SCE 594: Special Topics in Intelligent Automation & Robotics

Lecture 25: Impedance Control I

جامعة الملك فهد للبترول والمعادن
King Fahd University of Petroleum & Minerals

Ramy Rashad
Assistant Professor
CIE Department

Outline

- Recap last lecture
- From Motion Control to Impedance Control
- Impedance Control of a Point Mass
- Impedance Control of a n-link manipulator

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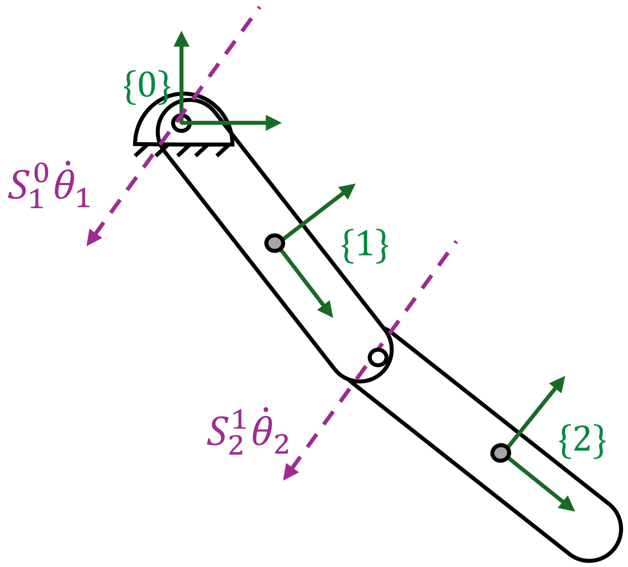
Recap: State Space Dynamics

- The governing equations of an n-link manipulator* with control torques τ are:

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + B(\dot{\theta})\dot{\theta} + g(\theta) = \tau$$

where $\theta \in Q = \mathbb{S}^1 \times \cdots \times \mathbb{S}^1 \cong \mathbb{T}^n$.

- We can cast it into state space form
  - $x = (x_1, x_2) = (\theta, \dot{\theta}) \in TQ$
  - $\begin{pmatrix}\dot{x}_1 \\ \dot{x}_2\end{pmatrix} = \begin{pmatrix} x_2 \\ -M^{-1}(x_1)\,[c(x) + b(x_2) + g(x_1)]\end{pmatrix} + \begin{pmatrix} 0 \\ M^{-1}(x_1)\end{pmatrix}\tau$



$c(x) \coloneqq C(x_1, x_2)x_2 \in \mathbb{R}^n, \qquad b(x_2) \coloneqq B(x_2)x_2 \in \mathbb{R}^n, \qquad g(x_1) \in \mathbb{R}^n$

Recap: Stabilization Control

- The control law

$$\tau = \tau_p + \tau_d$$
$$= -\nabla\Psi(x_1) - K_d x_2 = g(x_1) - K_p(x_1 - \theta_d) - K_d x_2$$

makes the closed loop system

$$\begin{pmatrix} \dot{x}_1 \\ M(x_1)\dot{x}_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ -c(x) - b(x_2) - K_p(x_1 - \theta_d) - K_d x_2 \end{pmatrix}$$

have a globally asymptotically equilibrium point at $x_d = (\theta_d, 0_2)$.

Lyapunov function: $V(x_1, x_2) = V_{OL}(x_1, x_2) + \Psi(x_1)$

$V_{OL}(x_1, x_2) = \frac{1}{2} x_2^\top M(x_1) x_2 + E_{pot}(x_1)$, $\Psi(x_1) = \frac{1}{2}(x_1 - \theta_d)^\top K_p (x_1 - \theta_d) - E_{pot}(x_1)$

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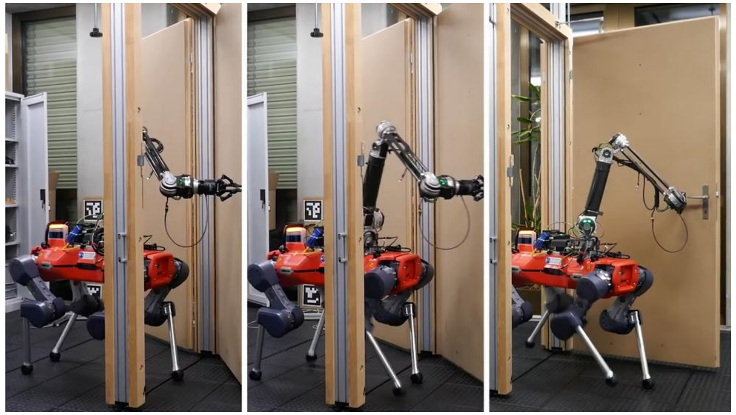
Paradigm Shift: Motion Control vs. Interaction Control

- **Motion Control**
- Main objective is to achieve stability and reject external disturbances to maximize performance.

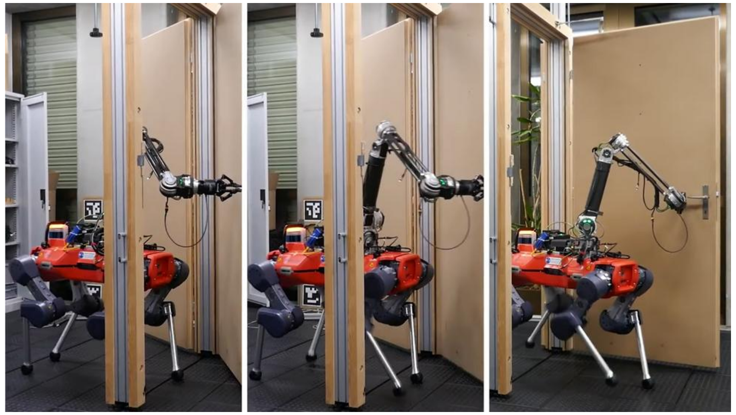
- **Interaction Control**
- Main objective is to be able to interact with an unknown environment in a stable & safe manner.



