

SCE 594: Special Topics in Intelligent Automation & Robotics

Lecture 14: Forward and Differential Kinematics



Outline

- Recap last lecture
- FK using Product of Exponentials (PoE) Formula
- DK using Geometric Jacobian
- Example



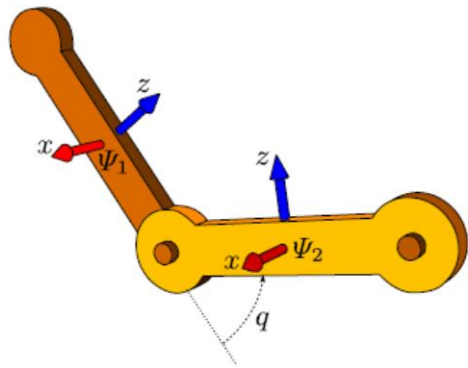
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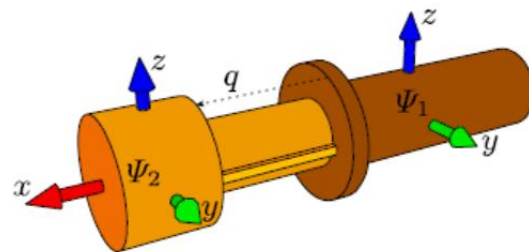


Recap: Motivation

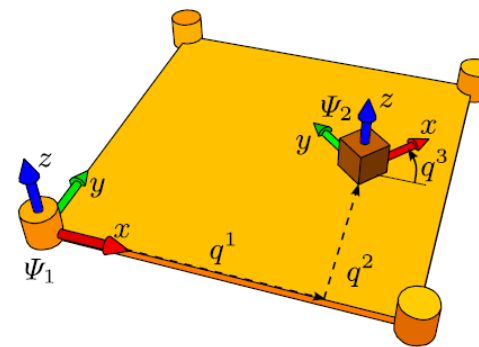
- An ideal joint (aka **kinematic pair**) is a purely kinematic relation between two rigid bodies restricting the relative twist $\mathcal{V}_1^{*,2}$.
- The degrees of freedom (**DoF**) of a joint is the number of independent coordinates of $\mathcal{V}_1^{*,2}$.



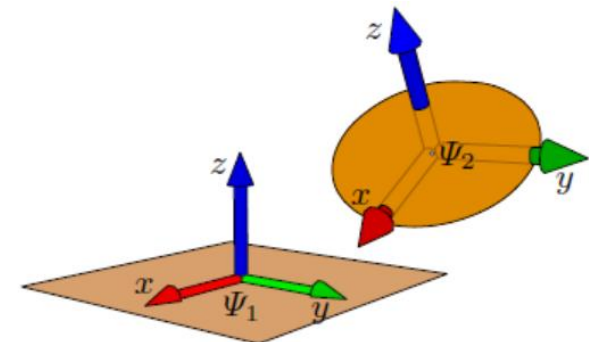
Revolute joint (1 DoF)



Prismatic joint (1 DoF)



Planar joint (3 DoF)

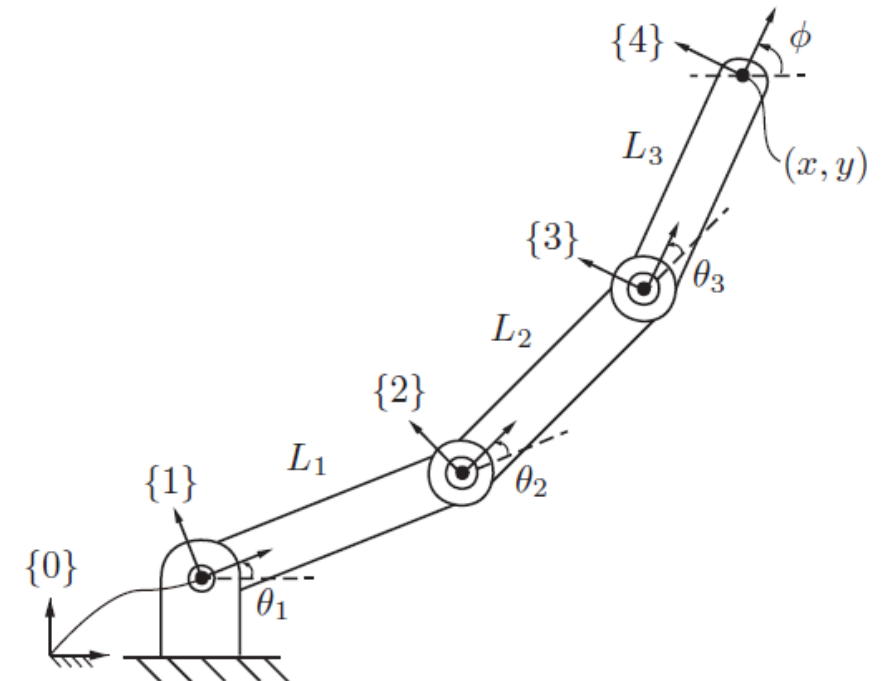
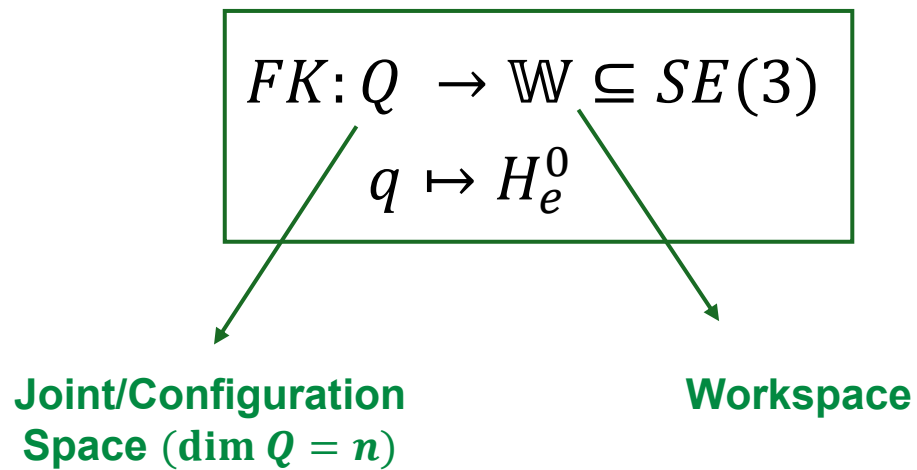


Free motion (6 DoF)



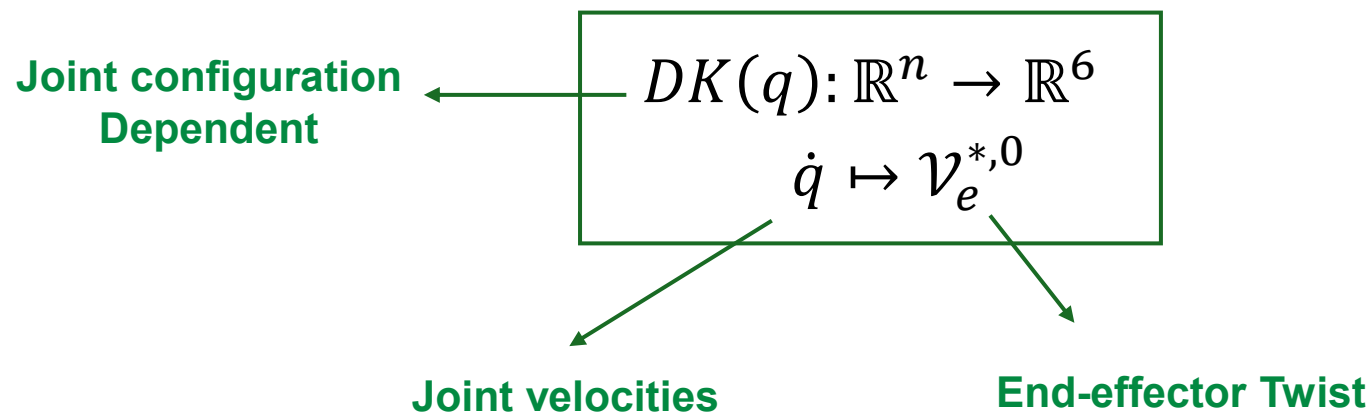
Recap: Forward Kinematics

- The **forward kinematics** of a robot refers to the calculation of the position and orientation of its end-effector frame given:
 - The robot's geometry (link lengths, joint types, etc.)
 - The joint configurations $q := (q_1, \dots, q_n) \in Q$



