SCE594: Special Topics in Intelligent Automation & Robotics

Assignment 3

Instructions:

- This is a **graded** assignment of 5%.
- You can do the assignment either **individually** or in a **group of 2**.
- You can write on the pdf directly with a tablet & electronic pen **or** print the pdf and answer with an ink-pen.
- Good handwriting and neatness makes your work easier to read and understand and is highly recommended.
- Upload your solution as a **single pdf file** on Blackboard by **5:00pm Tuesday 24 February 2026**.
- If you do the assignment in a group of two, both members should upload the pdf file.
- Plagiarism will **not tolerated** at all.
- The use of LLM Apps (e.g. ChatGPT) **is allowed** if you use it as an assistant to get hints or help you in intermediate steps you are unable to solve.
- It is very likely the assignment can be fully solved by AI if you use the correct tool. However, be aware that you are **sabotaging your own intellect** by doing so.

Student 1 Name:	
Student 1 ID:	

Student 2 Name:	
Student 2 ID:	

Question 1:
Prove the following identity

$$\frac{d}{dt}\left(\mathrm{Ad}_{H_i^k}\right) = \mathrm{Ad}_{H_i^k}\,\mathrm{ad}_{\mathcal{V}_i^{i,k}} \tag{1}$$

with $\mathrm{ad}_{\mathcal{V}} = \begin{pmatrix} \tilde{\omega} & 0_{3\times3} \\ \tilde{v} & \tilde{\omega} \end{pmatrix} \in \mathbb{R}^{6\times6}$ for $\mathcal{V} = \begin{pmatrix} \omega \\ v \end{pmatrix} \in \mathbb{R}^6$

Question 2:

The momentum balance equation for the generalized momentum of a rigid body with body fixed frame $\{b\}$ expressed in a stationary frame {s} is given by

$$\frac{d}{dt}(P^{s,b}) = \mathcal{W}^{s,b} \tag{2}$$

where $\mathcal{W}^{s,b} \in (\mathbb{R}^6)^*$ denotes the applied wrench to the body expressed in $\{s\}$.

a) Using Eq. (1) and the identity

$$\mathrm{ad}_{\mathcal{V}_i^{m,j}} = \mathrm{Ad}_{H_k^m}\, \mathrm{ad}_{\mathcal{V}_i^{k,j}} \mathrm{Ad}_{H_m^k}$$

to show that Eq. (2) is equivalent to

$$\frac{d}{dt}\left(P^{b,b}\right) = \mathrm{ad}^{\top}_{\mathcal{V}_b^{b,s}} P^{b,b} + \mathcal{W}^{b,b} \tag{3}$$

b) Show that Eq. (3) can be rewritten as

$$\frac{d}{dt}\left(P^{b,b}\right) = \mathcal{J}\left(P^{b,b}\right)\mathcal{V}_b^{b,s} + \mathcal{W}^{b,b} \tag{4}$$

where $\mathcal{J}(P^{b,b}) \in \mathbb{R}^{6\times 6}$ is the skew-symmetric matrix

$\mathcal{J}(P) = \begin{pmatrix} \tilde{P}_\omega & \tilde{P}_v \\ \tilde{P}_v & 0_{3\times 3} \end{pmatrix}$ for $P = \begin{pmatrix} P_\omega \\ P_v \end{pmatrix} \in (\mathbb{R}^6)^*$.

c) If $\{b\}$ is chosen to be placed at the body's center of mass, one has that $P^{b,b} = \mathfrak{I}^{b,b}\mathcal{V}_b^{b,s}$ can be written as

$$P^{b,b} = \begin{pmatrix} P_\omega^{b,b} \\ P_v^{b,b} \end{pmatrix} = \begin{pmatrix} J^{b,b}\,\omega_b^{b,s} \\ m\, v_b^{b,s} \end{pmatrix}.$$

Show that in this case, we can rewrite Eq. (3) as

$$J^{b,b}\dot{\omega}_b^{b,s} = -\omega_b^{b,s} \wedge \left(J^{b,b}\omega_b^{b,s}\right) + \tau^{b,b} \tag{5}$$

$$m\dot{v}_b^{b,s} = -m\omega_b^{b,s} \wedge v_b^{b,s} + f^{b,b} \tag{6}$$

where $\mathcal{W}^{b,b} = \begin{pmatrix} \tau^{b,b} \\ f^{b,b} \end{pmatrix}$.

d) Using the relation between $v_b^{b,s}$ and $\dot{\xi}_b^s$, show that Eq. (6) is equivalent to

$$\ddot{\xi}_b^s = \frac{1}{m} R_b^s f^{b,b}$$

Question 3:

Let $H_1, H_2 \in SE(3)$, and let $\mathrm{Ad}_H : \mathbb{R}^6 \to \mathbb{R}^6$ denote the matrix representation of the Adjoint map of $SE(3)$ then prove the following identities:

a) $\mathrm{Ad}_{H_1} \cdot \mathrm{Ad}_{H_2} \cdot \mathcal{V} = \mathrm{Ad}_{H_1 H_2} \cdot \mathcal{V}, \quad \forall\, \mathcal{V} \in \mathbb{R}^6$

b) $(\mathrm{Ad}_H)^{-\mathbf{1}} = \mathrm{Ad}_{H^{-1}}$