Paradigm Shift: Motion Control vs. Interaction Control

- **Motion Control**
- Closed dynamical system.
- Stability analysis requires closed loop model.

- **Interaction Control**
- Open dynamical system.
- Unknown environment needs to be incorporated in closed loop stability analysis.



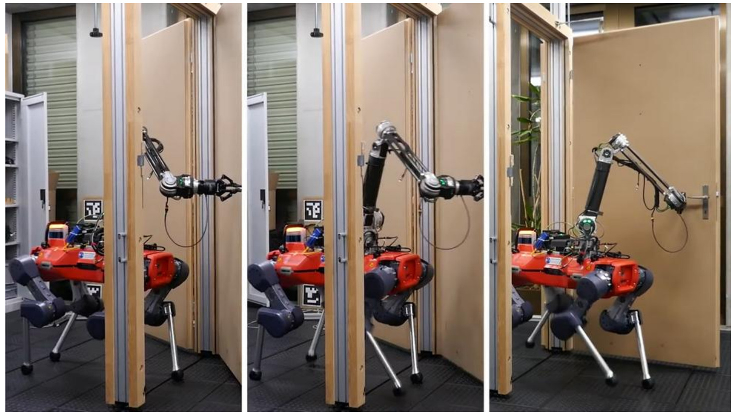
Paradigm Shift: Motion Control vs. Interaction Control

- **Motion Control**
- Based on unilateral signals.

e.g., control of position or velocity

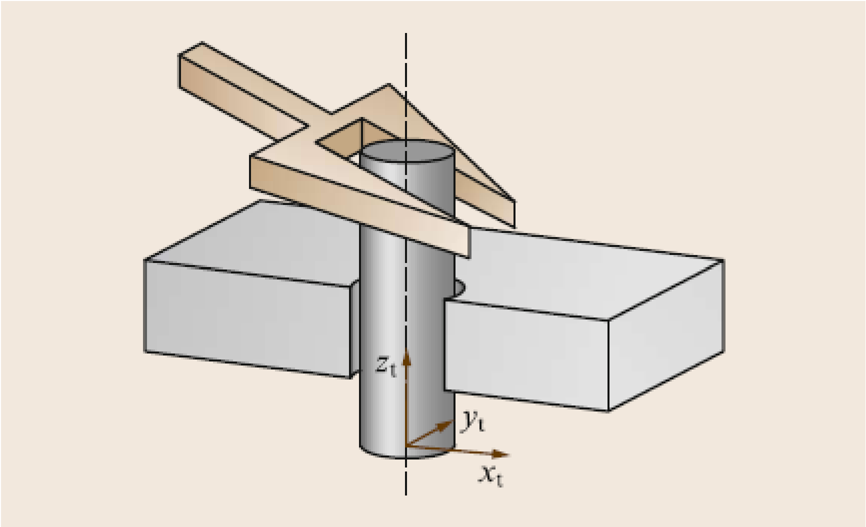
- **Interaction Control**
- Based on bilateral signals

e.g., control of relation between velocity and force.

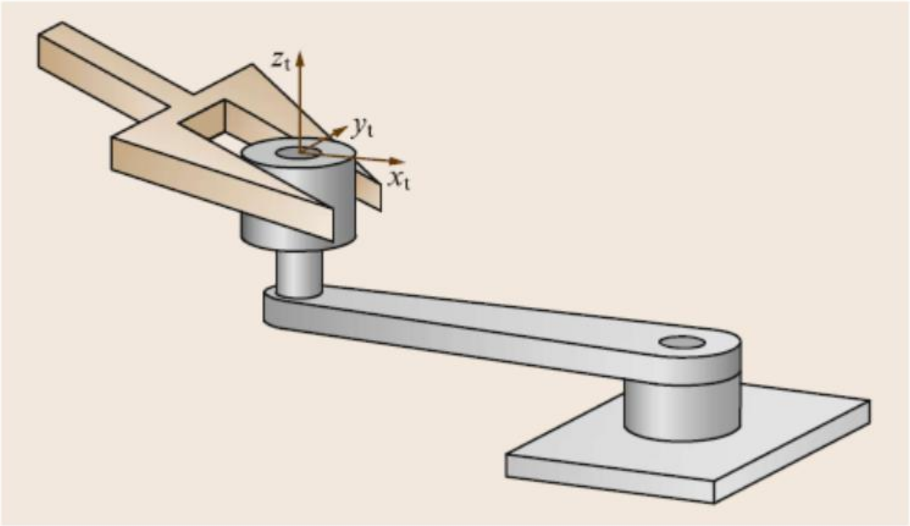


Interaction Control

- Controlling either the pose or wrench requires perfect knowledge of the task environment and when contact occurs or not, which is clearly not possible in practice.
- Instead, the behavior (relation between the wrench and pose) of the controlled robot could be modified independent of the environment.



