1. Group:

A group is a pair $\overbrace{(G, \circ)}^{\text{Group}}$ where G is a set &

$$\underset{\text{Group operation}}{\circ} : G \times G \longrightarrow G$$
$$(a, b) \longmapsto a \circ b$$

$\boxed{\begin{array}{l} \text{iv) } a \circ b = b \circ a \ , \forall a, b \in G \\ \quad (G, \circ) \text{ is an abelian group} \end{array}}$

such that:

i) $(a \circ b) \circ c = a \circ (b \circ c) \quad , \forall a, b, c \in G$

for all

ii) $\exists \underset{\text{identity element}}{e} \in G$ s.t. $g \circ e = e \circ g = g \quad , \forall g \in G$

iii) $\exists \underset{\text{inverse of } g}{g^{-1}} \in G$ s.t $g \circ g^{-1} = g^{-1} \circ g = e \quad , \forall g \in G$

2) Examples:

i) $(\mathbb{R}^n, +)$: $\underset{x}{\begin{pmatrix} a \\ b \\ c \end{pmatrix}} + \underset{y}{\begin{pmatrix} e \\ f \\ g \end{pmatrix}} := \begin{pmatrix} a+e \\ b+f \\ c+g \end{pmatrix}$

yes

Abelian group

$x^{-1} = \begin{pmatrix} -a \\ -b \\ -c \end{pmatrix}$ $\quad e = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

ii) $(\mathbb{R}^{n\times n}, \oplus)$

$$e = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

iii) $(\mathbb{R}^{n\times n}, \odot)$ $\qquad (A\odot B)\odot C = (A)\odot(B\odot C)$

$$\boxed{A^{-1}\odot A = e} \; ? \qquad e = \begin{pmatrix}1&&&&\\&1&&&\\&&1&&\\&&&\ddots&\\&&&&1\end{pmatrix}$$

General Linear n-dim. Group

iv) $$\boxed{GL(n,\mathbb{R})} := \{ M\in\mathbb{R}^{n\times n} \mid \det(M)\neq 0\} \subset \mathbb{R}^{n\times n}$$

$$(GL(n,\mathbb{R}), \odot) \Longrightarrow$$