1) Compute $d\Psi(R)$:

$$\frac{d}{dt}\left(A(t)\,B(t)\right) = \dot{A}B + A\dot{B}$$

- $\Psi(R) = \frac{1}{2}\operatorname{tr}(I - R_d^T R)$

- $\Psi(R_\varepsilon) = \frac{1}{2}\operatorname{tr}(I - R_d^T R_\varepsilon) = \frac{1}{2}\operatorname{tr}(I) - \frac{1}{2}\operatorname{tr}(R_d^T R_\varepsilon)$

- $\left.\frac{d}{d\varepsilon}\right|_{\varepsilon=0} = \frac{-1}{2}\left.\frac{d}{d\varepsilon}\right|_{\varepsilon=}\operatorname{tr}(R_d^T R_\varepsilon) = -\frac{1}{2}\operatorname{tr}\left[R_d^T \overbrace{\left(\left.\frac{d}{d\varepsilon}\right|_{\varepsilon=0} R_\varepsilon\right)}^{\delta R}\right] = -\frac{1}{2}\operatorname{tr}(R_d^T \delta R)$

- $\langle d\Psi(R) \mid \delta R\rangle := \operatorname{tr}(d\Psi(R)\cdot\delta R) \longrightarrow \boxed{d\Psi(R) = -\frac{1}{2}R_d^T}$

2) Convert $d\Psi(R) \longrightarrow \tau_p$

$$\begin{tikzcd} \delta\omega \quad \mathbb{R}^3 \arrow[r, bend left, "\beta_R"] & T_R SO(3) \quad \delta R \\ e_R \quad (\mathbb{R}^3)^* & T_R^* SO(3) \quad d\Psi(R) \arrow[l, bend left, "\beta_R^*"] \end{tikzcd}$$

$$\langle \alpha \mid A(u) \rangle$$

$$\langle d\Psi(R) \mid \delta R \rangle = \langle d\Psi(R) \mid \beta_R(\delta\omega) \rangle_{T_R SO(3)}$$

$$= \langle \beta_R^*(d\Psi(R)) \mid \delta\omega \rangle_{\mathbb{R}^3}$$

$$\langle A^*(\alpha) \mid u \rangle$$

$$\boxed{e_R := \beta_R^*(d\Psi(R)) \in (\mathbb{R}^3)^*}$$

$$\boxed{\begin{aligned} \delta R &:= \beta_R(\delta\omega) \\ &= R(\widetilde{\delta\omega}) \end{aligned}}$$

$$\langle \overset{1\times 3}{e_R} \mid \overset{3\times 1}{\delta\omega} \rangle_{\mathbb{R}^3} = \langle d\Psi(R) \mid \delta R \rangle_{T_R SO(3)} = \langle \tilde{e}_R \mid \widetilde{\delta\omega} \rangle_{so(3)}$$

$$e_R\,\delta\omega \qquad \mathrm{tr}(d\Psi(R)\ \delta R) \qquad \frac{1}{2}\mathrm{tr}(\tilde{e}_R\,\widetilde{\delta\omega})$$

$* \ e_R := \beta_R^*(d\Psi(R)) \in (\mathbb{R}^3)^*$

$* \ \delta R = R(\widetilde{\delta\omega})$

$* \ d\Psi(R) = -\frac{1}{2}R_d^T$

$$\langle d\Psi(R) \mid \delta R\rangle = -\frac{1}{2}\mathrm{tr}(\underbrace{R_d^T R}_{\in \mathbb{R}^{3\times 3}}\widetilde{\delta\omega})$$

$$= \underbrace{-\frac{1}{2}\mathrm{tr}(\mathrm{sym}(R_d^T R)\widetilde{\delta\omega})}_{=0} - \frac{1}{2}\mathrm{tr}(\mathrm{sk}(R_d^T R)\widetilde{\delta\omega})$$

$\mathrm{sym}(A) = \frac{A+A^T}{2}$

$\mathrm{sk}(A) = \frac{A-A^T}{2}$

$\forall A \in \mathbb{R}^{3\times 3}$, you have that

$A = \mathrm{sym}(A) + \mathrm{sk}(A)$

$e_R \in (\mathbb{R}^3)^*$

$\tilde{e}_R \in so^*(3)$

let $A \in \mathbb{R}^{3\times 3}_{sym}$, $B \in so(3)$

$\mathrm{tr}(AB) = 0$

$$\langle d\Psi(R) \mid S\omega \rangle_{T_R SO(3)} = -\frac{1}{2} tr\left(sk(R_d^T R)\, \widetilde{S\omega} \right)$$

$$\boxed{\tilde{e}_R = sk(R_d^T R)}$$

$\in so(3)$

$$\langle e_R \mid S\omega \rangle_{\mathbb{R}^3} = \langle \tilde{e}_R \mid \widetilde{S\omega} \rangle_{so(3)} = -\frac{1}{2} tr\left(\tilde{e}_R\, \widetilde{S\omega} \right)$$

$$* \quad \tilde{e}_R \xrightarrow{S^{-1}} e_R \longrightarrow \boxed{T_p = -k_p e_R}, \quad k_p \in \mathbb{R}_+$$