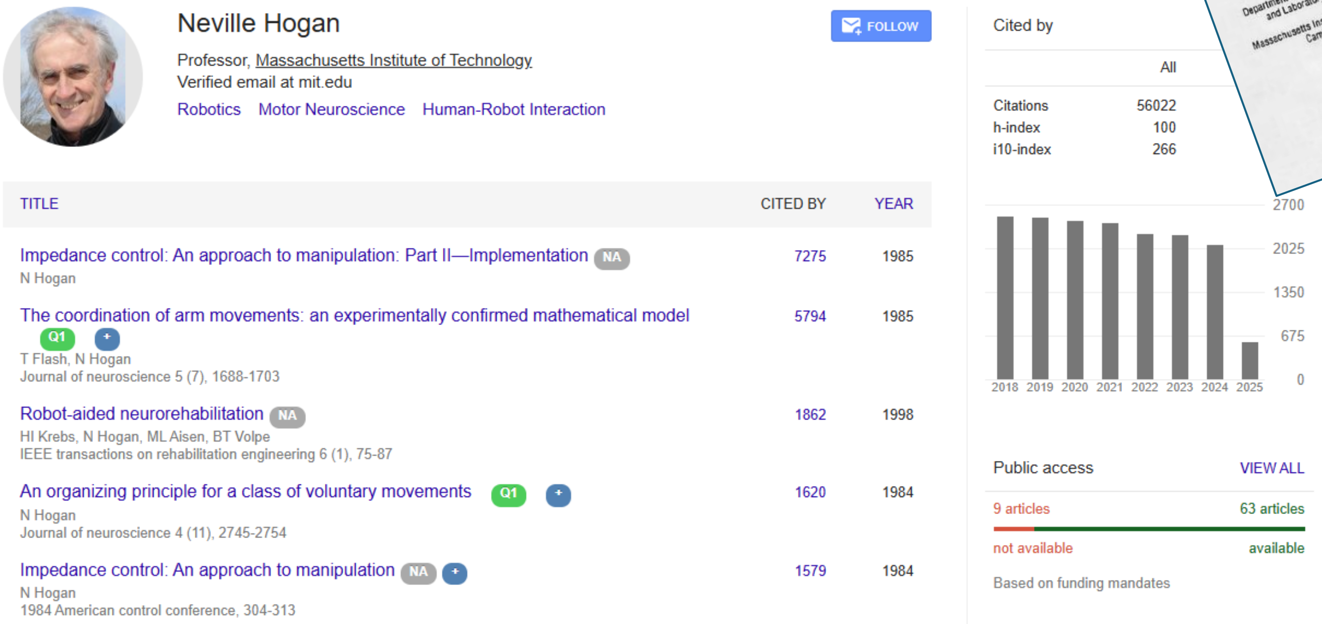
Insertion of a cylindrical peg into a hole



Turning a crank with an idle handle

Impedance Control

- The first one to address this issue in the field of robotics was Neville Hogan in his seminal trilogy.



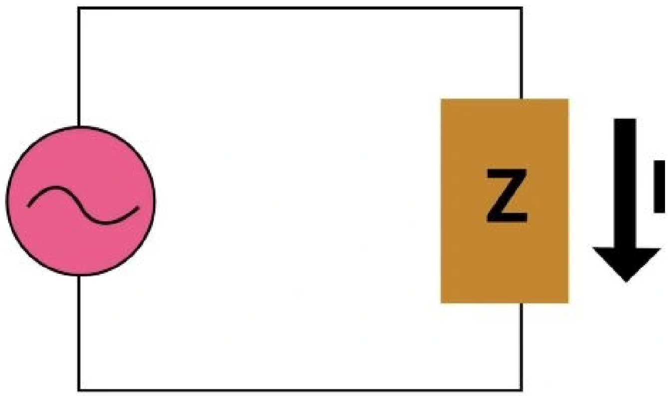
Electrical Impedance and Admittance

- In electrical engineering, impedance (Z) is the total opposition a circuit offers to the flow of alternating current.

$$\vec{Z} = \frac{\vec{V}}{\vec{I}} = R + j\,X$$

- Admittance (Y) is the reciprocal of impedance and is a measure of how easily a circuit or device will allow a current to flow.

$$\vec{Y} = \frac{\vec{I}}{\vec{V}} = G + j\,B$$



V: Voltage [V]
I: Current [A]

Phasors (frequency dependent):

$\vec{V} = |V| e^{j(\omega t + \phi_V)}$, $\vec{I} = |I| e^{j(\omega t + \phi_I)}$

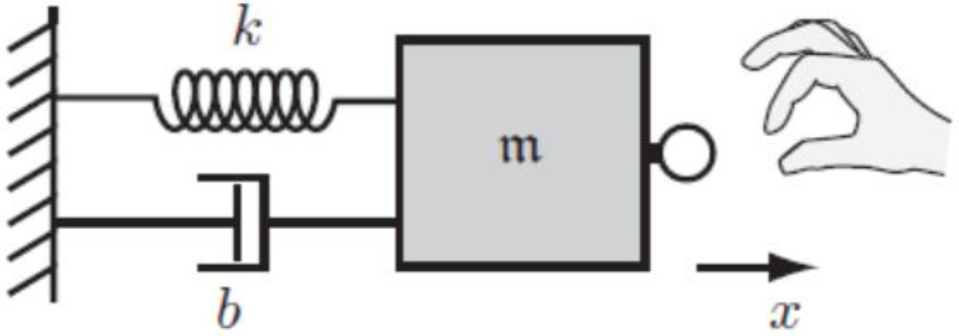
Mechanical Impedance and Admittance

- In mechanical engineering, impedance is a measure of how much a structure resists motion when subjected to a harmonic force.

