SCE 594: Special Topics in Intelligent Automation & Robotics

Lecture 26: Impedance Control II

جامعة الملك فهد للبترول والمعادن
King Fahd University of Petroleum & Minerals

Ramy Rashad
Assistant Professor
CIE Department

Outline

- Recap last lecture
- Impedance Control of a n-link manipulator

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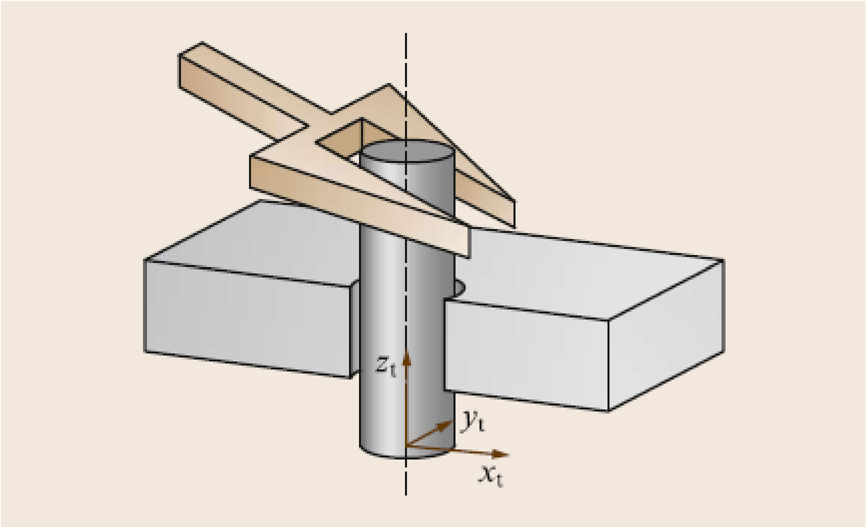
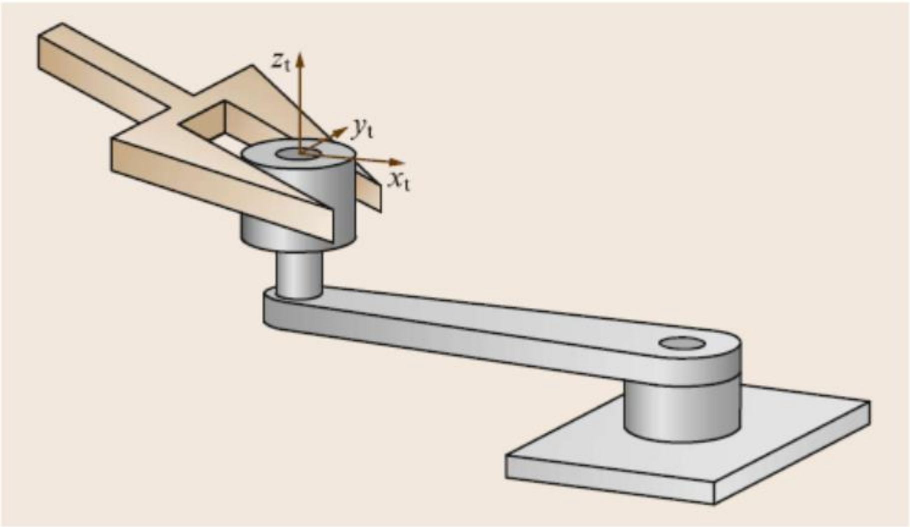
Recap: Interaction Control

- Controlling either the pose or wrench requires perfect knowledge of the task environment and when contact occurs or not, which is clearly not possible in practice.
- Instead, the behavior (relation between the wrench and pose) of the controlled robot could be modified independent of the environment.



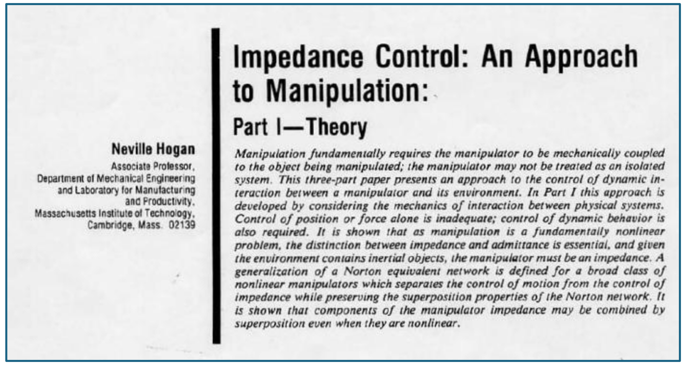
Insertion of a cylindrical peg into a hole



Turning a crank with an idle handle

Recap: Impedance Control

- Neville Hogan's idea of impedance control is that instead of commanding exact positions or forces, the robot is controlled to behave like a mechanical impedance:
  - It responds to external forces with a desired relationship between force, position, and velocity, like a spring-damper-mass system.
- The key is shaping the robot's dynamic behavior — its *stiffness*, *damping*, and *inertia* — to match the interaction task needs.

