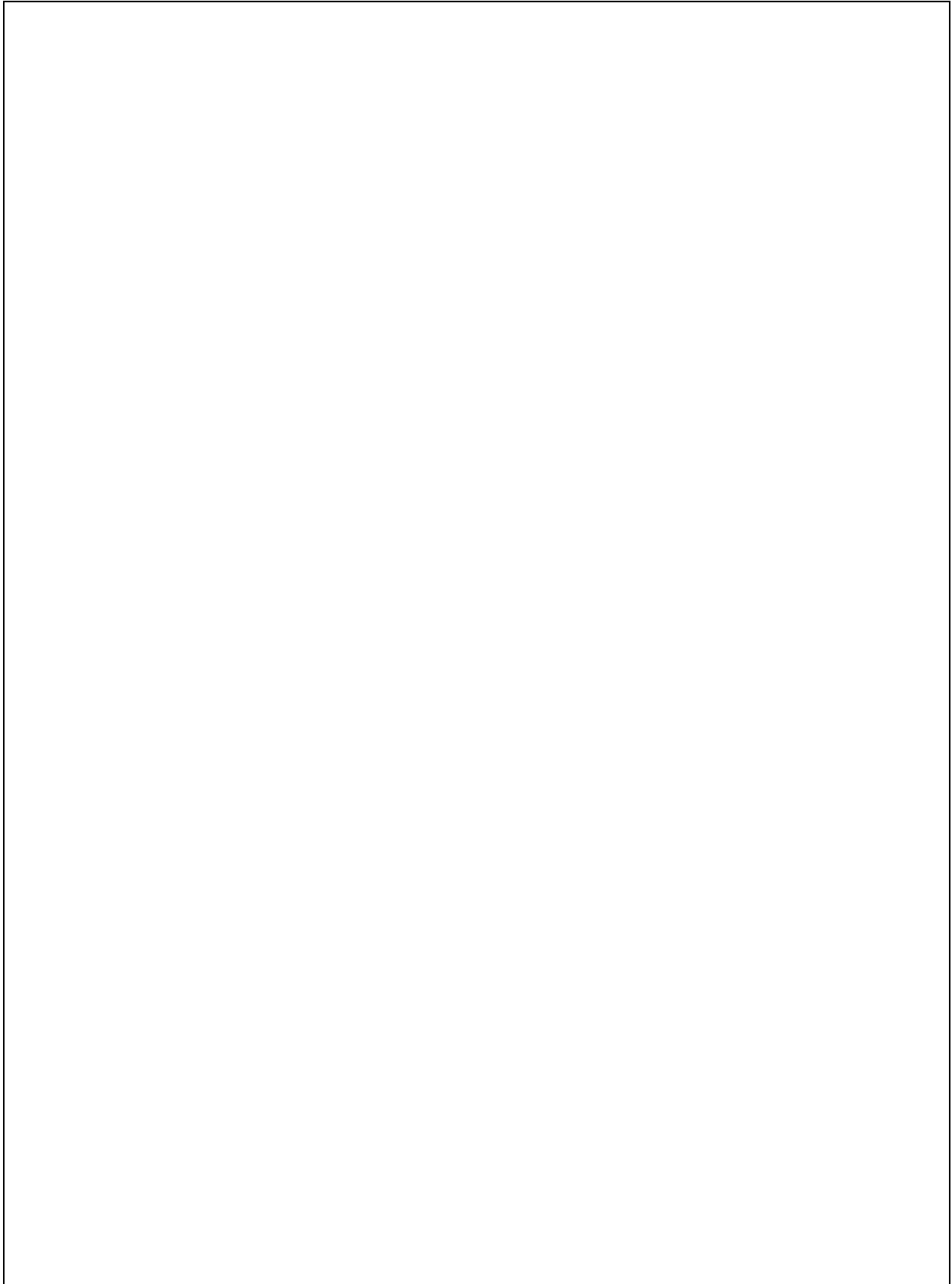
- $H_n^0 = H_1^0 H_2^1 \dots H_n^{n-1}$ ,
- $(AB)^{-1} = B^{-1}A^{-1}$ ,  $\forall A, B \in \mathbb{R}^{n \times n}$
- $(H_i^k)^{-1} = H_k^i$
- $\tilde{V}_i^{j,j} := \dot{H}_i^j H_j^i$

Show that

$$\tilde{V}_n^{0,0} = \tilde{V}_1^{0,0} + \tilde{V}_2^{0,1} + \dots + \tilde{V}_n^{0,n-1}$$

and explain why it follows that

$$\mathcal{V}_n^{0,0} = \mathcal{V}_1^{0,0} + \mathcal{V}_2^{0,1} + \dots + \mathcal{V}_n^{0,n-1}$$



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### Question 4:

Solve examples 4.3 and 4.6 (6R spatial open chain), 4.4 (RRPRRR spatial open chain), 4.5 (UR5 6R robot arm) in Lynch and Park Chapter 4.

**Question 5:**

For the 3R planar robot arm shown below:

- a) expand the 3 equations corresponding to

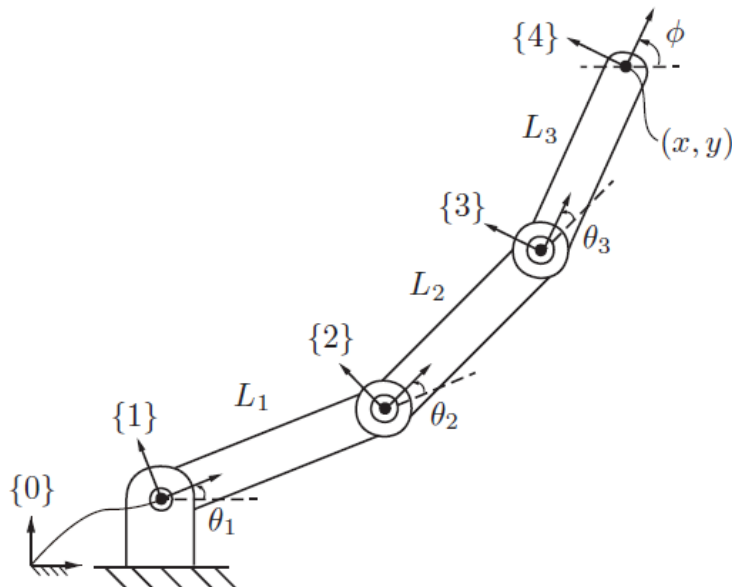
$$\mathcal{V}_i^{i,s} = \text{Ad}_{H_{i-1}^i} \mathcal{V}_{i-1}^{i-1,s} + S_i^{i,i-1} \dot{q}_i$$

- b) Solve the 3 equations recursively such that you can write the right hand side as some function of the form

$$\begin{aligned} \mathcal{V}_1^{1,s} &= f_1(q) \dot{q}, \\ \mathcal{V}_2^{2,s} &= f_2(q) \dot{q}, \\ \mathcal{V}_3^{3,s} &= f_3(q) \dot{q} \end{aligned}$$

with  $q := \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}$ .

- c) Then show you get the same result by using the compact formula  $\mathcal{V} = \mathcal{L}(q) \mathcal{S} \dot{q}$  defined in Lect. 15 slides 37 and 42.





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