$$\vec{Z} = \frac{\vec{F}}{\vec{v}}$$

- Mechanical admittance is the reciprocal of impedance.

$$\vec{Y} = \frac{\vec{v}}{\vec{F}}$$

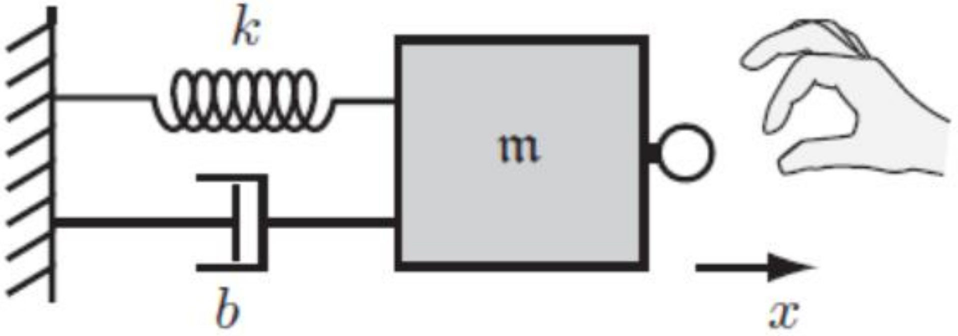


Impedance vs. Admittance

- **Impedance**
- $F(s) = Z(s)X(s)$
- $Z(s) = m\,s^2 + b\,s + k$
- Motion input, Force output

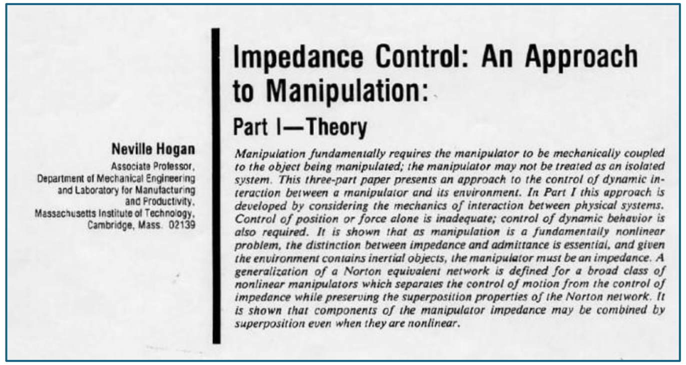
- **Admittance**
- $X(s) = Y(s)F(s)$
- $Y(s) = \frac{1}{m\,s^2+b\,s+k}$
- Force input, Motion output

$m\,\ddot{x} + b\,\dot{x} + k\,x = F(t)$



Impedance Control

- Neville Hogan's idea of impedance control is that instead of commanding exact positions or forces, the robot is controlled to behave like a mechanical impedance:
  - It responds to external forces with a desired relationship between force, position, and velocity, like a spring-damper-mass system.
- The key is shaping the robot's dynamic behavior — its *stiffness*, *damping*, and *inertia* — to match the interaction task needs.

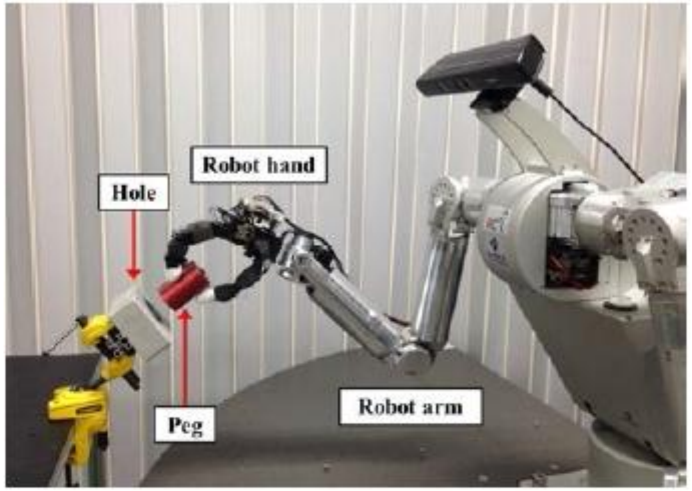


Impedance Control: An Approach to Manipulation:

Part I—Theory

Neville Hogan
Associate Professor,
Department of Mechanical Engineering
and Laboratory for Manufacturing
and Productivity,
Massachusetts Institute of Technology,
Cambridge, Mass. 02139

Manipulation fundamentally requires the manipulator to be mechanically coupled to the object being manipulated; the manipulator may not be treated as an isolated system. This three-part paper presents an approach to the control of dynamic interaction between a manipulator and its environment. In Part I this approach is developed by considering the mechanics of interaction between physical systems. Control of position or force alone is inadequate; control of dynamic behavior is also required. It is shown that as manipulation is a fundamentally nonlinear problem, the distinction between impedance and admittance is essential, and given the environment contains inertial objects, the manipulator must be an impedance. A generalization of a Norton equivalent network is defined for a broad class of nonlinear manipulators which separates the control of motion from the control of impedance while preserving the superposition properties of the Norton network. It is shown that components of the manipulator impedance may be combined by superposition even when they are nonlinear.