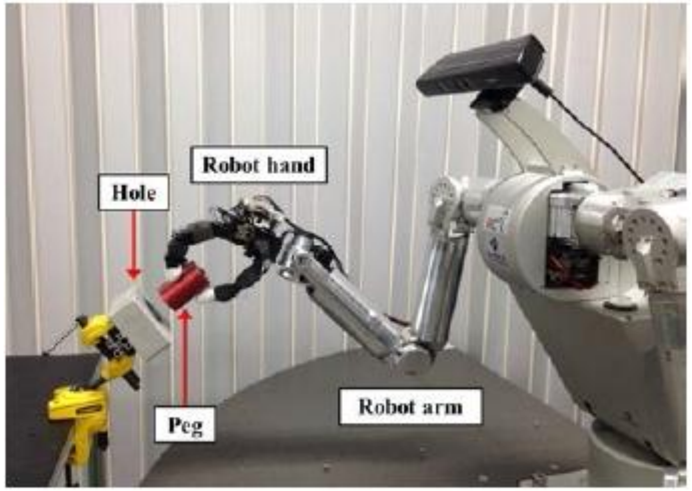


Impedance Control: An Approach to Manipulation:

Part I—Theory

Neville Hogan
Associate Professor,
Department of Mechanical Engineering
and Laboratory for Manufacturing
and Productivity,
Massachusetts Institute of Technology,
Cambridge, Mass. 02139

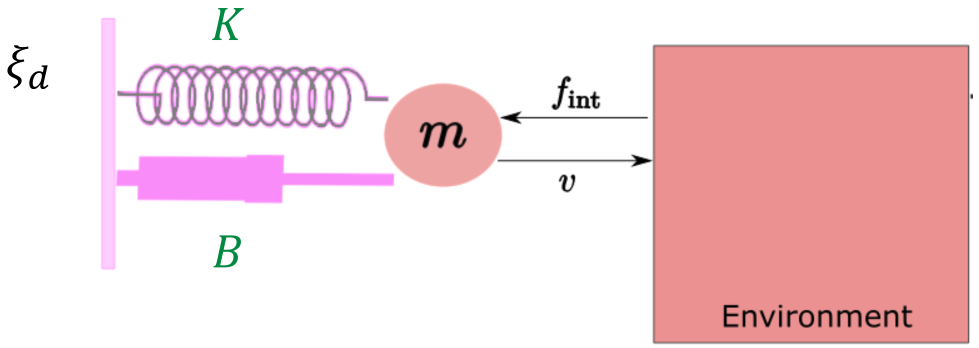
Manipulation fundamentally requires the manipulator to be mechanically coupled to the object being manipulated; the manipulator may not be treated as an isolated system. This three-part paper presents an approach to the control of dynamic interaction between a manipulator and its environment. In Part I this approach is developed by considering the mechanics of interaction between physical systems. Control of position or force alone is inadequate; control of dynamic behavior is also required. It is shown that as manipulation is a fundamentally nonlinear problem, the distinction between impedance and admittance is essential, and given the environment contains inertial objects, the manipulator must be an impedance. A generalization of a Norton equivalent network is defined for a broad class of nonlinear manipulators which separates the control of motion from the control of impedance while preserving the superposition properties of the Norton network. It is shown that components of the manipulator impedance may be combined by superposition even when they are nonlinear.



