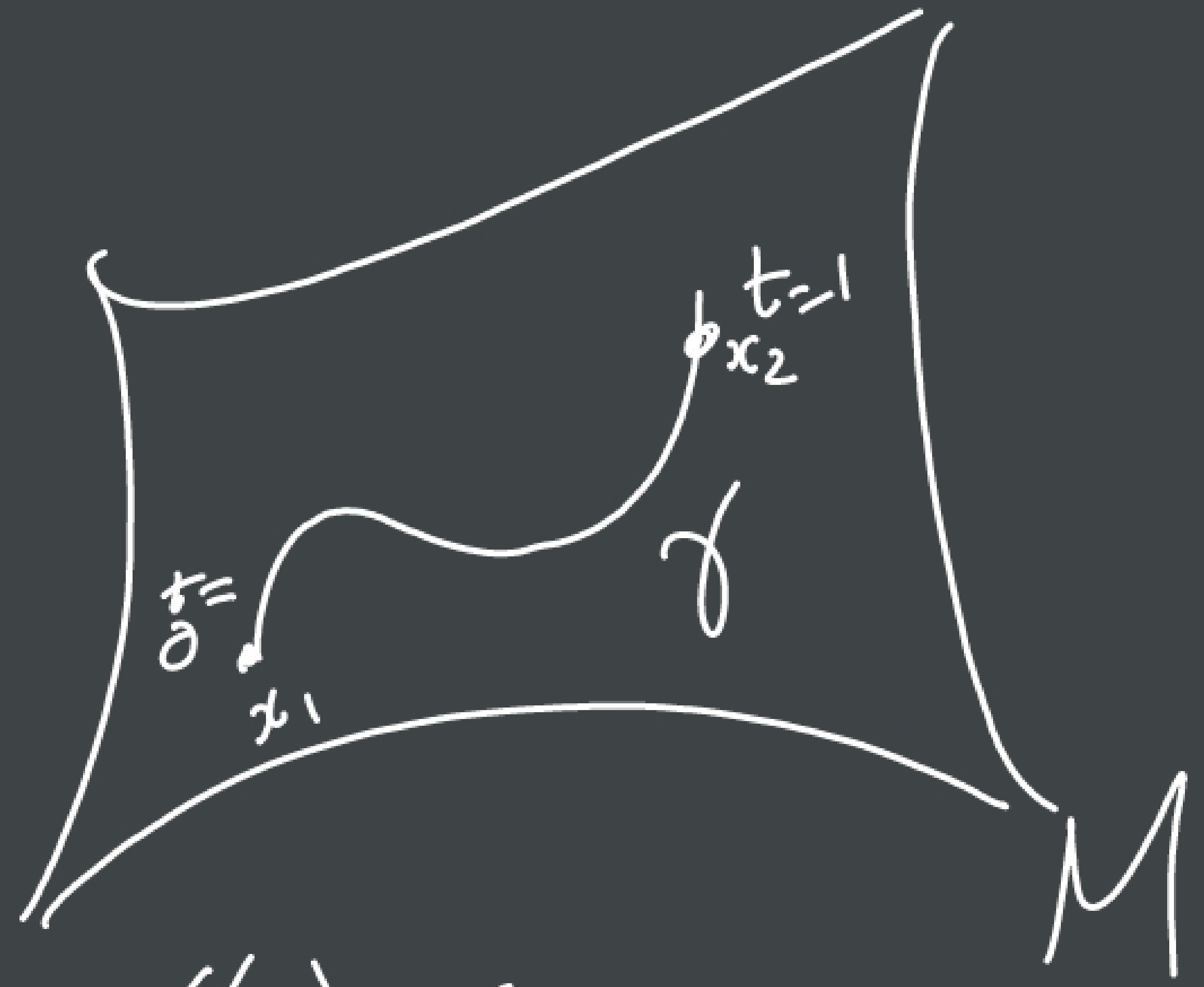
1) Curve on a manifold:

$$\gamma : I \subset \mathbb{R} \longrightarrow M$$
$$t \longmapsto \gamma(t)$$



$$\gamma(0) = x_1$$
$$\gamma(1) = x_2$$

2) Function on the manifold: $\boxed{\dim M = n}$

$\Longrightarrow f : M \longrightarrow \mathbb{R}$ $\boxed{\text{is of class } C^k}$

$$\begin{tikzcd}
U \subset M \arrow[r, "f"] \arrow[d, shift right, "\phi"'] & \mathbb{R} \\
\mathbb{R}^n \arrow[u, shift right, "\phi^{-1}"'] \arrow[ur, "f \circ \phi^{-1}"'] &
\end{tikzcd}$$

$f_\phi := \Longrightarrow f \circ \phi^{-1} \Longrightarrow$ local representative of f in the chart (U, ϕ)

$$f \circ \phi^{-1} : \mathbb{R}^n \longrightarrow \mathbb{R}$$

$$\boxed{f \in C^k(M)}$$

3) Maps between Manifolds:

We call f a *diffeomorphism* if $f \in C^\infty(M)$ & also f^{-1} exists & $f^{-1} \in C^\infty(M)$.

$$
\begin{tikzcd}
U \subset M \arrow[r, "f"] \arrow[d, "\phi_M"'] & N \supset V \arrow[l, bend left, "f^{-1}"] \arrow[d, "\phi_N"] \\
\mathbb{R}^n \arrow[r, "\phi_N \circ f \circ \phi_M^{-1}"'] & \mathbb{R}^m
\end{tikzcd}
$$

4) Summary:

Lie Groups

Set	$S \overset{f}{\underset{\text{bijections}}{\longleftrightarrow}} T$	$S \cong T$
Group	$(G, \odot) \overset{f}{\underset{\text{group isomorph.}}{\longleftrightarrow}} (H, \diamond)$	$G \cong H$
Vector Space	$(V, \oplus, \odot) \overset{f}{\underset{\text{linear isomorph.}}{\longleftrightarrow}} (W, \diamond, \diamond)$	$V \cong W$
Manifold	$(M, \mathcal{O}_M, \mathcal{A}_M) \overset{f}{\underset{\text{diffeomorphisms}}{\longleftrightarrow}} (N, \mathcal{O}_N, \mathcal{A}_N)$	$M \cong N$