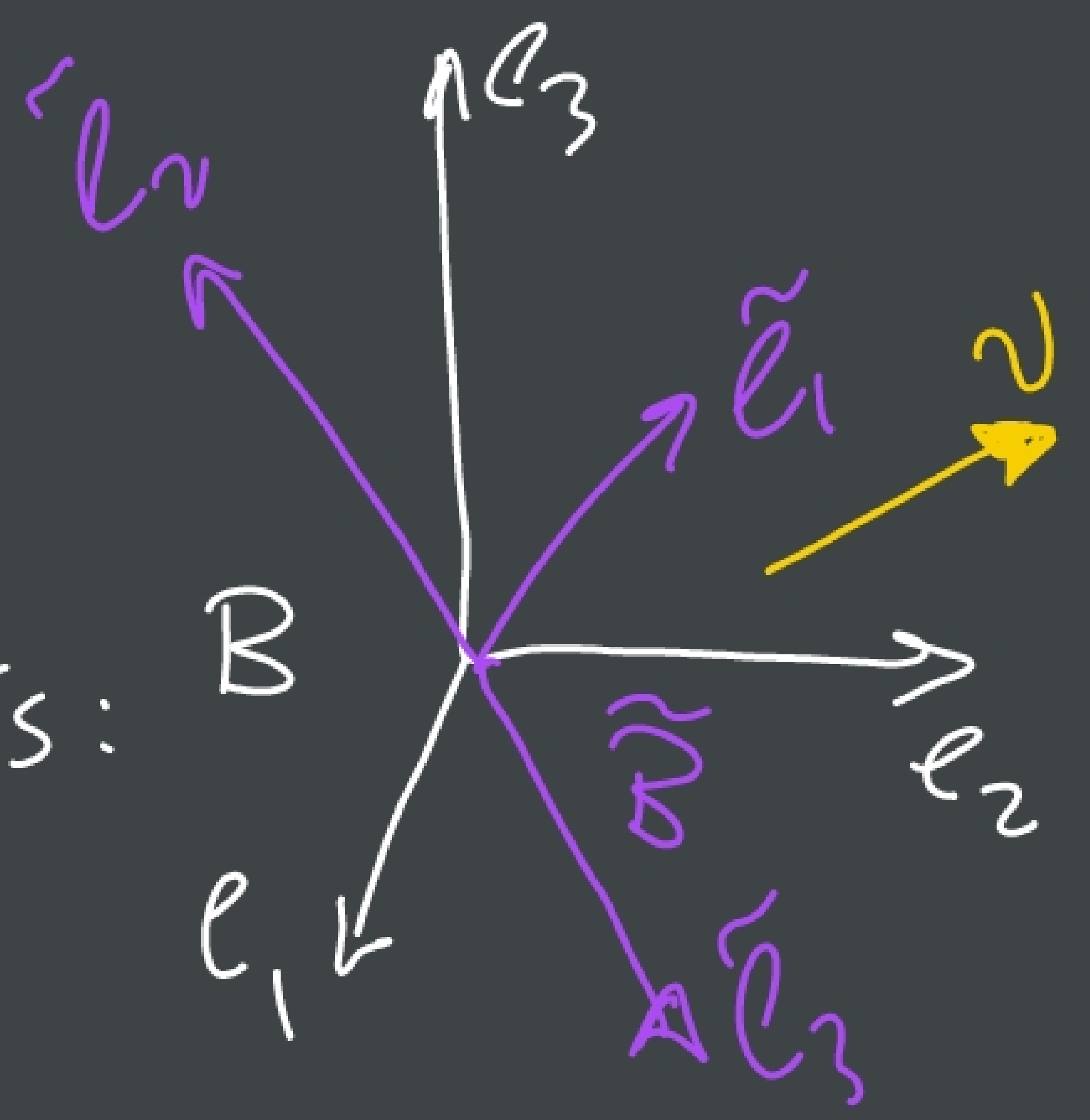
1) Change of Basis: $\quad P \in GL(n, \mathbb{R})$ $\qquad V$

$$\Rightarrow [v]_B = \underset{n\times n}{P}\,[v]_{\tilde{B}} \qquad (1)$$

$\Rightarrow \alpha(v) \in \mathbb{R}$, it is indpt. from the choice of basis: B



$$\alpha(v) = [\alpha]_B^T\,[v]_B \qquad (2)$$

$$= [\alpha]_{\tilde{B}}^T\,[v]_{\tilde{B}} \qquad (3)$$

* Subst. (1) into (2):

$$\alpha(v) = [\alpha]_B^T P [v]_{\tilde{B}} = \left(P^T[\alpha]_B\right)^T [v]_{\tilde{B}} \qquad (4)$$

Unless $P^{-1} = P^T$

* Compare (4) to (3)

$$\boxed{[\alpha]_{\tilde{B}} = P^T[\alpha]_B} \qquad \boxed{[v]_{\tilde{B}} = P^{-1}[v]_B}$$

$$\begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{pmatrix} \qquad \begin{pmatrix} v^1 \\ \vdots \\ v^n \end{pmatrix}$$

2) Components of an endomorphism:

$$[A] = \begin{pmatrix} A^1_1 & \cdots & A^1_n \\ \vdots & \ddots & \\ A^n_1 & \sim & A^n_n \end{pmatrix}$$

A^k_i ← row (k), ← col. (i)

Consider $A : V \to V$

$v \mapsto A(v)$

$\boxed{[v] \in \mathbb{R}^n}$

$$\underset{\substack{\cap \\ V}}{A(e_i)} = A^1_i e_1 + \cdots + A^n_i e_n = \sum_{k=1}^{n} A^k_i e_k$$

$$\overset{\Psi}{A(v)} = A(v^1 e_1 + \cdots + v^n e_n) = v^1 A(e_1) + \cdots + v^n A(e_n)$$

$$= v^1 \sum_k A^k_1 + \cdots + v^n \sum_k A^k_n e_k$$

$$= \sum_{i=1}^{n} \sum_{k=1}^{n} A^k_i v^i e_k$$

$$\boxed{[A(v)] = [A] \overset{\text{M.-Mult.}}{\otimes} [v]}$$

3) Change of Basis

a) $$\boxed{[A]_{\tilde{B}} = P^{-1}\,[A]_B\,P}$$

$$[A] = \begin{bmatrix} A^1_1 & \cdots & A^1_n \\ \vdots & \ddots & \vdots \\ A^n_1 & \cdots & A^n_n \end{bmatrix}$$

$A \in End(V)$
$A : V \to V$

b) $$\boxed{[\beta]_{\tilde{B}} = P^T\,[\beta]_B\,P}$$

$$[\beta] = \begin{bmatrix} \beta_{11} & \cdots & \beta_{1n} \\ \vdots & \ddots & \vdots \\ \beta_{n1} & \cdots & \beta_{nn} \end{bmatrix}$$

$\beta \in B\ell(V)$
$\beta : V \to V^*$