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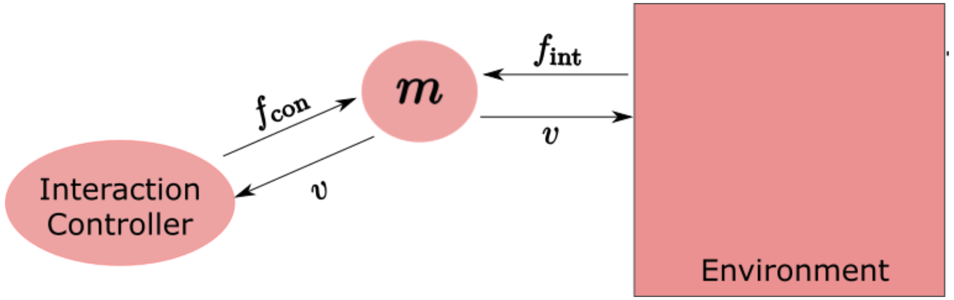
Point Mass Dynamics

- To provide some intuition, let's start in a simple Euclidean space $\mathbb{R}^n$.
- The governing equations of a point mass (with no gravity) are:
  - $\dot{\xi} = v, \qquad m\dot{v} = f_{\text{con}} + f_{\text{int}}$



Impedance Control of a Point Mass

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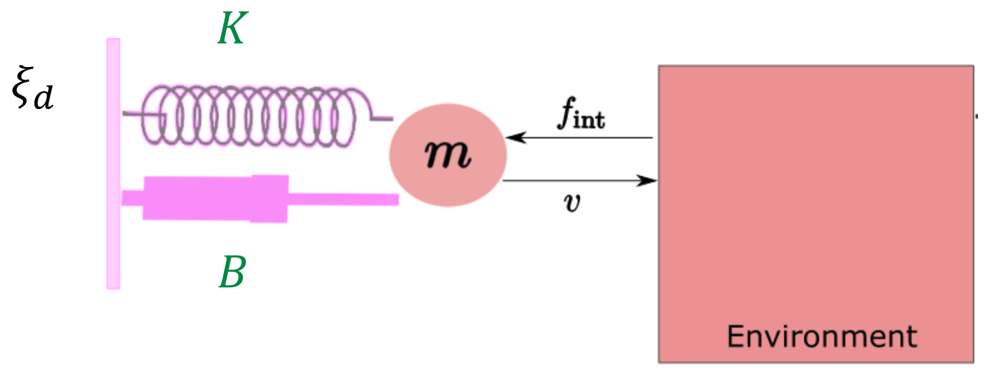
- The governing equations of a point mass (with no gravity) are:
  - $\dot{\xi} = v,\qquad m\dot{v} = f_{\text{con}} + f_{\text{int}}$
- The goal of impedance control is to implement the task-space behavior
  - $m\,\ddot{q} + B\dot{q} + Kq = f_{\text{int}}\,,\qquad q \coloneqq \xi - \xi_d,$



Impedance Control of a Point Mass

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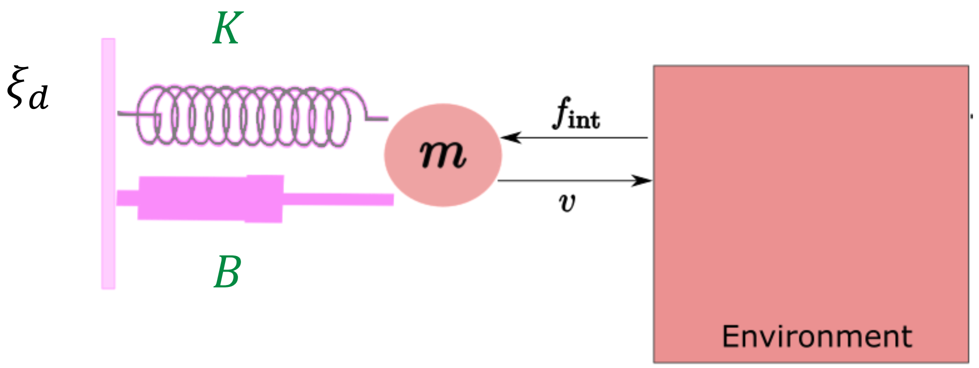
- The governing equations of a point mass (with no gravity) are:
  - $\dot{\xi} = v, \qquad m\dot{v} = f_{\text{con}} + f_{\text{int}}$
- The goal of impedance control is to implement the task-space behavior
  - $m\,\ddot{q} + B\dot{q} + Kq = f_{\text{int}}\,, \qquad q \coloneqq \xi - \xi_d,$
- The impedance control law then takes the form

$$f_{\text{con}} = m\ddot{\xi}_d - B(v - \dot{\xi}_d) - K(\xi - \xi_d)$$



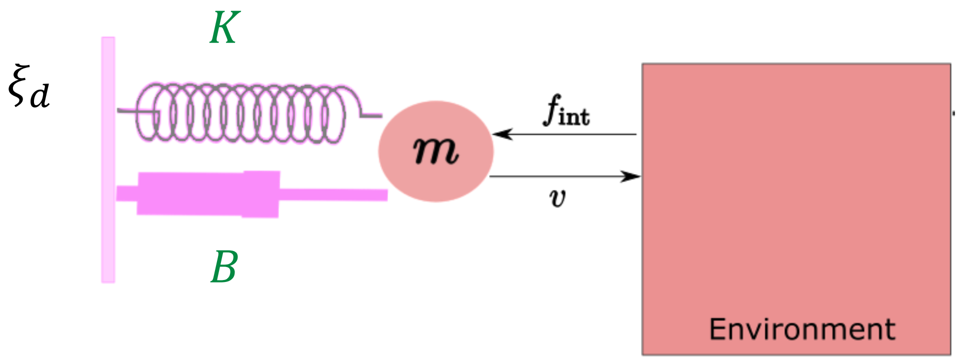
Impedance Control of a Point Mass

- During static interaction ($\ddot{q} = \dot{q} = 0$), the impedance behavior then becomes