Recap: Differential Kinematics

- The differential/velocity kinematics is the differential of this map which is a mapping from $\dot{q} \in T_q Q$ to $\dot{H}_e^0 \in T_{H_e^0} SE(3)$.
- **Geometrically**, we can equivalently represent it as a map from $\dot{q} \in T_q Q \cong \mathbb{R}^n$ to the end effector's twist $\mathcal{V}_e^{*,0} \in \mathbb{R}^6$

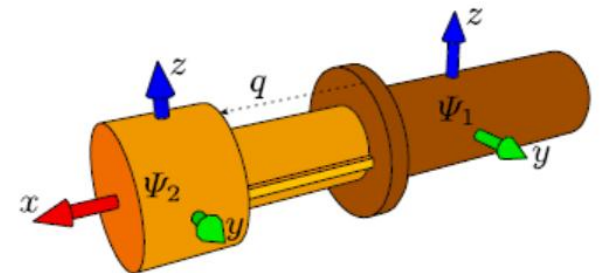


Recap: Displacement Subgroup

- Consider a joint connecting two rigid bodies with frames Ψ_i and Ψ_j .
- The **displacement subgroup** of $SE(3)$ associated with the joint connecting body i to body j is denoted by

$$G_c^p \subseteq SE(3)$$

Represents the set of all possible relative poses of child body c with respect to parent body p that are allowed by the joint.



b DoF joint constraints $n - b$ DoFs



Recap: Ideal Joint Modeling

- The joint configuration is mapped to the relative pose by

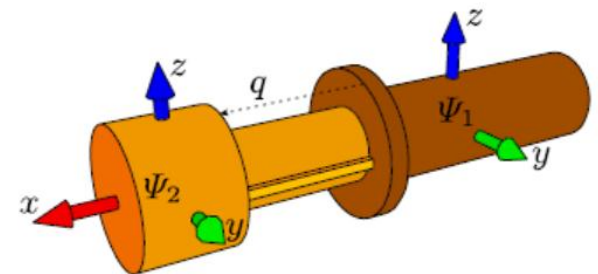
$$\begin{aligned}\varphi_c: G_c &\rightarrow G_c^p \\ q_c &\mapsto H_c^p\end{aligned}$$

- The induced map on the velocities is given by

$$\begin{aligned}l_c: \mathfrak{g}_c &\rightarrow \mathfrak{g}_c^p \\ v_c &\mapsto \mathcal{V}_c^{*,p}\end{aligned}$$

- The joint velocity is related to its configuration by

$$\begin{aligned}\chi_{q_c}: \mathfrak{g}_c &\rightarrow T_{q_c} G_c \\ v_c &\mapsto \dot{q}_c\end{aligned}$$



b DoF joint constraints $n - b$ DoFs



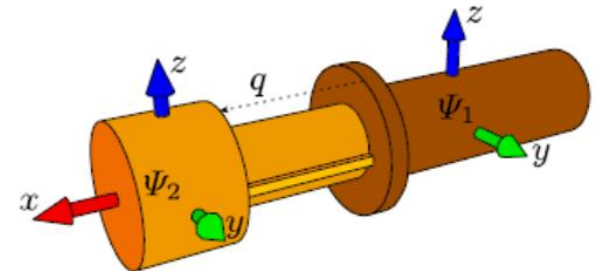
Recap: Exponential Map

- The maps

$$\begin{aligned} \varphi_c: G_c &\rightarrow G_c^p \\ q_c &\mapsto H_c^p \end{aligned}$$

$$\begin{aligned} \iota_c: \mathfrak{g}_c &\rightarrow \mathfrak{g}_c^p \\ v_c &\mapsto \mathcal{V}_c^{*,p} \end{aligned}$$

can be geometrically (and elegantly) represented using the **exponential map** of a Lie group.



b DoF joint constraints $n - b$ DoFs



Recap: Exponential Coordinates of Rotation

- Angular velocity - \mathbb{R}^3

$$\omega_c^{*,p} = \hat{n}_c^{*,p} \dot{\theta}_c$$

- Rotation matrix - $SO(3)$

$$R_c^p(\theta_c) = e^{\tilde{n}_c^{p,p} \theta_c} R_c^p(0) \quad \text{or} \quad R_c^p(\theta_c) = R_c^p(0) e^{\tilde{n}_c^{c,p} \theta_c}$$

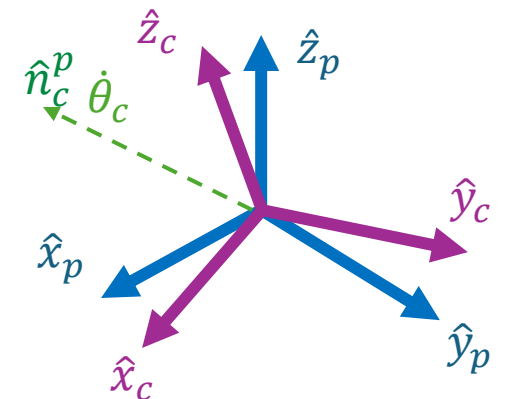
- Exponential map

$$\text{exp: } so(3) \rightarrow SO(3)$$

$$\tilde{n}\theta \mapsto e^{\tilde{n}\theta}$$

Rodrigues's formula (Closed form expression for exp)

$$e^{\tilde{n}\theta} = I_3 + \sin \theta \tilde{n} + (1 - \cos \theta) \tilde{n}^2$$



Recap: Exponential Coordinates of R.B. Motion

- Twist - \mathbb{R}^6

$$\mathcal{V}_c^{*,p} = \mathcal{S}_c^{*,p} \dot{q}_c$$

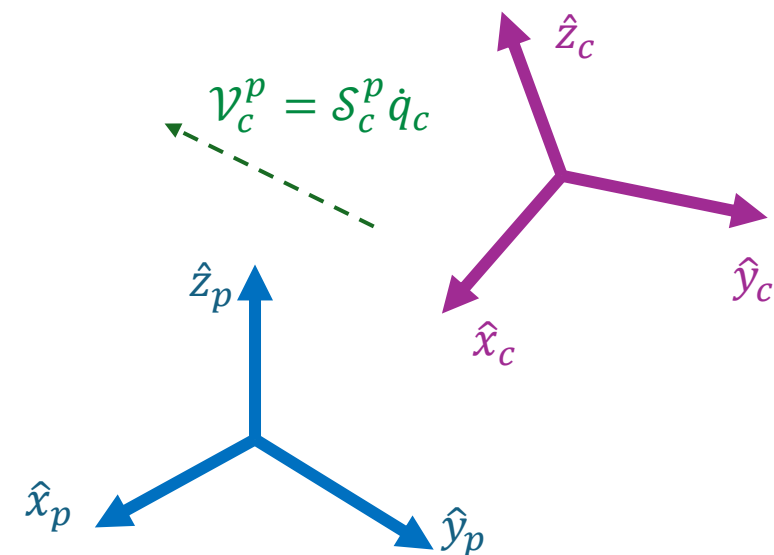
- Homogeneous transformation - $SE(3)$

$$H_c^p(q_c) = e^{\tilde{\mathcal{S}}_c^{p,p} q_c} H_c^p(0) \quad \text{or} \quad H_c^p(q_c) = H_c^p(0) e^{\tilde{\mathcal{S}}_c^{c,p} q_c}$$

- Exponential map

$$\exp: se(3) \rightarrow SE(3)$$

$$\tilde{\mathcal{S}}q \mapsto e^{\tilde{\mathcal{S}}q}$$



Recap: Exponential Map on SE(3)

- The exponential map

$$\begin{aligned}\exp: se(3) &\rightarrow SE(3) \\ \tilde{S}q &\mapsto e^{\tilde{S}q}\end{aligned}$$

Case 1: (Pure translation)

Let $S := (0, v) \in \mathbb{R}^6$ be a screw axis with $\|v\| = 1$. Then for any $q \in \mathbb{R}$ along that screw axis, we have that

$$e^{\tilde{S}q} = \begin{pmatrix} I & vq \\ 0 & 1 \end{pmatrix}$$

Case 2: (Pure rotation)

Let $S := (\omega, v) \in \mathbb{R}^6$ be a screw axis with $\|\omega\| = 1$. Then for any $q \in \mathbb{R}$ along that screw axis, we have that

