SCE 594: Special Topics in Intelligent Automation & Robotics

Lecture 27: Impedance Control III

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Outline

- Recap last lecture
- Desired Impedance in 6D
- Admittance Control

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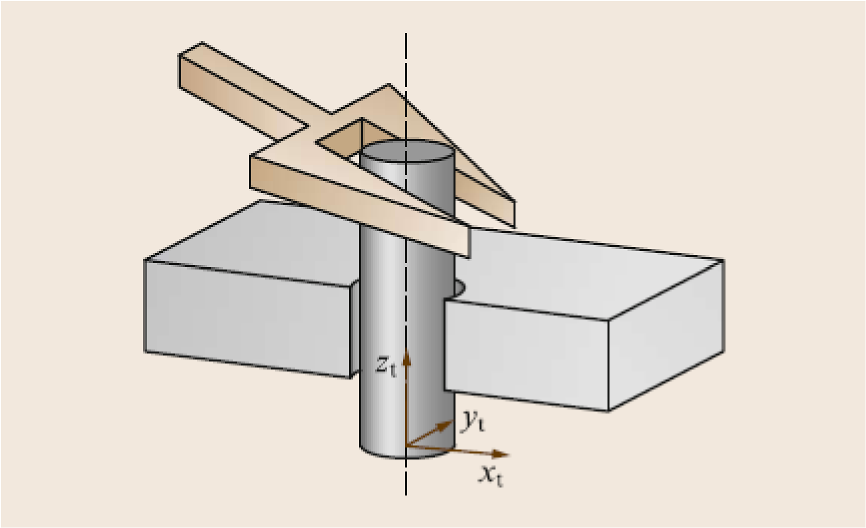
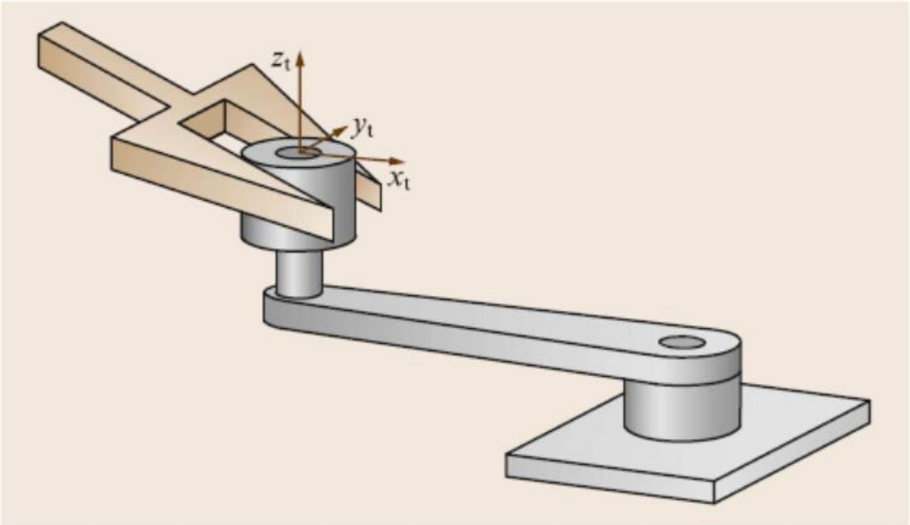
Recap: Interaction Control

- Controlling either the pose or wrench requires perfect knowledge of the task environment and when contact occurs or not, which is clearly not possible in practice.
- Instead, the behavior (relation between the wrench and pose) of the controlled robot could be modified independent of the environment.



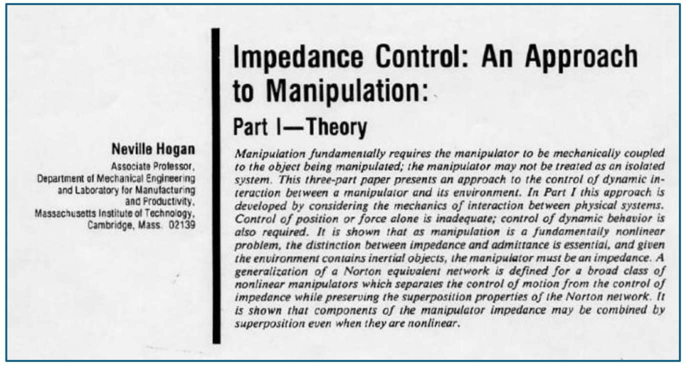
Insertion of a cylindrical peg into a hole



Turning a crank with an idle handle

Recap: Impedance Control

- Neville Hogan's idea of impedance control is that instead of commanding exact positions or forces, the robot is controlled to behave like a mechanical impedance:
  - It responds to external forces with a desired relationship between force, position, and velocity, like a spring-damper-mass system.
- The key is shaping the robot's dynamic behavior — its *stiffness*, *damping*, and *inertia* — to match the interaction task needs.