Recap: Impedance Control of a Point Mass

- To provide some intuition, let's start in a simple Euclidean space $\mathbb{R}^n$.
- The governing equations of a point mass (with no gravity) are:
  - $\dot{\xi} = v, \qquad m\dot{v} = f_{\text{con}} + f_{\text{int}}$
- The goal of impedance control is to implement the task-space behavior

$$m\,\ddot{q} + B\dot{q} + Kq = f_{\text{int}}\,, \qquad q := \xi - \xi_d,$$



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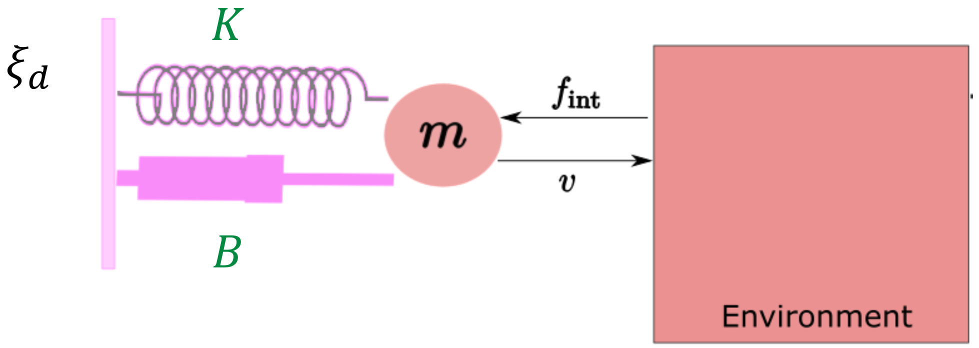
$$m\ddot{q} + B\dot{q} + Kq = f_{\text{int}}, \qquad q := \xi - \xi_d,$$

- The impedance control law that yields this impedance behavior is

$$f_{\text{con}} = m\ddot{\xi}_d - B(v - \dot{\xi}_d) - K(\xi - \xi_d)$$

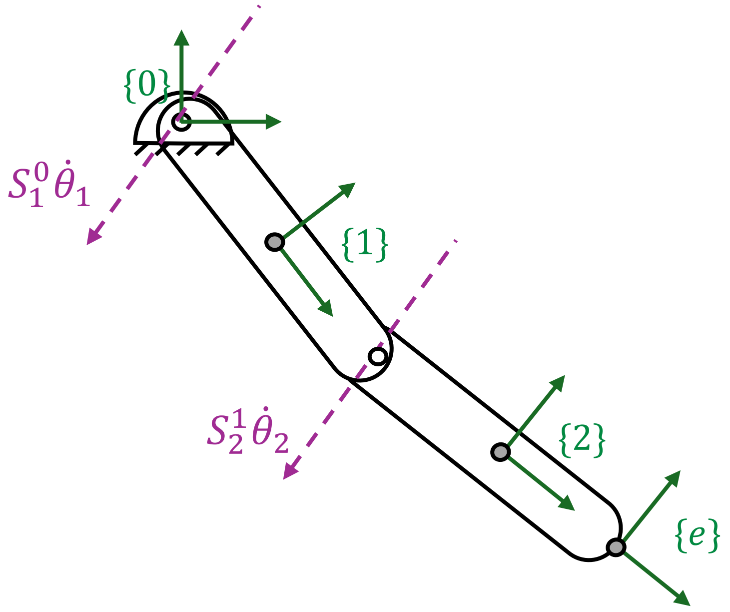


Recap: Joint-Space Dynamics

- Now we consider the impedance control problem of a n-link manipulator governed by

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + B(\dot{\theta})\dot{\theta} + g(\theta) = \tau_{\text{con}} + \tau_{\text{int}}$$

- where $\tau_{\text{con}} \in \mathbb{R}^n$ are joint control torques and $\tau_{\text{int}} \in \mathbb{R}^n$ are torques due to interaction with the environment.



Recap: Geometric Jacobian

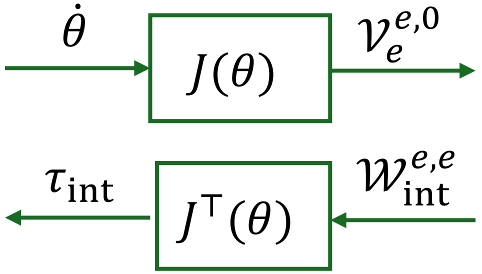
- The geometric Jacobian is a matrix that linearly maps the joint velocities to the end-effector's twist

$$\mathcal{V}_e^{e,0} = J(\theta)\dot{\theta}$$

- The dual (or simply transpose) of the geometric Jacobian linearly maps any wrench at the end effector to joint torques

