SCE594: Special Topics in Intelligent Automation & Robotics

Assignment 2

Instructions:

- This is a **graded** assignment of 5%.
- You can do the assignment either **individually** or in a **group of 2**.
- You can write on the pdf directly with a tablet & electronic pen **or** print the pdf and answer with an ink-pen.
- Good handwriting and neatness makes your work easier to read and understand and is highly recommended.
- Upload your solution as a **single pdf file** on Blackboard by **5:00pm Tuesday 10 February 2026**.
- If you do the assignment in a group of two, both members should upload the pdf file.
- Plagiarism will **not tolerated** at all.
- The use of LLM Apps (e.g. ChatGPT) **is allowed** if you use it as an assistant to get hints or help you in intermediate steps you are unable to solve.
- It is very likely the assignment can be fully solved by AI if you use the correct tool. However, be aware that you are **sabotaging your own intellect** by doing so.

Student 1 Name:	
Student 1 ID:	

Student 2 Name:	
Student 2 ID:	

Question 1:

Let $M \in O(n)$. Show that the determinant of M must be either +1 or -1.

Question 2:

Let $R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \in SO(3)$

a) Write down the conditions $R^\top R = I$, $\det R = +1$ in terms of the components of R.

b) How many degrees of freedom does R have?

Question 3:

Let $\phi_h: \mathbb{R}^n \to \mathbb{R}^n$ be identified by $(M, \xi) \in O(n) \times \mathbb{R}^n$ such that $\phi_h(x) = M\, x + \xi,\ \ \forall x \in \mathbb{R}^n$.
Show that ϕ_h an isometry on $\mathbb{R}^n$

Hint: An isometry on $\mathbb{R}^n$ preserves the Euclidean metric of $\mathbb{R}^n$, see Lec6 slide 15.

Question 4:

Let $\phi_{h_1}: \mathbb{R}^n \to \mathbb{R}^n$ be identified by $(M_1, \xi_1) \in O(n) \times \mathbb{R}^n$ and $\phi_{h_2}: \mathbb{R}^n \to \mathbb{R}^n$ be identified by $(M_2, \xi_2) \in O(n) \times \mathbb{R}^n$ such that $\phi_{h_i}(x) = M_i\, x + \xi_i,\ \ \forall x \in \mathbb{R}^n, i \in \{1,2\}$.

Let $\phi_{h_3} \coloneqq \phi_{h_2} \circ \phi_{h_1} : \mathbb{R}^n \to \mathbb{R}^n$ to be the composition of isometries ϕ_{h_1} and ϕ_{h_2}.
Show that the orthogonal matrix and translation vector that correspond to ϕ_{h_3} are given by

$$M_3 = M_2 M_1, \quad \xi_3 = M_2 \xi_1 + \xi_2$$

Question 5:

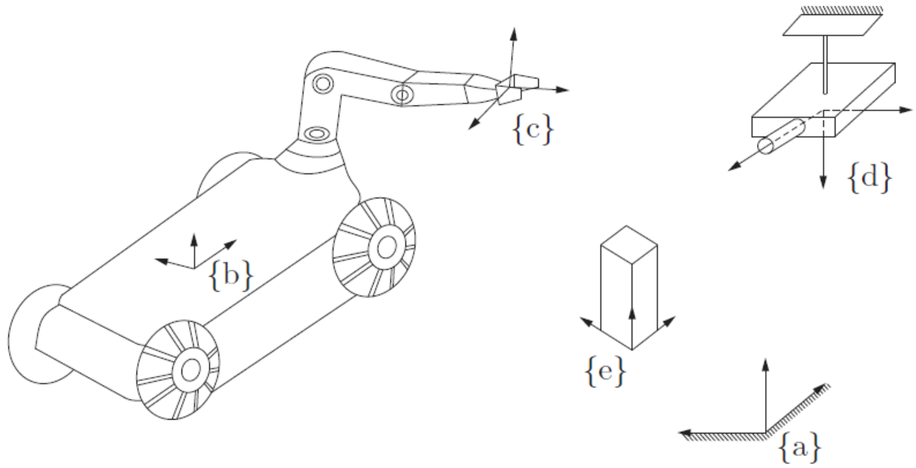
Prove that the composition $\phi_{h_3} \coloneqq \phi_{h_2} \circ \phi_{h_1}$ of isometries ϕ_{h_1} and ϕ_{h_2} is another isometry.

Hint: Show that ϕ_{h_3} preserves the Euclidean metric of $\mathbb{R}^n$.

Question 6:

Let $\phi_h^{-1}: \mathbb{R}^n \rightarrow \mathbb{R}^n$ be the isometry defined such that $\phi_h^{-1} \circ \phi_h = e$, where $e: \mathbb{R}^n \rightarrow \mathbb{R}^n$ is the identity map ($e(x) = x, \ \forall x \in \mathbb{R}^n$. Show that if ϕ_h is identified by $(M, \xi) \in O(n) \times \mathbb{R}^n$ then ϕ_h^{-1} is identified by $(M^\top, -M^\top \xi)$.

Question 7:
The figure below shows a robot arm mounted on a wheeled mobile platform moving in a room, and a camera fixed to the ceiling. Frames {b} and {c} are respectively attached to the wheeled platform and the end-effector of the robot arm, and frame {d} is attached to the camera. A fixed frame {a} has been established, and the robot must pick up an object with body frame {e}.



Suppose that the transformations H_b^d and H_e^d can be calculated from measurements obtained with the camera. The transformation H_c^b can be calculated using the arm's joint-angle measurements. The transformation H_d^a is assumed to be known in advance from the room's schematics. Suppose these calculated and known transformations are given as follows:

$$H_b^d = \begin{bmatrix} 0 & 0 & -1 & 2.5 \\ 0 & -1 & 0 & -1.5 \\ -1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad H_e^d = \begin{bmatrix} 0 & 0 & -1 & 3 \\ 0 & -1 & 0 & 1 \\ -1 & 0 & 0 & 1.2 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$H_d^a = \begin{bmatrix} 0 & 0 & -1 & 4 \\ 0 & -1 & 0 & 0.5 \\ -1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad H_c^b = \begin{bmatrix} 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0.3 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -0.4 \\ 1 & 0 & 0 & 0.25 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

To calculate how to move the robot arm so as to pick up the object, show using the given information that the configuration of the object relative to the robot hand (H_e^c) should be:

$$H_e^c = \begin{bmatrix} 0 & 0 & 1 & -0.75 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & -\frac{2.6}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & \frac{1.6}{\sqrt{2}} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$