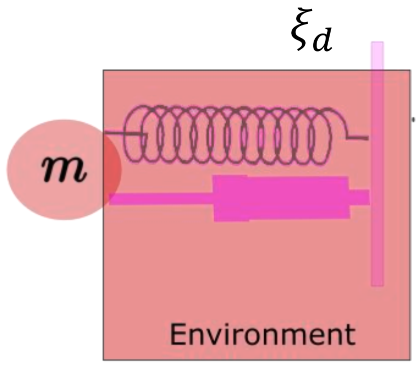
$$f_{\text{int}} = K(\xi - \xi_d)$$

Motion input,
Force Output !

- Therefore, K plays the role of an active stiffness.
- The force applied to the environment depends on K and the virtual setpoint ξ_d.



Before Interaction



During Interaction

Impedance Control vs. PD Control

Aspect	PD Control	Impedance Control
Primary goal	Track a desired position (no interaction considered)	Shape the dynamic interaction (force vs motion)
External forces	Seen as disturbances to reject	Part of the behavior to regulate
If environment pushes	Robot tries hard to return to position (may push back aggressively)	Robot "complies" according to set stiffness/damping
Output behavior	Stiff behavior unless you manually tune gains	Adjustable compliance via virtual mass-spring-damper model

$$f_{\text{pd}} = -B(v) - K(\xi - \xi_d)$$

$$f_{\text{imp}} = m\ddot{\xi}_d - B\left(v - \dot{\xi}_d\right) - K(\xi - \xi_d)$$

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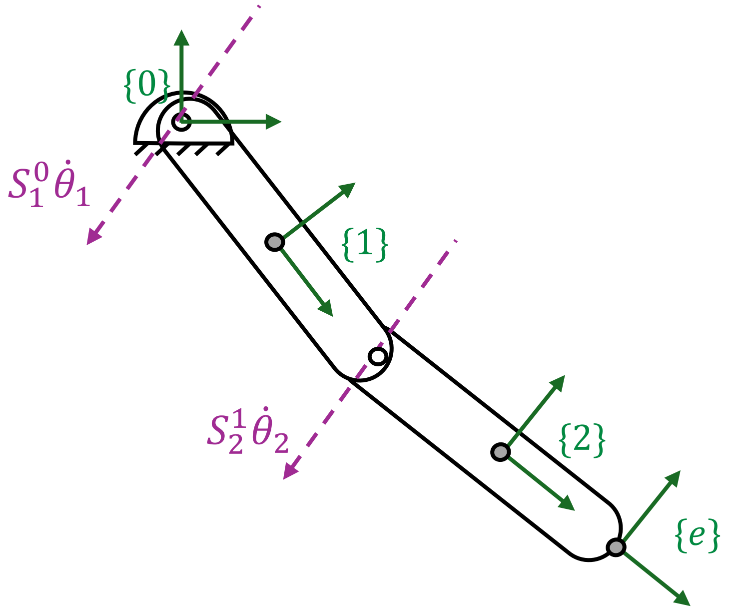
    - Task Space Dynamics

Impedance Control of n-link Manipulator

- Now we consider the impedance control problem of a n-link manipulator governed by

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + B(\dot{\theta})\dot{\theta} + g(\theta) = \tau_{\text{con}} + \tau_{\text{int}}$$

- where $\tau_{\text{con}} \in \mathbb{R}^n$ are joint control torques and $\tau_{\text{int}} \in \mathbb{R}^n$ are torques due to interaction with the environment.

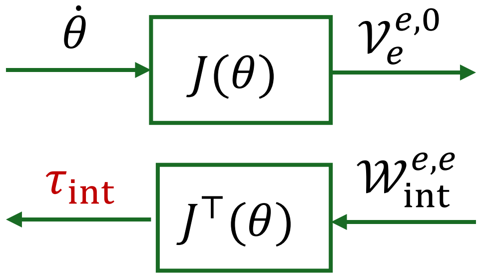


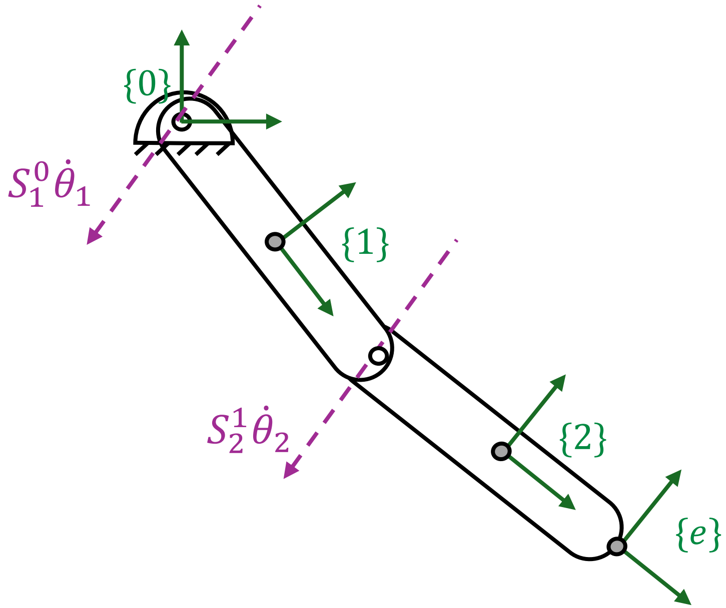
Impedance Control of n-link Manipulator

- Now we consider the impedance control problem of a n-link manipulator governed by

$$M(\theta)\ddot{\theta} + C\big(\theta,\dot{\theta}\big)\dot{\theta} + B\big(\dot{\theta}\big)\dot{\theta} + g(\theta) = \tau_{\text{con}} + \tau_{\text{int}}$$