$$\tau_{\text{int}} = J^\top(\theta)\,\mathcal{W}_{\text{int}}^{e,e}$$

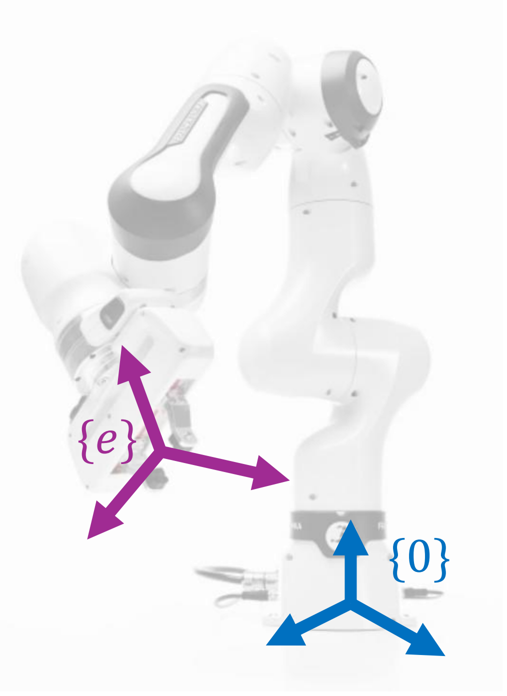
- It provides an instantaneous kinematic relation between the joint space and the task space.



Recap: Task-Space Dynamics

- A human interacting with an-unactuated robot arm ($\tau_{\text{con}} = 0$) will experience the behavior given by

$$\underbrace{\Lambda(\theta)}_{\text{Apparent inertia}} \dot{\mathcal{V}}_e^{e,0} + \underbrace{\eta\left(\theta, \mathcal{V}_e^{e,0}\right)}_{\text{Apparent nonlinear friction}} \mathcal{V}_e^{e,0} + \underbrace{\gamma(\theta)}_{\text{Apparent gravitational forces}} = \mathcal{W}_{\text{int}}^{e,e}$$



$$\Lambda(q) \coloneqq \left(J(q) M^{-1}(q) J^\top(q)\right)^{-1}$$

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Manipulator Dynamics in the Task Space

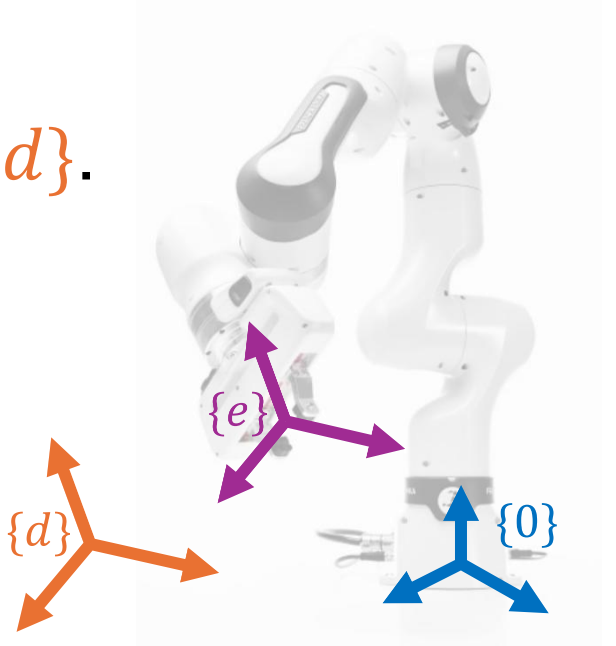
- With the actuator torques τ_{con}, we have that

$$\Lambda(\theta)\dot{\mathcal{V}}_e^{e,0} + \eta(\theta, \mathcal{V}_e^{e,0})\mathcal{V}_e^{e,0} + \gamma(\theta) = J^{-\top}(\theta)\tau_{\text{con}} + \mathcal{W}_{\text{int}}^{e,e}$$

- We can now shape the task-space dynamics to be our desired behavior

$$\mathfrak{M}\,\ddot{q} + \mathfrak{B}\dot{q} + \mathfrak{K}q = \mathcal{W}_{\text{int}}^{e,e}$$

where $q \in \mathbb{R}^6$ characterizes the error* between $\{e\}$ & $\{d\}$.



