1) PD Control on Q: (Joint space)

* objective stabilize @ $x_d = (\theta_d, 0)$

* Control law $\Rightarrow$ $\tau = \tau_p + \tau_d$

* Error between θ & θ_d (Joint space) $\Rightarrow$ $\theta_e := \theta - \theta_d \in \mathbb{R}^n$

* Potential function $\Psi(\theta) = \frac{1}{2}(\theta - \theta_d)^T K_p (\theta - \theta_d) - \mathcal{E}_{Pot}(\theta)$

* Proportional torque $\boxed{\tau_p = -\nabla_\theta \Psi(\theta) = -K_p(\theta - \theta_d) + g(\theta)}$

2) Lyapunov Analysis:

$$\boxed{V : TQ \to \mathbb{R}}$$

$x_d = (\theta_d, \dot{\theta}_d = 0)$

- $$V(x) = V_{OL}(x) + \Psi(x_1)$$
  $$= \frac{1}{2} x_2^T M(x_1) x_2 + E_{Pot}(x_1) + \Psi(x_1)$$
  $$= \frac{1}{2} x_2^T M(x_1) x_2 + \underbrace{\frac{1}{2} (x_1 - x_{1,d})^T K_p (x_1 - x_{1,d})} + \underbrace{E_{Pot}(\theta) - E_{Pot}(\theta)}_{= 0}$$

- $$\overset{\overset{0}{\parallel}}{V}(x_d) = 0 + \cancel{E_{Pot}(x_{1,d})} + 0$$

- $$\nabla_\theta E_{Pot}(\theta) = g(\theta)$$

* Lyapv. Analysis:

$\boxed{B(x_2) > 0}$ friction

- $\dot{V}(x) = (\nabla_{x_1} V)^T \dot{x}_1 + (\nabla_{x_2} V)^T \dot{x}_2$

$= - x_2^T B(x_2) x_2 + x_2^T \tau_d = - x_2^T (B(x_2) + K_d) x_2$

≤ 0

- la Salle $\Rightarrow$ $(x = x_d)$ is G.A.S

$\boxed{\tau_d = -K_d x_2}$

damping term