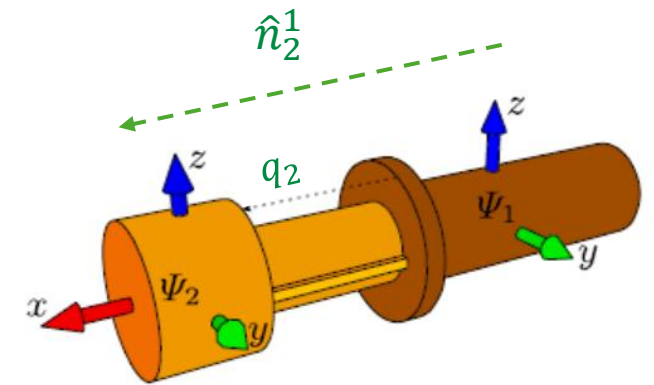
$$e^{\tilde{S}q} = \begin{pmatrix} e^{\tilde{\omega}q} & (Iq + (1 - \cos q)\tilde{\omega} + (q - \sin q)\tilde{\omega}^2)v \\ 0 & 1 \end{pmatrix}$$



Recap: Modeling Ideal Joints

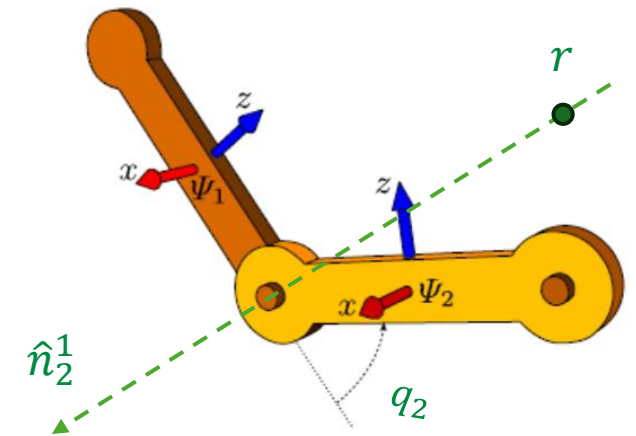
- Prismatic joints

- $\mathcal{S}_c^{*,p} = \begin{pmatrix} 0 \\ \hat{n}_c^{*,p} \end{pmatrix} \in \mathbb{R}^6$,
- $\hat{n}_c^{*,p}$ is the translation axis
- \dot{q}_c is the linear velocity along the screw axis



- Revolute joints

- $\mathcal{S}_c^{*,p} = \begin{pmatrix} \hat{n}_c^{*,p} \\ -\hat{n}_c^{*,p} \wedge r^* \end{pmatrix} \in \mathbb{R}^6$,
- \hat{n}_c^p is the rotation axis
- \dot{q}_c is the angular velocity around the screw axis
- r is any point on the screw axis



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Forward Kinematics: 3R Robot

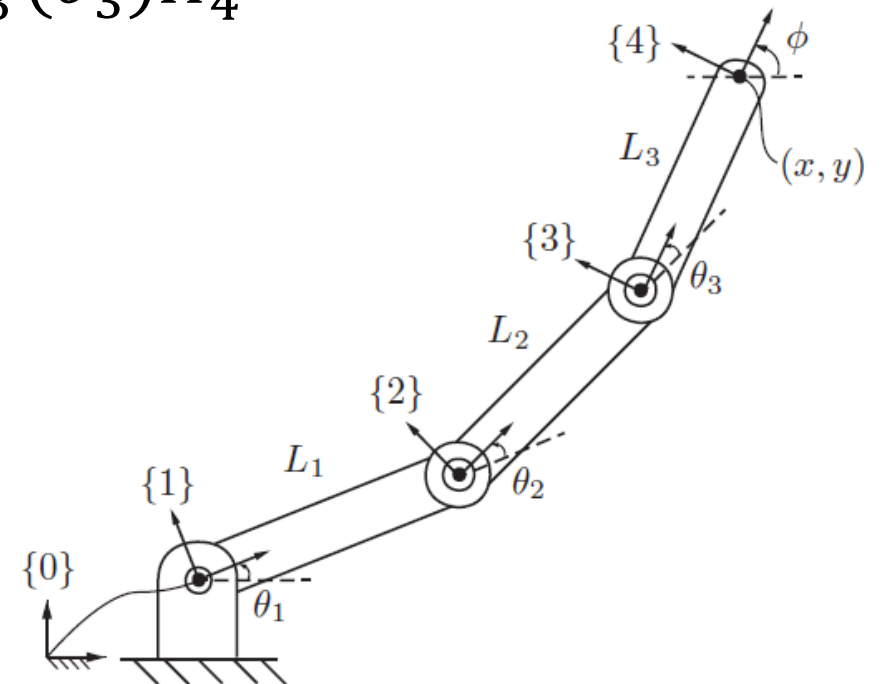
- A systematic way to derive the forward kinematics map

$$(\theta_1, \theta_2, \theta_3) \mapsto H_4^0$$

is to attach a reference frame to each link {1}, {2}, {3}.

- Then we have that:

$$H_4^0(\theta) = H_1^0(\theta_1)H_2^1(\theta_2)H_3^2(\theta_3)H_4^3$$



Forward Kinematics: 3R Robot

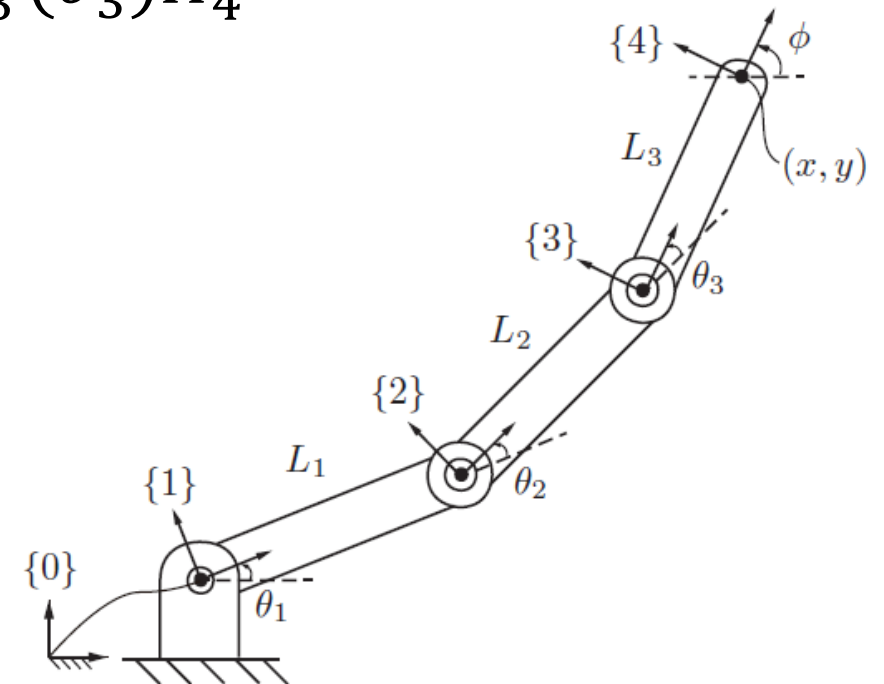
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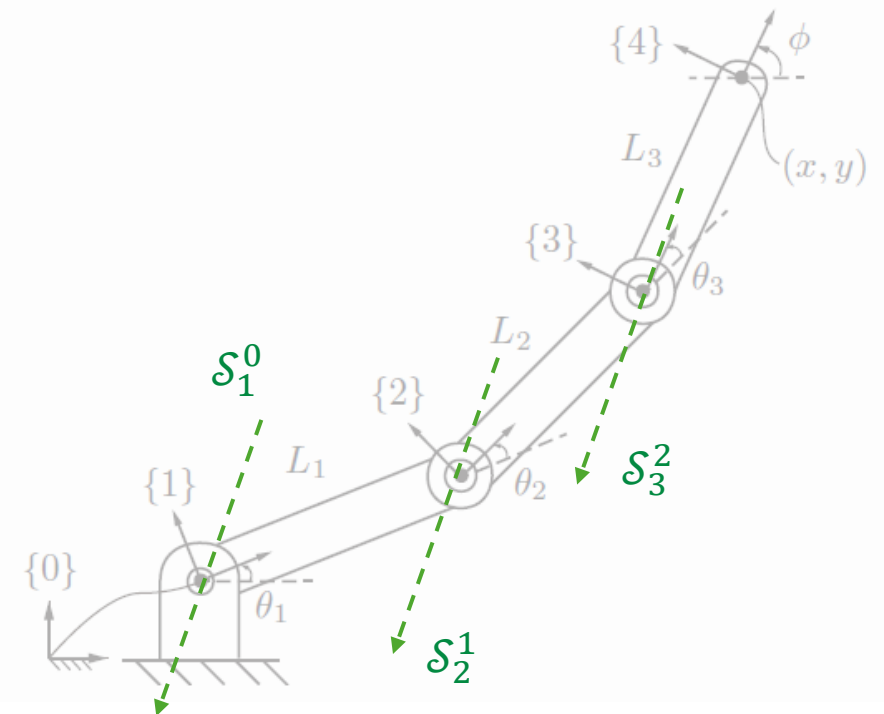
A widely used convention for choosing the body-fixed frames $\{1\}, \{2\}, \{3\}$ is based on the so-called Denavit–Hartenberg parameters (D–H parameters). We will use a more powerful approach based on the **exponential map**.



