SCE594: Special Topics in Intelligent Automation & Robotics

Assignment 1

Instructions:

- This is a **graded** assignment of 5%.
- You can do the assignment either **individually** or in a **group of 2**.
- You can write on the pdf directly with a tablet & electronic pen **or** print the pdf and answer with an ink-pen.
- Good handwriting and neatness makes your work easier to read and understand and is highly recommended.
- Upload your solution as a **single pdf file** on Blackboard by **5:00pm Sunday 25 January 2026**.
- If you do the assignment in a group of two, both members should upload the pdf file.
- Plagiarism will **not tolerated** at all.
- The use of LLM Apps (e.g. ChatGPT) **is allowed** if you use it as an assistant to get hints or help you in intermediate steps you are unable to solve.
- It is very likely the assignment can be fully solved by AI if you use the correct tool. However, be aware that you are **sabotaging your own intellect** by doing so.

Student 1 Name:	
Student 1 ID:	

Student 2 Name:	
Student 2 ID:	

Question 1:

Consider the set $so(3) \coloneqq \{\Omega \in \mathbb{R}^{3\times 3} \mid \Omega^\top = -\Omega\}$ of all 3×3 skew symmetric matrices. Show that $(so(3),\oplus,\odot)$ has a vector space structure over the field $(\mathbb{R}, +, \cdot)$ where $+$ denotes standard addition, $\cdot$ denotes standard multiplication and $\oplus$ denotes standard matrix addition, and $\odot$ denotes scalar multiplication by a matrix.

(Hint: To prove $so(3)$ has a vector space structure, verify that it satisfies all axioms (rules) that a vector space should satisfy.)

Question 2:

Consider the set $O(3) \coloneqq \{R \in \mathbb{R}^{3\times 3} \mid R^{\top}R = I_3\}$ of all 3×3 orthogonal matrices which have their inverse equal to its transpose. Show that $(O(3), \oplus, \odot)$ does not have a vector space structure over the field $(\mathbb{R}, +, \cdot)$. Write down all axioms that are not satisfied.

Question 3:

Prove that the set $SO(3) \coloneqq \{R \in O(3) \mid \det R = +1\}$ equipped with matrix multiplication forms a Group. Check also whether it is an Abelian group or not.