- where $\tau_{\text{con}} \in \mathbb{R}^n$ are joint control torques and $\tau_{\text{int}} \in \mathbb{R}^n$ are torques due to interaction with the environment.
- Recall that:





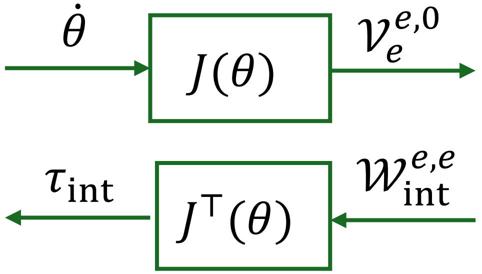
where $J(\theta)$ is the geometric Jacobian.

Geometric Jacobian Revisited

- The geometric Jacobian is a matrix that linearly maps the joint velocities to the end-effector's twist

$$\mathcal{V}_e^{e,0} = \begin{pmatrix} \omega_e^{e,0} \\ v_e^{e,0} \end{pmatrix} = J(\theta)\dot{\theta}$$

- It provides an instantaneous kinematic relation between the joint space and the task space.



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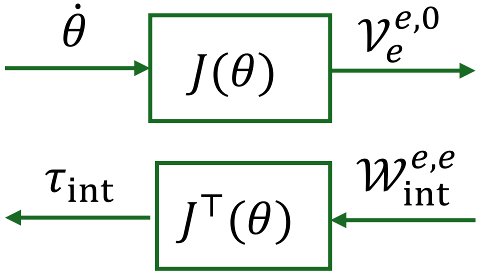
- It provides an instantaneous kinematic relation between the joint space and the task space.
- Each column of $J(\theta)$ is a joint screw axis represented in $\{e\}$ frame.

$$J(\theta) \coloneqq \left(Ad_{H_0^e(\theta)} \mathcal{S}_1^{0,0} \ , Ad_{H_1^e(\theta)} \mathcal{S}_2^{1,1}, \cdots, Ad_{H_{n-1}^e(\theta)} \mathcal{S}_n^{n-1,n-1} \right)$$

$J(\theta) \in \mathbb{R}^{6\times n}$



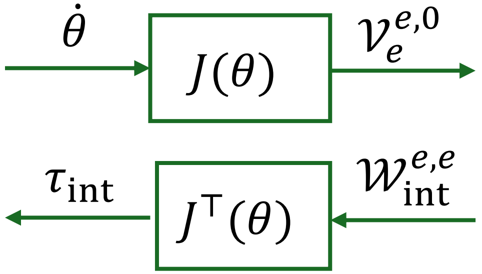
Geometric Jacobian Revisited

- The dual (or simply transpose) of the geometric Jacobian linearly maps any wrench at the end effector to joint torques

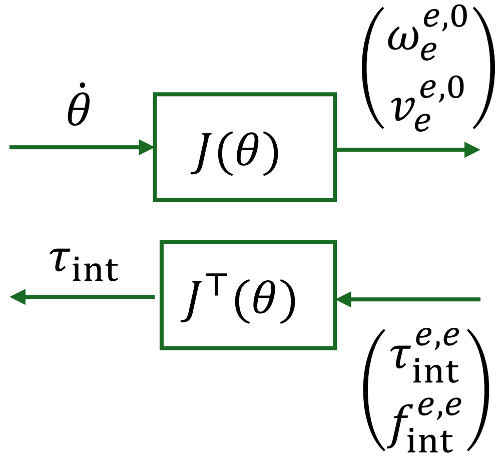
$$\tau_{\text{int}} = J^\top(\theta)\, \mathcal{W}^{e,e}_{\text{int}} = J^\top(\theta) \begin{pmatrix} \tau^{e,e}_{\text{int}} \\ f^{e,e}_{\text{int}} \end{pmatrix}$$

- Essential to utilize in interaction control, human-robot collaboration, and teleoperation.
- Analyzing properties of this matrix is essential for identifying kinematic singularities of the robot.

$J^\top(\theta) \in \mathbb{R}^{n\times 6}$



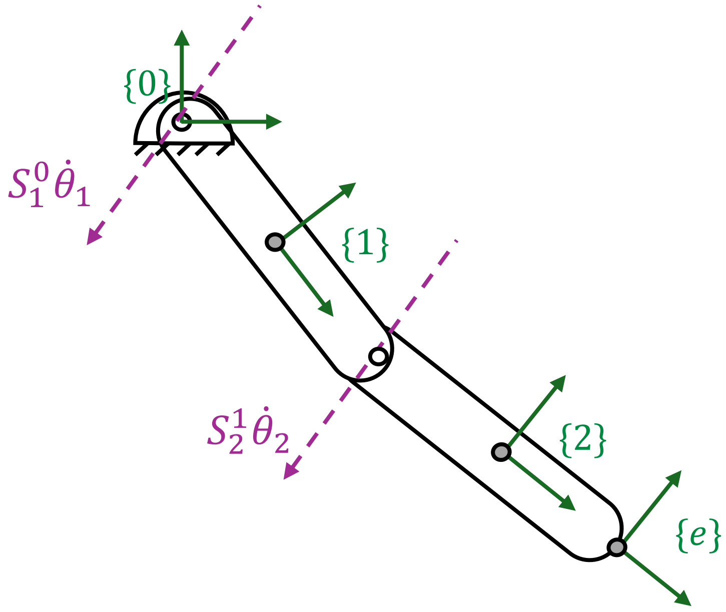
Example: 2-Link Manipulator



- Recall our two-link manipulator example.
- The geometric Jacobian takes the form