Forward Kinematics: 3R Robot

- Using the exponential coordinates, we have that

$$\begin{aligned}
 H_4^0(\theta) &= H_1^0(\theta_1)H_2^1(\theta_2)H_3^2(\theta_3)H_4^3 \\
 &= e^{\tilde{\mathcal{S}}_1^{0,0}\theta_1}H_1^0(0)e^{\tilde{\mathcal{S}}_2^{1,1}\theta_2}H_2^1(0)e^{\tilde{\mathcal{S}}_3^{2,2}\theta_3}H_3^2(0)H_4^3
 \end{aligned}$$



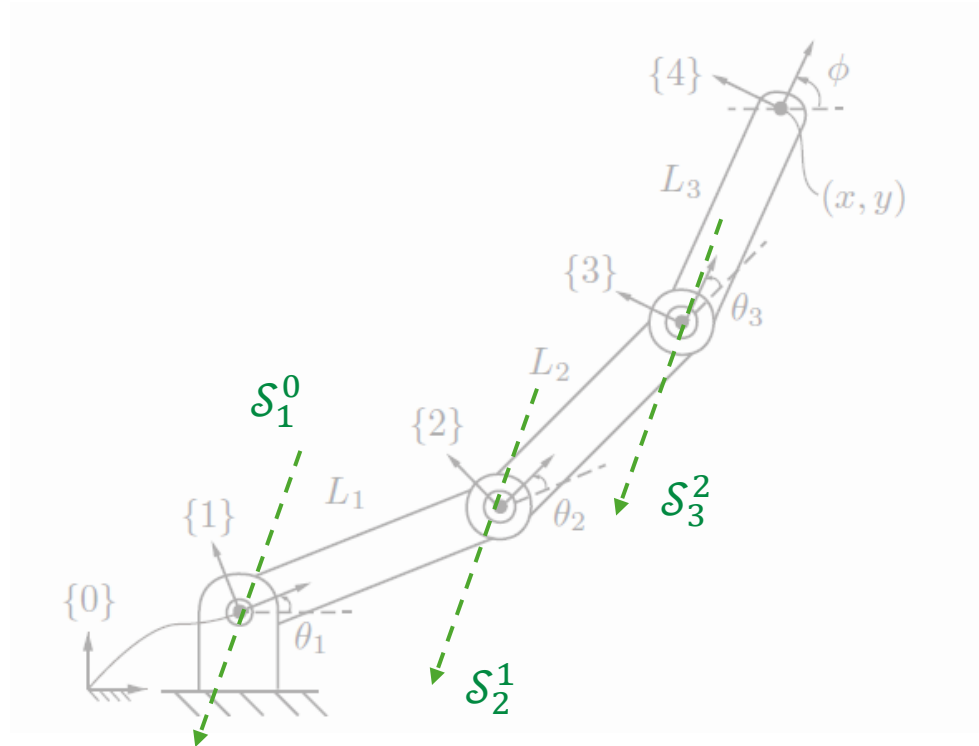
$$H_c^p(q_c) = e^{\tilde{\mathcal{S}}_c^{p,p}q_c}H_c^p(0) \quad \text{or} \quad H_c^p(q_c) = H_c^p(0)e^{\tilde{\mathcal{S}}_c^{c,p}q_c}$$



Forward Kinematics: 3R Robot

- Using the exponential coordinates, we have that

$$\begin{aligned} H_4^0(\theta) &= H_1^0(\theta_1)H_2^1(\theta_2)H_3^2(\theta_3)H_4^3 \\ &= e^{\tilde{\mathcal{S}}_1^{0,0}\theta_1}H_1^0(0)e^{\tilde{\mathcal{S}}_2^{1,1}\theta_2}H_2^1(0)e^{\tilde{\mathcal{S}}_3^{2,2}\theta_3}H_3^2(0)H_4^3 \\ &\stackrel{*}{=} e^{\tilde{\mathcal{S}}_1^{0,0}\theta_1}e^{\tilde{\mathcal{S}}_2^{0,1}\theta_2}e^{\tilde{\mathcal{S}}_3^{0,2}\theta_3}H_4^0(0) \end{aligned}$$



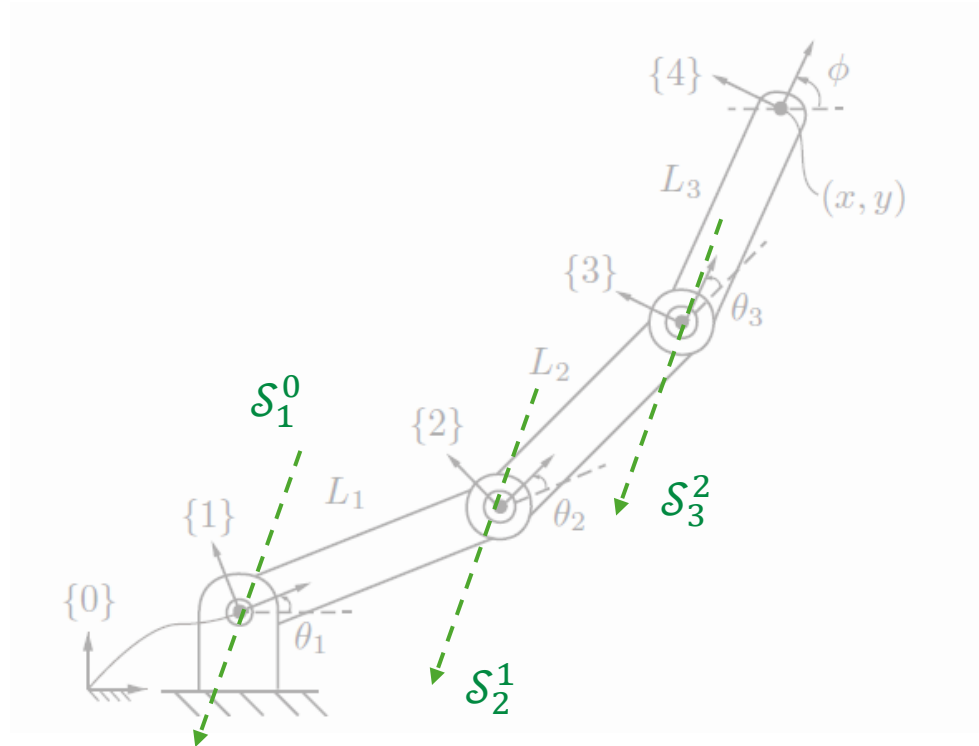
* Homework Problem



Forward Kinematics: 3R Robot

- Alternatively, we also have that

$$\begin{aligned}
 H_4^0(\theta) &= H_1^0(\theta_1)H_2^1(\theta_2)H_3^2(\theta_3)H_4^3 \\
 &= H_1^0(0)e^{\tilde{\mathcal{S}}_1^{1,0}\theta_1}H_2^1(0)e^{\tilde{\mathcal{S}}_2^{2,1}\theta_2}H_3^2(0)e^{\tilde{\mathcal{S}}_3^{3,2}\theta_3}H_4^3 \\
 &\stackrel{*}{=} H_4^0(0)e^{\tilde{\mathcal{S}}_1^{4,0}\theta_1}e^{\tilde{\mathcal{S}}_2^{4,1}\theta_2}e^{\tilde{\mathcal{S}}_3^{4,2}\theta_3}
 \end{aligned}$$



