SCE594: Special Topics in Intelligent Automation & Robotics

Assignment 4

Instructions:

- This is a **graded** assignment of 5%.
- This is an **individual assignment.**
- You can write on the pdf directly with a tablet & electronic pen **or** print the pdf and answer with an ink-pen.
- Good handwriting and neatness makes your work easier to read and understand and is highly recommended.
- Upload your solution as a **single pdf file** on Blackboard by **11:59pm Thursday 16 April 2026.**
- Plagiarism will **not tolerated** at all.
- The use of LLM Apps (e.g. ChatGPT) **is allowed** if you use it as an assistant to get hints or help you in intermediate steps you are unable to solve.
- It is very likely the assignment can be fully solved by AI if you use the correct tool. However, be aware that you are **sabotaging your own intellect** by doing so.

Student Name:	
Student ID:	

Question 1:

Compute the Hessian of the following functions and check which ones of these is locally positive definite around the origin.

a) $f(x_1, x_2) = x_1^2 + 3x_2^2 + 2x_1x_2$

b) $f(x_1, x_2) = x_1^3x_2 + x_1x_2^2$

c) $f(x_1, x_2) = \ln(x_1^2 + x_2^2)$

Question 2:

Consider the system

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -2(x_1 + x_2) - 4x_1^3$$

Use the function

$$V(x) = 4x_1^2 + 2\,x_2^2 + 4\,x_1^4$$

To show that the origin is globally asymptotically stable using Lyapunov's direct method.

Question 3:

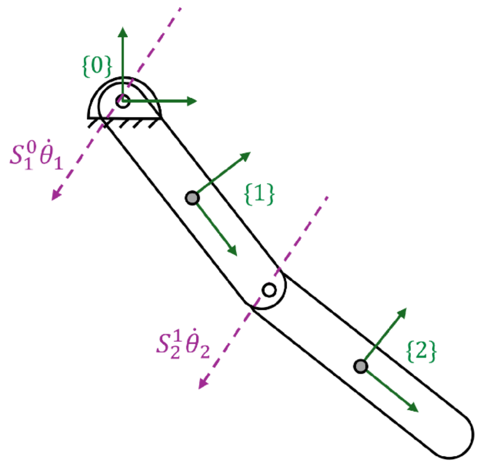
Consider the state space model of the pendulum discussed in Lecture 17 class notes. Let all constants be 1 for simplicity ($m = L = b = g = 1$). Consider the candidate Lyapunov function

$$V(x_1, x_2) = \frac{1}{2}\, x_2^2 + (1 - \cos x_1).$$

Evaluate the stability of the equilibrium point $x_* = (0,0)$ using Lyapunov's direct method.

Question 4:

Consider the dynamic equations of the two-link manipulator shown below:



which can be shown to be equal to:

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + g(\theta) = \tau$$

with

$$M(\theta) := \begin{pmatrix} \alpha + 2\,\beta\cos\theta_2 & \delta + \beta\cos\theta_2 \\ \delta + \beta\cos\theta_2 & \delta \end{pmatrix}, \qquad C(\theta,\dot{\theta}) := \begin{pmatrix} -\beta\dot{\theta}_2\sin\theta_2 & -\beta(\dot{\theta}_1+\dot{\theta}_2)\sin\theta_2 \\ \beta\dot{\theta}_1\sin\theta_2 & 0 \end{pmatrix}$$

$$g(\theta) = \begin{pmatrix} \gamma_1\sin\theta_1 + \gamma_2\sin(\theta_1+\theta_2) \\ \gamma_2\sin(\theta_1+\theta_2) \end{pmatrix}$$

where $\theta := (\theta_1,\theta_2) \in (-\pi,\pi] \times (-\pi,\pi]$ and $\dot{\theta} := (\dot{\theta}_1,\dot{\theta}_2) \in \mathbb{R}\times\mathbb{R}$, while $\alpha,\beta,\delta,\gamma_1,\gamma_2 \in \mathbb{R}_+$ are constants related to the manipulator parameters. There numerical values are given in the table below:

α	β	δ	γ_1	γ_2
3.4021	0.124	1.0396	8.5347	2.43288

Note:

If you want to understand how these equations were derived, you can check the MATLAB Tutorial Series.

Your tasks in this assignment are:

1) Write the dynamics of this system in state space form:

$$\dot{x} = \begin{pmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \\ f_4(x) \end{pmatrix}, \qquad x = (\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2) \in \mathcal{X}$$

2) Show that the equilibrium points of this nonlinear system must satisfy $g(\theta) = 0$ under zero control torque.
3) Show that the system has only 4 physically valid equilibrium points and state them explicitly in the table below:

$x_{*,1}$	$x_{*,2}$	$x_{*,3}$	$x_{*,4}$

4) [**Bonus**] Assess the stability of each equilibrium point using Lyapunov's indirect method (i.e., by linearizing around each equilibrium point and examining the Jacobian's eigenvalues). **Tip: Use MATLAB/Python to solve the bonus part.**