$$J(\theta) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ j_1(\theta) & 0 \\ j_2(\theta) & l_2 \\ 0 & 0 \end{pmatrix}$$

where j_1, j_2 are scalar functions of θ.



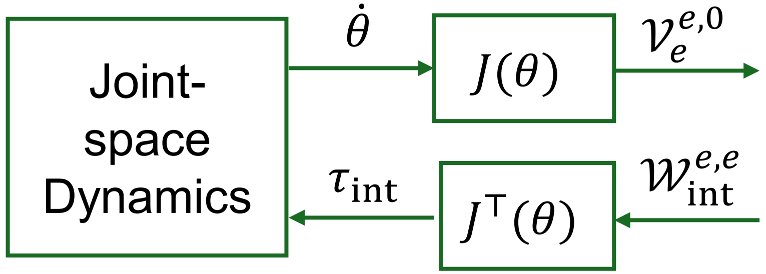
$$J(\theta) := \left(Ad_{H_0^e(\theta)} \mathcal{S}_1^{0,0}\ , Ad_{H_1^e(\theta)} \mathcal{S}_2^{1,1}\right) \in \mathbb{R}^{6\times 2}$$

Manipulator Dynamics in the Task Space

- If we do not apply any actuator torques ($\tau_{\text{con}} = 0$), the dynamics in the joint-space are given by

$$M(\theta)\ddot{\theta} + c(\theta, \dot{\theta}) + b(\dot{\theta}) + g(\theta) = J^{\top}(\theta)\, \mathcal{W}_{\text{int}}^{e,e}$$

- An interesting question is what does the dynamics "feel like" seen from the robot's end-effector perspective?



Manipulator Dynamics in the Task Space

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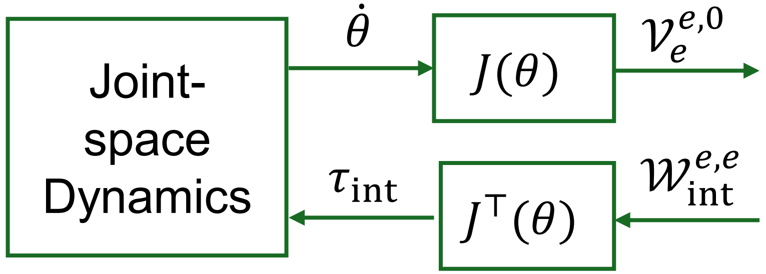
- An interesting question is what does the dynamics "feel like" seen from the robot's end-effector perspective?
- In other words, someone is pushing/pulling the robot's end-effector, will feel the dynamics in the task-space given by*

$$\Lambda(\theta)\dot{\mathcal{V}}_e^{e,0} + \eta(\theta, \mathcal{V}_e^{e,0})\mathcal{V}_e^{e,0} + \gamma(\theta) = \mathcal{W}_{\text{int}}^{e,e}$$



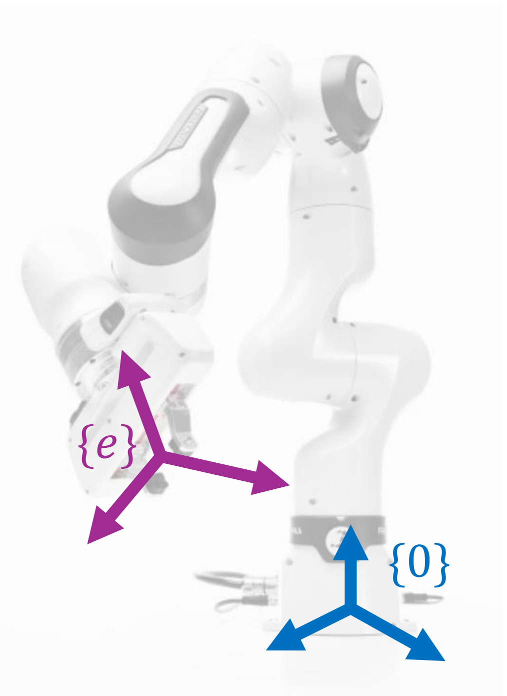
*Homework Problem

Manipulator Dynamics in the Task Space

- Therefore, a human interacting with an-unactuated robot arm* will experience the behavior given by

$$\underbrace{\Lambda(\theta)}_{\text{Apparent inertia}}\dot{\mathcal{V}}_e^{e,0} + \underbrace{\eta\left(\theta, \mathcal{V}_e^{e,0}\right)}_{\text{Apparent nonlinear friction}}\mathcal{V}_e^{e,0} + \underbrace{\gamma(\theta)}_{\text{Apparent gravitational forces}} = \mathcal{W}_{\text{int}}^{e,e}$$

Apparent inertia | Apparent nonlinear friction | Apparent gravitational forces



* This is only possible in so called back-drivable robots