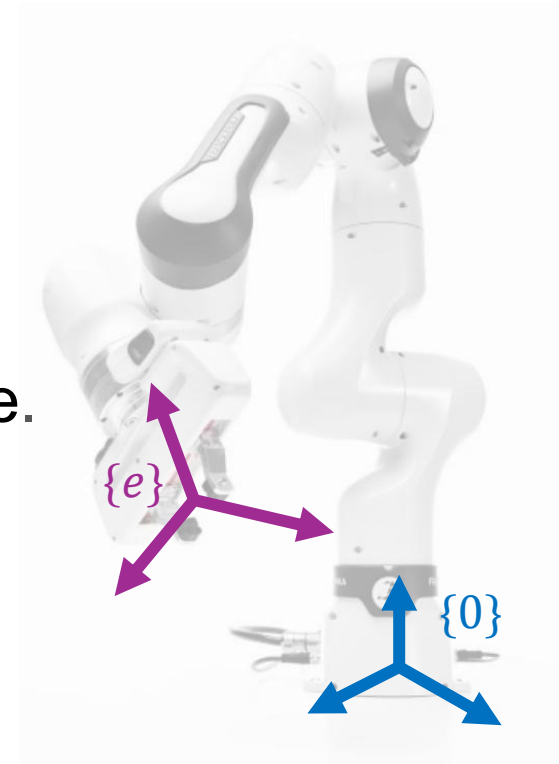
$$H_c^p(q_c) = e^{\tilde{\mathcal{S}}_c^{p,p}q_c}H_c^p(0) \quad \text{or} \quad H_c^p(q_c) = H_c^p(0)e^{\tilde{\mathcal{S}}_c^{c,p}q_c}$$

Product of Exponentials (PoE) Formula

- Thus, the forward kinematics usually has the **spatial** form

$$H_e^0(q) = e^{\tilde{\mathcal{S}}_1^{0,0} q_1} e^{\tilde{\mathcal{S}}_2^{0,1} q_2} \dots e^{\tilde{\mathcal{S}}_n^{0,n-1} q_n} H_e^0(0)$$

- This is called the product of exponentials formula.
- You need to provide:
 - The end-effector pose $H_e^0(0)$ when the robot is at its *home* configuration.
 - The screw axes $\mathcal{S}_1^0, \mathcal{S}_2^1, \dots, \mathcal{S}_n^{n-1}$, corresponding to the joint motions at *home* configuration, expressed in base $\{0\}$ frame.
 - The joint configurations q .

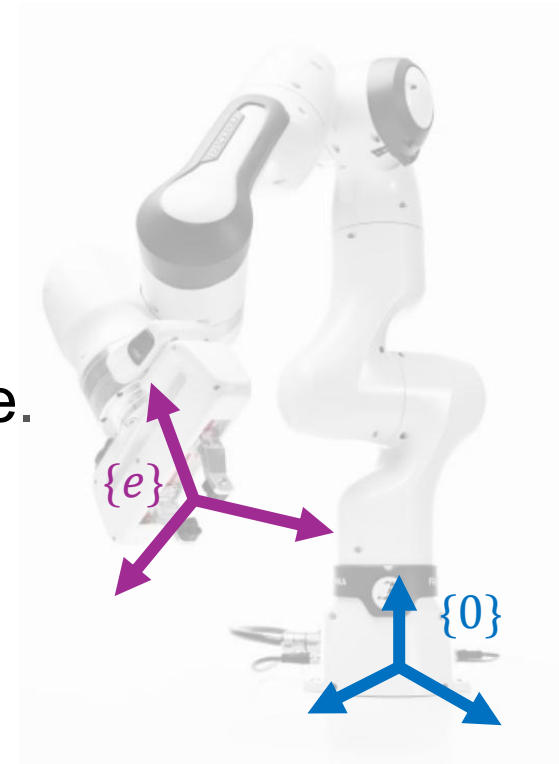


Product of Exponentials (PoE) Formula

- Alternatively, the forward kinematics also has the **end-effector** form

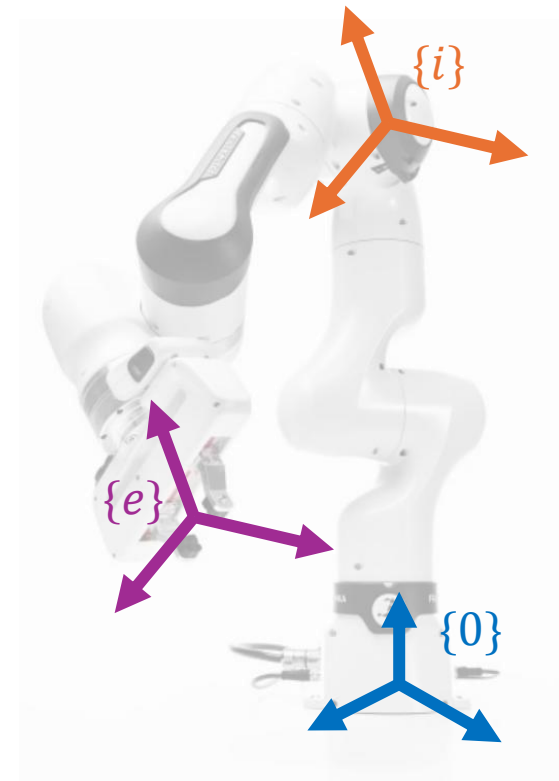
$$H_e^0(q) = H_e^0(0) e^{\tilde{\mathcal{S}}_1^{e,0} q_1} e^{\tilde{\mathcal{S}}_2^{e,1} q_2} \dots e^{\tilde{\mathcal{S}}_n^{e,n-1} q_n}$$

- This is called the product of exponentials formula.
- You need to provide:
 - The end-effector pose $H_e^0(0)$ when the robot is at its *home* configuration.
 - The screw axes $\mathcal{S}_1^0, \mathcal{S}_2^1, \dots, \mathcal{S}_n^{n-1}$, corresponding to the joint motions at *home* configuration, expressed in base $\{e\}$ frame.
 - The joint configurations q .



Pose of Link i

- To compute forward kinematics using PoE formula, you need stationary frame $\{0\}$ and a frame at the end-effector $\{e\}$ only.
- However, it is common to define a frame at every link.
- Typically, either at:
 - Parent Joint
 - Link's CoM

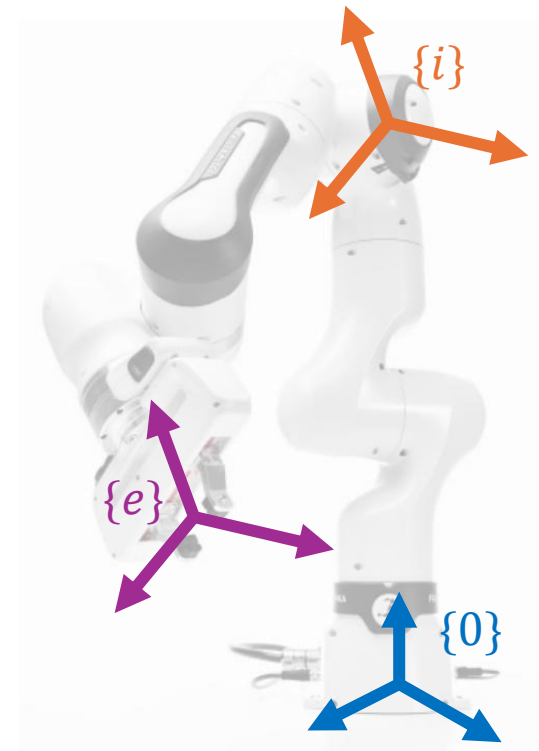


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- To compute forward kinematics using PoE formula, you need stationary frame $\{0\}$ and a frame at the end-effector $\{e\}$ only.
- However, it is common to define a frame at every link.
- Typically, either at:
 - Parent Joint
 - Link's CoM
- The pose of link $\{i\}$ can be computed by

$$H_i^0(q_{1:i}) = e^{\tilde{\mathcal{S}}_1^{0,0} q_1} e^{\tilde{\mathcal{S}}_2^{0,1} q_2} \dots e^{\tilde{\mathcal{S}}_i^{0,i-1} q_i} H_i^0(0)$$

$$q_{1:i} := (q_1, \dots, q_i)$$



Outline

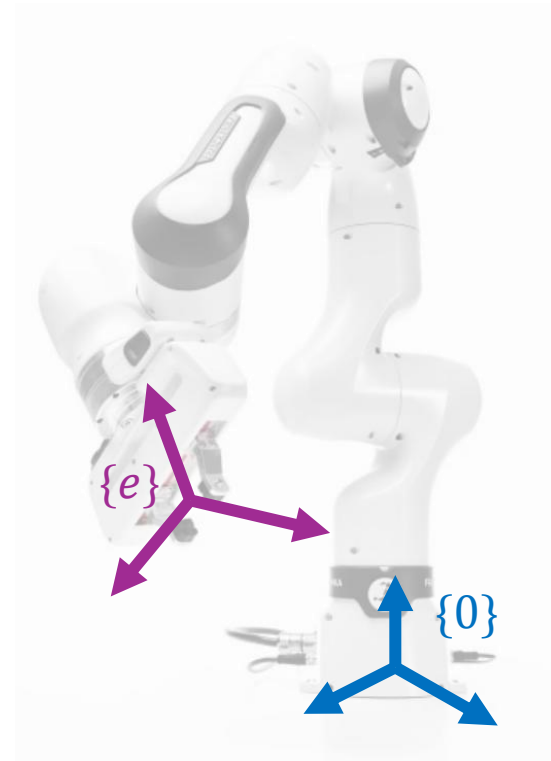
- Recap last lecture
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- **DK using Geometric Jacobian**
- Example