*this error will be defined later

Impedance Control of n-link manipulator

- Therefore, the control law that will yields the desired impedance behavior at the end effector

$$\mathfrak{M}\,\ddot{q} + \mathfrak{B}\dot{q} + \mathfrak{K}q = \mathcal{W}_{\text{int}}^{e,e}$$

will be given by

$$\tau_{\text{con}} = J^{\top}(\theta)\left[\underbrace{\Lambda(\theta)\dot{\mathcal{V}}_e^{e,0} + \eta\left(\theta, \mathcal{V}_e^{e,0}\right)\mathcal{V}_e^{e,0} + \gamma(\theta)}_{\text{task-dynamics compensation}} - \underbrace{\left(\mathfrak{M}\,\ddot{x}_d + \mathfrak{B}\dot{q} + \mathfrak{K}q\right)}_{\text{desired behavior}}\right]$$

Alternative formulations 1

- Impedance Control law

$$\tau_{\text{con}} = J^{\top}(\theta)\left[\underbrace{\Lambda(\theta)\dot{\mathcal{V}}_e^{e,0} + \eta\left(\theta, \mathcal{V}_e^{e,0}\right)\mathcal{V}_e^{e,0} + \gamma(\theta)}_{\text{task-dynamics compensation}} - \underbrace{\left(\mathfrak{M}\ddot{x}_d + \mathfrak{B}\dot{q} + \mathfrak{K}q\right)}_{\text{desired behavior}}\right]$$

- One can also rewrite the impedance control law as

$$\tau_{\text{con}} = \underbrace{M(\theta)\ddot{\theta} + c\left(\theta, \dot{\theta}\right) + b\left(\dot{\theta}\right) + g(\theta)}_{\text{joint-dynamics compensation}} - J^{\top}(\theta)\underbrace{\left(\mathfrak{M}\ddot{x}_d + \mathfrak{B}\dot{q} + \mathfrak{K}q\right)}_{\text{desired behavior}}$$

Alternative formulations 2

- Impedance Control law

$$\tau_{\text{con}} = J^\top(\theta)\left[\Lambda(\theta)\dot{\mathcal{V}}_e^{e,0} + \eta\left(\theta, \mathcal{V}_e^{e,0}\right)\mathcal{V}_e^{e,0} + \gamma(\theta) - (\mathfrak{M}\ddot{x}_d + \mathfrak{B}\dot{q} + \mathfrak{K}q)\right]$$

- The above controller requires measurement of accelerations of the end effector $\left(\dot{\mathcal{V}}_e^{e,0}, \ddot{q}\right)$ which might be noisy in practice.

Alternative formulations 2

- A common alternative is

$$\tau_{\text{con}} = J^{\top}(\theta)\left[\eta\left(\theta, \mathcal{V}_e^{e,0}\right)\mathcal{V}_e^{e,0} + \gamma(\theta) - (\mathfrak{B}\dot{q} \ + \mathfrak{K}q)\right]$$

- The impedance behavior is given in this case* by

$$\Lambda(\theta)\ddot{q} + \mathfrak{B}\dot{q} \ + \mathfrak{K}q \ = \mathcal{W}_{\text{int}}^{e,e}$$

- Therefore, the actual physical inertia $\Lambda(\theta)$ of the manipulator will be apparent** to the user at the end-effector.

**Unless the robot is very lightweight

*Assuming that $\ddot{x}_d \approx 0$

Alternative formulations 3

- In slow interaction tasks, another common alternative than

$$\tau_{\text{con}} = J^{\top}(\theta)\left[\Lambda(\theta)\dot{\mathcal{V}}_e^{e,0} + \eta\left(\theta, \mathcal{V}_e^{e,0}\right)\mathcal{V}_e^{e,0} + \gamma(\theta) - (\mathfrak{M}\,\ddot{q} + \mathfrak{B}\dot{q} + \mathfrak{K}q)\right]$$

is not to compensate the full robot nonlinearities but only the gravity terms:

$$\tau_{\text{con}} = J^{\top}(\theta)[\gamma(\theta) - (\mathfrak{B}\dot{q} + \mathfrak{K}q)]$$

or equivalently

$$\tau_{\text{con}} = g(\theta) - J^{\top}(\theta)(\mathfrak{B}\dot{q} + \mathfrak{K}q)$$

Similar to PD controller
+ gravity compensation !

Comparison

Impedance Control 3

- Control Law:

$$\tau_{\text{con}} = g(\theta) - J^{\top}(\theta)(\mathfrak{B}\dot{q} + \mathfrak{K}q)$$

- Task-space controller
- Uses Geometric Jacobian
- Goal: Regulate end-effector to x_d with compliant behavior
- End-effector behaves like Cartesian spring-damper

PD + Gravity Compensation

- Control Law:

$$\tau = g(\theta) - K_p(\theta - \theta_d) - K_d\dot{\theta}$$

- Joint-space controller
- Does not Use Geometric Jacobian
- Goal: Regulate joints to θ_d
- Each joint behaves like spring-damper

Impedance Control of Franka Panda

https://youtu.be/XwiX2vv14Qs?si=pgXHW3qEkEbhrNMp