1) $\tilde{\omega}$: skew-symmetric matrices:

$$\tilde{\omega} \in so(3)$$

$$\overset{\text{ii}}{}$$

$$\begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix}$$

$$\uparrow S$$

$$\omega := \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} \in \mathbb{R}^3$$

$$\tilde{\omega} \in so(2)$$

$$\overset{\text{ii}}{}$$

$$\begin{pmatrix} 0 & -\omega \\ \omega & 0 \end{pmatrix}$$

$$\uparrow$$

$$\omega \in \mathbb{R}$$

$$R^T \dot{R} = \tilde{\omega}^b \rightarrow \omega^b$$

Adjoint

$$\dot{R} R^T = \tilde{\omega}^s \rightarrow \omega^s$$

2) Twists ($se(3)$)

$$\tilde{\mathcal{V}}_b^{b,s} := H_s^b \dot{H}_b^s = \begin{pmatrix} R_s^b & -R_s^b \xi_b^s{}_{3\times 1} \\ 0_{1\times 3} & 1 \end{pmatrix} \begin{pmatrix} \dot{R}_b^s & \dot{\xi}_b^s \\ 0_{1\times 3} & 0 \end{pmatrix}$$

$$H_b^s = \begin{pmatrix} R_b^s & \xi_b^s \\ 0 & 1 \end{pmatrix} \qquad = \begin{pmatrix} R_s^b \dot{R}_b^s & R_s^b \dot{\xi}_b^s \\ 0_{1\times 3} & 0 \end{pmatrix} = \begin{pmatrix} \tilde{\omega}_b^{b,s} & v_b^{b,s} \\ 0 & 0 \end{pmatrix} \in se(3)$$

$$\dot{H}_b^s = \begin{pmatrix} \dot{R}_b^s & \dot{\xi}_b^s \\ 0 & 0 \end{pmatrix}$$

$$H_s^b = (H_b^s)^{-1} = \begin{pmatrix} R_s^b & -R_s^b \xi_b^s \\ 0 & 1 \end{pmatrix}$$

$$\boxed{\tilde{\mathcal{V}}_b^{s,s} = \begin{pmatrix} \tilde{\omega}_b^{s,s} & v_b^{s,s} \\ 0 & 0 \end{pmatrix} \in se(3)}$$

$$
\begin{tikzcd}
se(3) \ni \tilde{\mathcal{V}}_b^{b,s} = \begin{pmatrix} \tilde{\omega}_b^{b,s} & v_b^{b,s} \\ 0 & 0 \end{pmatrix} \arrow[r, squiggly, mapsto] & (\tilde{\omega}_b^{b,s}, v_b^{b,s}) \arrow[r, squiggly, mapsto] & (\omega_b^{b,s}, v_b^{b,s}) \arrow[r, mapsto] & \mathcal{V}_b^{b,s} \arrow[dd, leftrightarrow, bend left, "\text{Adjoint map}"'] \\
 & SO(3) \times \mathbb{R}^3 \arrow[u, phantom, sloped, "\in"] & \mathbb{R}^3 \times \mathbb{R}^3 \arrow[u, phantom, sloped, "\in"] & \mathbb{R}^6 \arrow[u, phantom, sloped, "\in"] \\
se(3) \ni \tilde{\mathcal{V}}_b^{s,s} = \begin{pmatrix} \tilde{\omega}_b^{s,s} & v_b^{s,s} \\ 0 & 0 \end{pmatrix} \arrow[rrr, squiggly, mapsto] & & & \mathcal{V}_b^{s,s} \\
 & & & \mathbb{R}^6 \arrow[u, phantom, sloped, "\in"]
\end{tikzcd}
$$