Differential Kinematics: 3R Robot

- Geometrically, the differential kinematics as a map from joint velocities $\dot{q} \in \mathbb{R}^n$ to the end effector's twist $\mathcal{V}_e^{*,0} \in \mathbb{R}^6$.
- This is represented by the so-called **geometric Jacobian**:

$$\mathcal{V}_e^{*,0} = J_e^{*,0}(q) \dot{q}$$



$$J_e^{*,0}(q) \in \mathbb{R}^{6 \times n}$$



Differential Kinematics: 3R Robot

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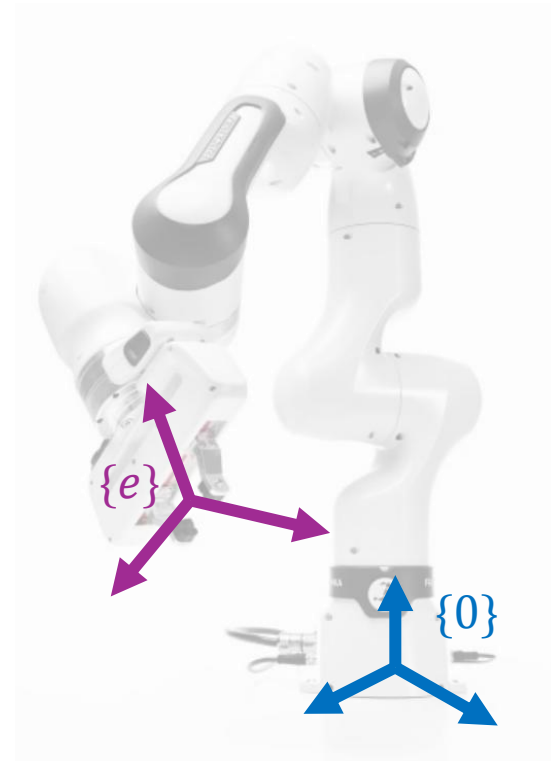
$$\mathcal{V}_e^{*,0} = J_e^{*,0}(q) \dot{q}$$

$$\mathcal{V}_e^{0,0} = J_e^{0,0}(q) \dot{q}$$

Spatial Jacobian

$$\mathcal{V}_e^{e,0} = J_e^{e,0}(q) \dot{q}$$

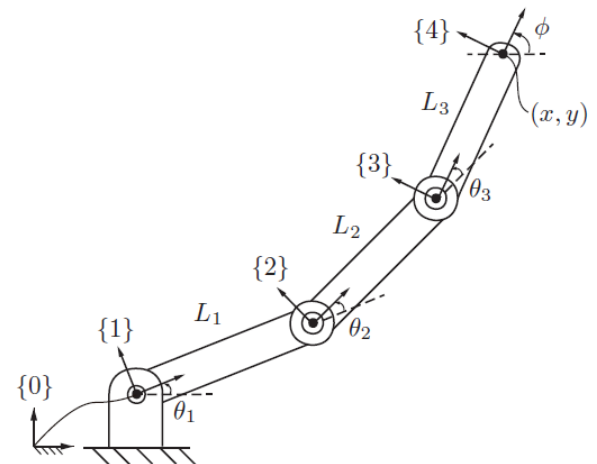
End-effector/Body Jacobian



End effector spatial twist

- Recall that relative twists can be added:

$$\mathcal{V}_e^{0,0} = \mathcal{V}_1^{0,0} + \mathcal{V}_2^{0,1} + \dots + \mathcal{V}_n^{0,n-1} + \mathcal{V}_e^{0,n}$$

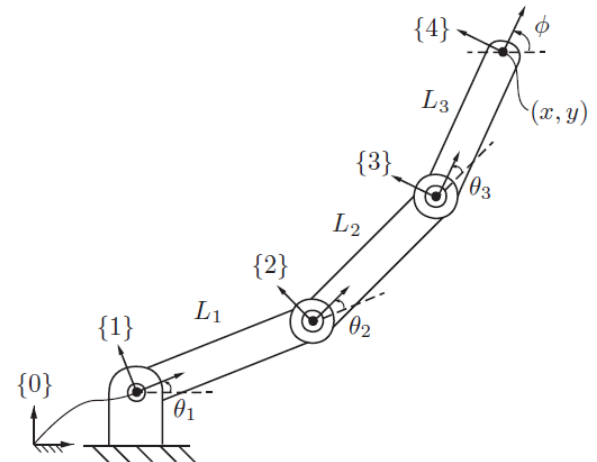


End effector spatial twist

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$$\mathcal{V}_e^{0,0} = \mathcal{V}_1^{0,0} + \mathcal{V}_2^{0,1} + \dots + \mathcal{V}_n^{0,n-1} + \mathcal{V}_e^{0,n}$$

- First, $\mathcal{V}_e^{0,n} = 0$, since both are rigidly attached to the n -th link.



End effector spatial twist

- Recall that relative twists can be added:

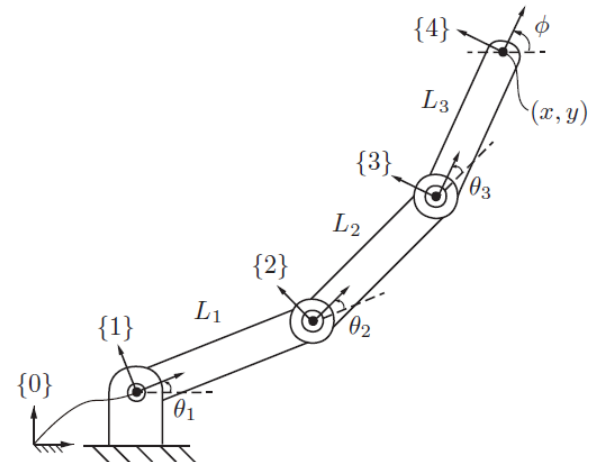
$$\mathcal{V}_e^{0,0} = \mathcal{V}_1^{0,0} + \mathcal{V}_2^{0,1} + \dots + \mathcal{V}_n^{0,n-1}$$

- We have that:

- $\mathcal{V}_c^{0,p} = \text{Ad}_{H_p^0(q_{1:p})} \mathcal{V}_c^{p,p}$

- $\mathcal{V}_c^{p,p} = \mathcal{S}_c^{p,p} \dot{q}_c$

- $H_p^0(q_{1:p}) = e^{\tilde{\mathcal{S}}_1^{0,0} q_1} e^{\tilde{\mathcal{S}}_2^{0,1} q_2} \dots e^{\tilde{\mathcal{S}}_p^{0,p-1} q_p} H_p^0(0)$



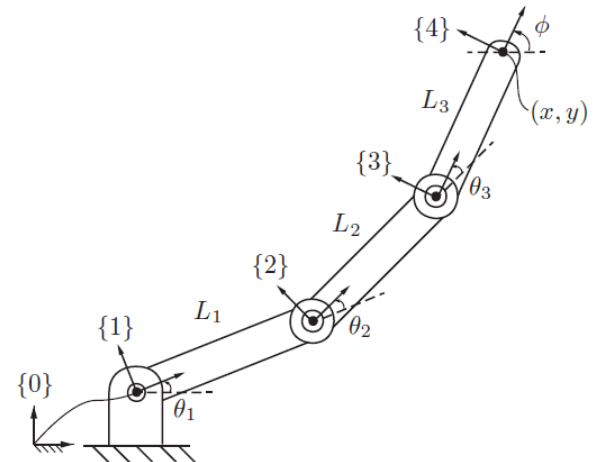
End effector spatial twist

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$$\mathcal{V}_e^{0,0} = \mathcal{V}_1^{0,0} + \mathcal{V}_2^{0,1} + \dots + \mathcal{V}_n^{0,n-1}$$