Impedance Control: An Approach to Manipulation:
Part I—Theory

Neville Hogan
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Massachusetts Institute of Technology,
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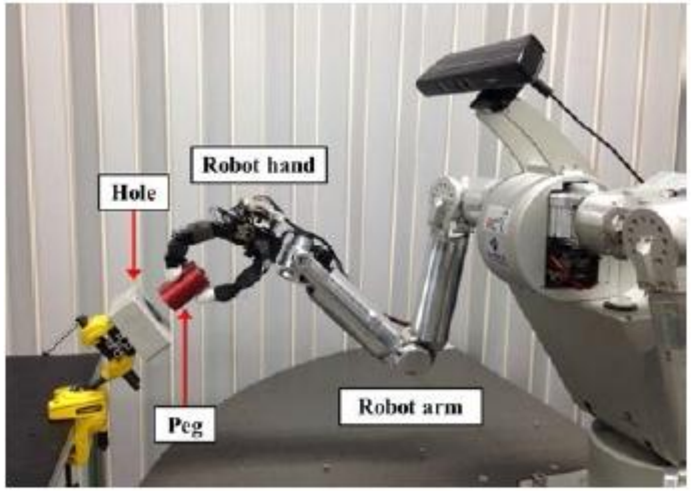
Manipulation fundamentally requires the manipulator to be mechanically coupled to the object being manipulated; the manipulator may not be treated as an isolated system. This three-part paper presents an approach to the control of dynamic interaction between a manipulator and its environment. In Part I this approach is developed by considering the mechanics of interaction between physical systems. Control of position or force alone is inadequate; control of dynamic behavior is also required. It is shown that as manipulation is a fundamentally nonlinear problem, the distinction between impedance and admittance is essential, and given the environment contains inertial objects, the manipulator must be an impedance. A generalization of a Norton equivalent network is defined for a broad class of nonlinear manipulators which separates the control of motion from the control of impedance while preserving the superposition properties of the Norton network. It is shown that components of the manipulator impedance may be combined by superposition even when they are nonlinear.

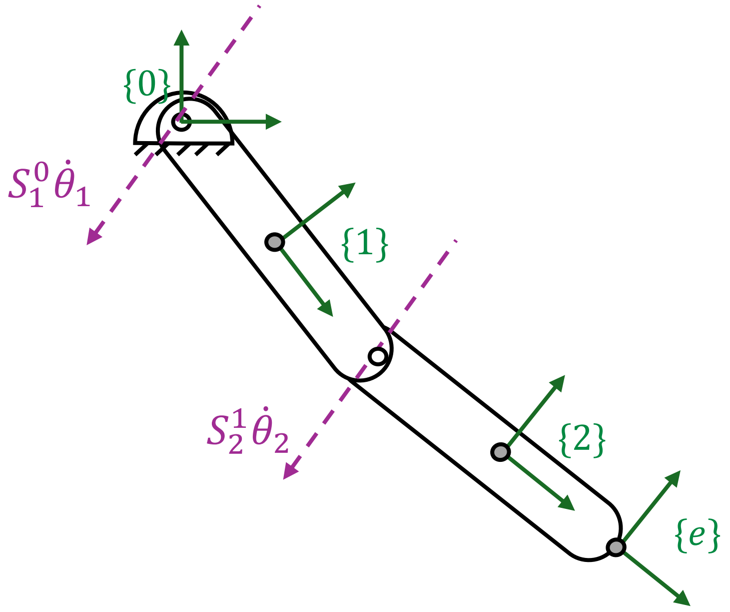


Recap: Joint-Space Dynamics

- Now we consider the impedance control problem of a n-link manipulator governed by

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + B(\dot{\theta})\dot{\theta} + g(\theta) = \tau_{\text{con}} + \tau_{\text{int}}$$

- where $\tau_{\text{con}} \in \mathbb{R}^n$ are joint control torques and $\tau_{\text{int}} \in \mathbb{R}^n$ are torques due to interaction with the environment.



Recap: Geometric Jacobian

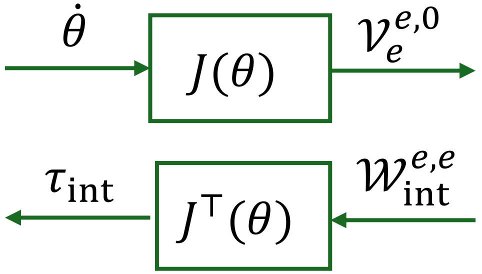
- The geometric Jacobian is a matrix that linearly maps the joint velocities to the end-effector's twist

$$\mathcal{V}_e^{e,0} = J(\theta)\dot{\theta}$$

- The dual (or simply transpose) of the geometric Jacobian linearly maps any wrench at the end effector to joint torques

$$\tau_{\text{int}} = J^\top(\theta)\,\mathcal{W}_{\text{int}}^{e,e}$$

- It provides an instantaneous kinematic relation between the joint space and the task space.



