1) Pendulum:

$$\Rightarrow x = (x_1, x_2) := (\theta, \dot{\theta})$$

$$\Rightarrow S^1 \times \mathbb{R}^1 \cong [-\pi, \pi) \times \mathbb{R} =: X$$

$$\Rightarrow \dot{x} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ -\frac{g}{l}\sin(x_1) - \frac{b}{ml^2} x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1/ml^2 \end{pmatrix} \tau$$

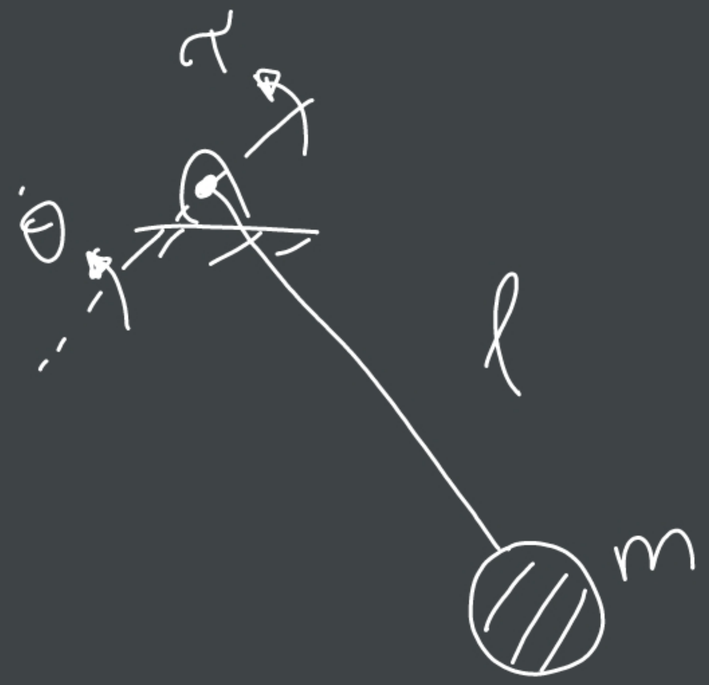
$\ddot{\theta} =$

$f(x)$ $\qquad$ g

$$\Rightarrow \boxed{\dot{x} = f(x) + g\tau}$$

$$\Rightarrow \boxed{ml^2\ddot{\theta} + b\dot{\theta} + mgl\sin\theta = \tau}$$

$\theta \in [-\pi, \pi) \approx S^1$, $\dot{\theta} \in \mathbb{R}$

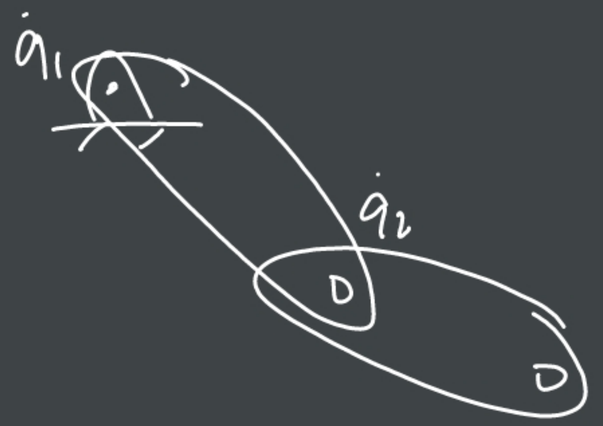


2) n-link manipulator:

$$x := (x_1, x_2) = (q, \dot{q})$$

$$X = TQ \rightarrow \dim X = 2n$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \underbrace{\begin{pmatrix} x_2 \\ -M^{-1}(x_1)\left[C(x_1, x_2) + g(x_1)\right] \end{pmatrix}}_{f(x)} + \underbrace{\begin{pmatrix} 0 \\ M^{-1}(x_1) \end{pmatrix}}_{g(x)} \tau$$



$$M(q)\ddot{q} + C(q, \dot{q}) + g(q) = \tau$$

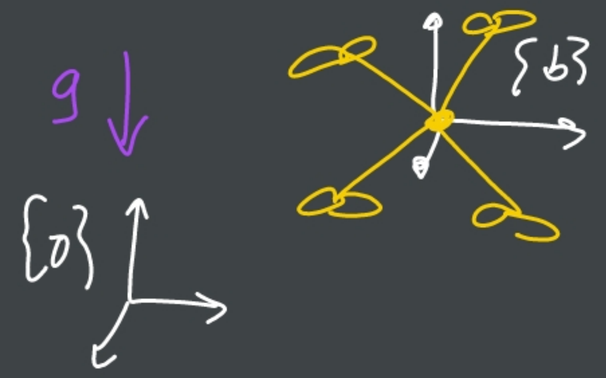
$q \in Q$ (n-dim), $\dot{q} \in T_q Q \cong \mathbb{R}^n$ (n-dim)

5) MAV:

$x = (x_1, x_2) = (H, \nu)$

$\mathcal{X} = SE(3) \times \mathbb{R}^6 \cong TSE(3)$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \underbrace{\begin{pmatrix} B(x_1, x_2) \\ \mathcal{I}^{-1}\left(ad^T_{x_2}\mathcal{I}x_2 + W_{grv}(x_1)\right) \end{pmatrix}}_{f(x)} + \underbrace{\begin{pmatrix} 0 \\ \mathcal{I}^{-1} \end{pmatrix}}_{g} W_{prop}$$



$$\dot{H} = H\tilde{\nu}$$

$$\mathcal{I}\dot{\nu} = ad^T_{\nu}\mathcal{I}\nu + W_{prop} + W_{grv}(R)$$

$H \in SE(3)$

$\nu \in \mathbb{R}^6 \rightsquigarrow \dot{H} \in T_H SE(3)$