- We have that:

$$\begin{aligned} \bullet \mathcal{V}_c^{0,p} &= \left(\text{Ad}_{e^{\tilde{s}_1^{0,0} q_1} e^{\tilde{s}_2^{0,1} q_2} \dots e^{\tilde{s}_p^{0,p-1} q_p} H_p^0(0)} \right) \mathcal{S}_c^{p,p} \dot{q}_c \\ &= \left(\text{Ad}_{e^{\tilde{s}_1^{0,0} q_1} e^{\tilde{s}_2^{0,1} q_2} \dots e^{\tilde{s}_p^{0,p-1} q_p}} \right) \left(\text{Ad}_{H_p^0(0)} \right) \mathcal{S}_c^{p,p} \dot{q}_c \end{aligned}$$



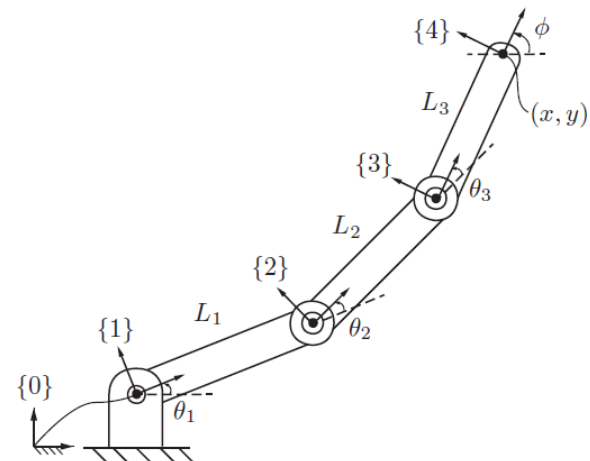
End effector spatial twist

- Recall that relative twists can be added:

$$\mathcal{V}_e^{0,0} = \mathcal{V}_1^{0,0} + \mathcal{V}_2^{0,1} + \dots + \mathcal{V}_n^{0,n-1}$$

- We have that:

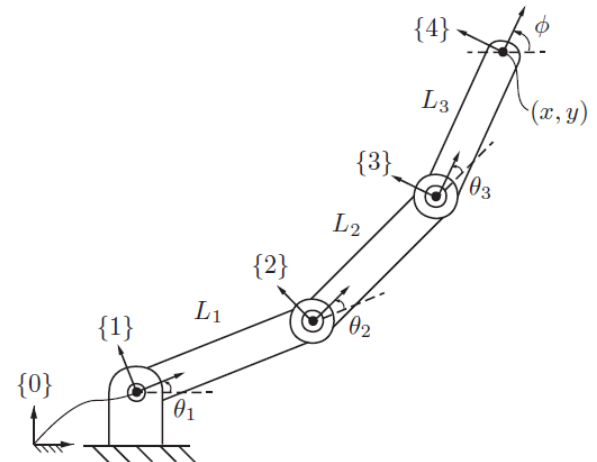
$$\begin{aligned} \bullet \mathcal{V}_c^{0,p} &= \left(\text{Ad}_{e^{\tilde{\mathcal{S}}_1^{0,0} q_1} e^{\tilde{\mathcal{S}}_2^{0,1} q_2} \dots e^{\tilde{\mathcal{S}}_p^{0,p-1} q_p} H_p^0(0)} \right) \mathcal{S}_c^{p,p} \dot{q}_c \\ &= \left(\text{Ad}_{e^{\tilde{\mathcal{S}}_1^{0,0} q_1} e^{\tilde{\mathcal{S}}_2^{0,1} q_2} \dots e^{\tilde{\mathcal{S}}_p^{0,p-1} q_p}} \right) \left(\text{Ad}_{H_p^0(0)} \right) \mathcal{S}_c^{p,p} \dot{q}_c \\ &= \left(\text{Ad}_{e^{\tilde{\mathcal{S}}_1^{0,0} q_1} e^{\tilde{\mathcal{S}}_2^{0,1} q_2} \dots e^{\tilde{\mathcal{S}}_p^{0,p-1} q_p}} \right) \mathcal{S}_c^{0,p} \dot{q}_c \end{aligned}$$



End effector spatial twist

- Therefore, we have that

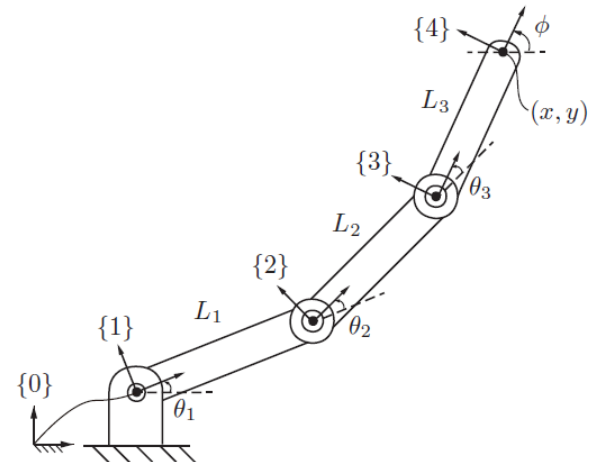
$$\begin{aligned}\mathcal{V}_e^{0,0} &= \mathcal{V}_1^{0,0} + \mathcal{V}_2^{0,1} + \dots + \mathcal{V}_n^{0,n-1} \\ &= \mathcal{V}_1^{0,0} + Ad_{H_1^0(q_1)} \mathcal{V}_2^{1,1} + \dots + Ad_{H_{n-1}^0(q_{1:n-1})} \mathcal{V}_n^{n-1,n-1} \\ &= \mathcal{S}_1^{0,0} \dot{q}_1 + Ad_{e^{\tilde{\mathcal{S}}_1^{0,0} q_1}} \mathcal{S}_2^{0,1} \dot{q}_2 + \dots + Ad_{e^{\tilde{\mathcal{S}}_1^{0,0} q_1} e^{\tilde{\mathcal{S}}_2^{0,1} q_2} \dots e^{\tilde{\mathcal{S}}_{n-1}^{0,n-2} q_{n-1}}} \mathcal{S}_n^{0,n-1} \dot{q}_n\end{aligned}$$



End effector spatial twist

- Therefore, we have that

$$\begin{aligned}
 \mathcal{V}_e^{0,0} &= \mathcal{V}_1^{0,0} + \mathcal{V}_2^{0,1} + \dots + \mathcal{V}_n^{0,n-1} \\
 &= \mathcal{V}_1^{0,0} + Ad_{H_1^0(q_1)} \mathcal{V}_2^{1,1} + \dots + Ad_{H_{n-1}^0(q_{1:n-1})} \mathcal{V}_n^{n-1,n-1} \\
 &= \mathcal{S}_1^{0,0} \dot{q}_1 + Ad_{e^{\tilde{\mathcal{S}}_1^{0,0} q_1}} \mathcal{S}_2^{0,1} \dot{q}_2 + \dots + Ad_{e^{\tilde{\mathcal{S}}_1^{0,0} q_1} e^{\tilde{\mathcal{S}}_2^{0,1} q_2} \dots e^{\tilde{\mathcal{S}}_{n-1}^{0,n-2} q_{n-1}}} \mathcal{S}_n^{0,n-1} \dot{q}_n \\
 &= J_1^0 \dot{q}_1 + J_2^0(q_1) \dot{q}_2 + \dots + J_n^0(q_{1:n-1}) \dot{q}_n
 \end{aligned}$$



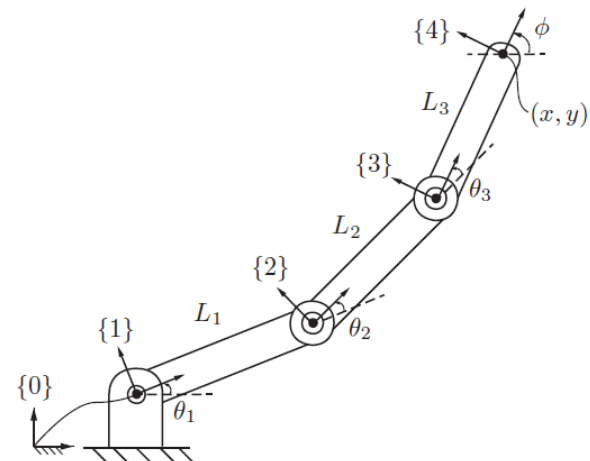
End effector spatial twist

- Therefore, we have that

$$\begin{aligned}
 \mathcal{V}_e^{0,0} &= \mathcal{V}_1^{0,0} + \mathcal{V}_2^{0,1} + \dots + \mathcal{V}_n^{0,n-1} \\
 &= \mathcal{V}_1^{0,0} + Ad_{H_1^0(q_1)} \mathcal{V}_2^{1,1} + \dots + Ad_{H_{n-1}^0(q_{1:n-1})} \mathcal{V}_n^{n-1,n-1} \\
 &= \mathcal{S}_1^{0,0} \dot{q}_1 + Ad_{e^{\tilde{\mathcal{S}}_1^{0,0} q_1}} \mathcal{S}_2^{0,1} \dot{q}_2 + \dots + Ad_{e^{\tilde{\mathcal{S}}_1^{0,0} q_1} e^{\tilde{\mathcal{S}}_2^{0,1} q_2} \dots e^{\tilde{\mathcal{S}}_{n-1}^{0,n-2} q_{n-1}}} \mathcal{S}_n^{0,n-1} \dot{q}_n \\
 &= J_1^0 \dot{q}_1 + J_2^0(q_1) \dot{q}_2 + \dots + J_n^0(q_{1:n-1}) \dot{q}_n \\
 &= \begin{pmatrix} J_1^0 & J_2^0(q_1) & \dots & J_n^0(q_{1:n-1}) \end{pmatrix} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{pmatrix} \\
 &= J_e^{0,0}(q) \dot{q}
 \end{aligned}$$

1st column
2nd column
n-th column

The geometric Jacobian expressed in the stationary frame (aka the spatial Jacobian)



Summary

- The forward kinematics of an n -link open chain manipulator is expressed by

$$H_e^0(q) = e^{\tilde{\mathcal{S}}_1^{0,0} q_1} e^{\tilde{\mathcal{S}}_2^{0,1} q_2} \dots e^{\tilde{\mathcal{S}}_n^{0,n-1} q_n} H_e^0(0)$$

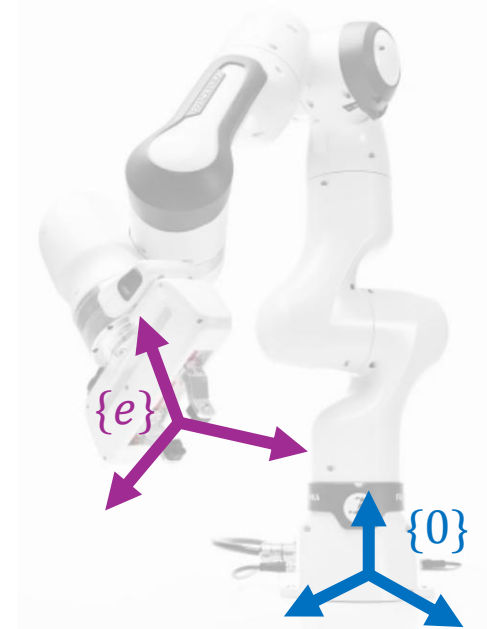
- The spatial Jacobian $J_e^{0,0}(q) \in \mathbb{R}^{6 \times n}$ relates the joint rates $\dot{\theta} \in \mathbb{R}^n$ to the spatial end effector's twist $\mathcal{V}_n^{0,0} \in \mathbb{R}^6$ by

$$\mathcal{V}_e^{0,0} = J_e^{0,0}(q) \dot{q}$$

- The i -th column of $J_e^{0,0}(q)$ is given for $i \in \{2, \dots, n\}$ by

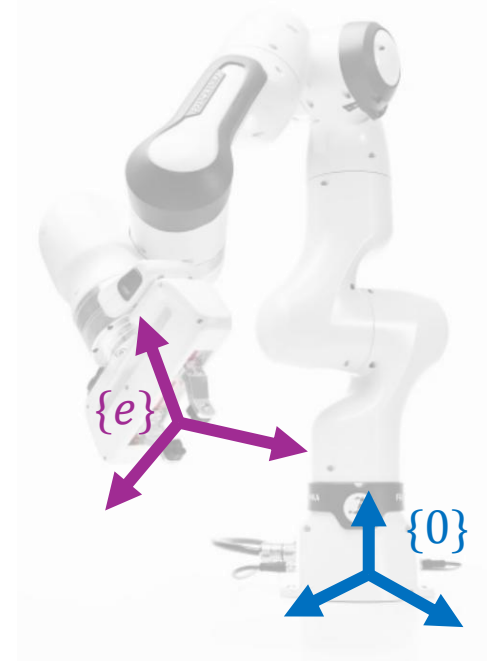
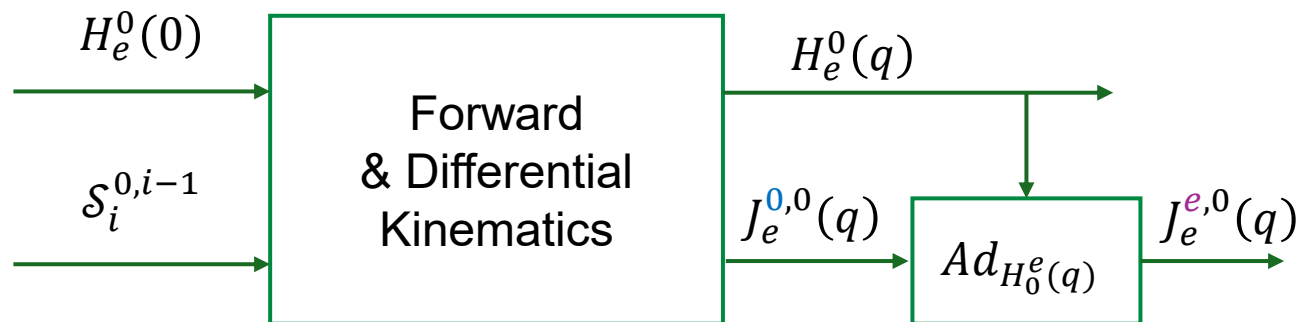
$$J_i^0(q_{1:i-1}) = \text{Ad}_{e^{\tilde{\mathcal{S}}_1^{0,0} q_1} e^{\tilde{\mathcal{S}}_2^{0,1} q_2} \dots e^{\tilde{\mathcal{S}}_{i-1}^{0,i-2} q_{i-1}}} \mathcal{S}_i^{0,i-1}$$

with the 1st column $J_1^0 = \mathcal{S}_1^{0,0}$.



Summary

- The above approach is highly systematic and can be easily programmable.
- The only inputs needed is the end-effector initial poses $H_e^0(0)$ and the constant screw axes $\mathcal{S}_i^{0,i-1}$ for joints in the home configuration.
- Since each column of the spatial Jacobian $J_e^{0,0}(q)$ is a twist expressed in $\{0\}$, we can compute the end-effector Jacobian $J_e^{e,0}(q)$ simply by the Adjoint transformation $Ad_{H_0^e(q)}$.

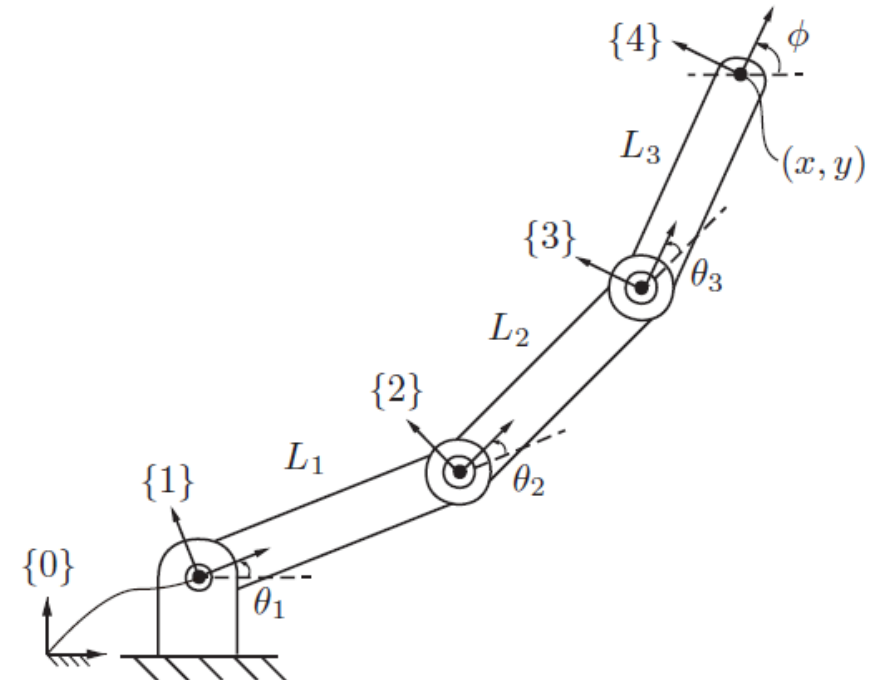


Outline

- Recap last lecture
- FK using Product of Exponentials (PoE) Formula
- DK using Geometric Jacobian
- **Example**



Example 1: 3R Planar Robot



Example 2: 3R Spatial Robot