Recap: Task-Space Dynamics

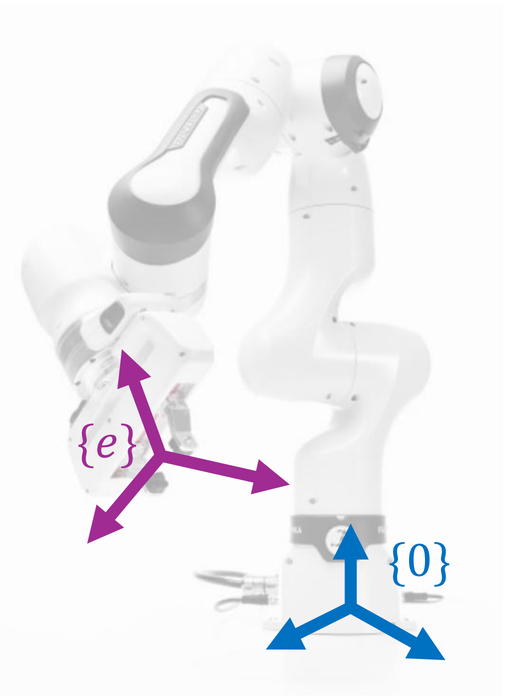
- A human interacting with a robot arm will experience the behavior given by

$$\underbrace{\Lambda(\theta)}\,\dot{\mathcal{V}}_e^{e,0} + \underbrace{\eta\left(\theta, \mathcal{V}_e^{e,0}\right)}\,\mathcal{V}_e^{e,0} + \underbrace{\gamma(\theta)} = J^{-\top}(\theta)\tau_{\mathrm{con}} + \mathcal{W}_{\mathrm{int}}^{e,e}$$

Apparent inertia

Apparent nonlinear friction

Apparent gravitational forces



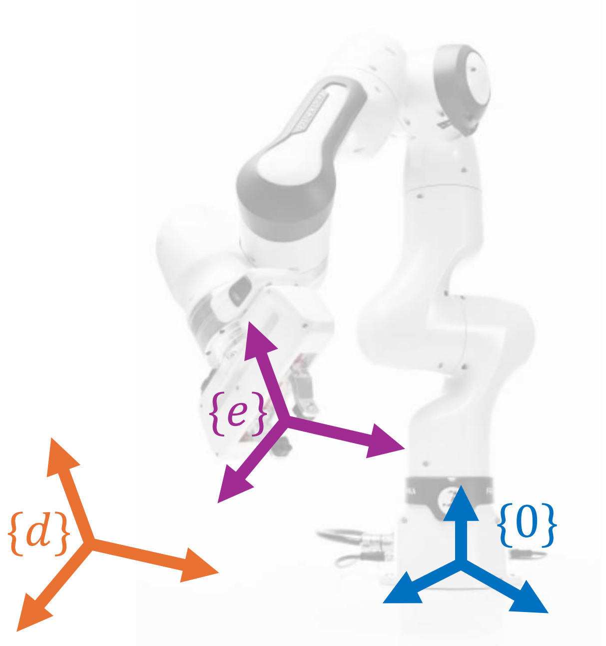
$$\Lambda(q) \coloneqq \left(J(q)M^{-1}(q)J^{\top}(q)\right)^{-1}$$

Recap: Desired Task-Space Dynamics

- We can now shape the task-space dynamics to be our desired behavior

$$\mathfrak{M}\,\ddot{q} + \mathfrak{B}\dot{q} + \mathfrak{K}q = \mathcal{W}_{\mathrm{int}}^{e,e}$$

where $q \in \mathbb{R}^6$ characterizes the error* between $\{e\}$ & $\{d\}$.



*this error will be defined later

Recap: Alternative formulations

- Impedance Control law 1

$$\tau_{\text{con}} = J^{\top}(\theta)\left[\underbrace{\Lambda(\theta)\dot{\mathcal{V}}_e^{e,0} + \eta\left(\theta, \mathcal{V}_e^{e,0}\right)\mathcal{V}_e^{e,0} + \gamma(\theta)}_{\text{task-dynamics compensation}} - \underbrace{(\mathfrak{M}\ddot{x}_d + \mathfrak{B}\dot{q} + \mathfrak{K}q)}_{\text{desired behavior}}\right]$$

- Impedance Control law 2

$$\tau_{\text{con}} = J^{\top}(\theta)\left[\eta\left(\theta, \mathcal{V}_e^{e,0}\right)\mathcal{V}_e^{e,0} + \gamma(\theta) - (\mathfrak{B}\dot{q} + \mathfrak{K}q)\right]$$

- Impedance Control law 3

$$\tau_{\text{con}} = J^{\top}(\theta)[\gamma(\theta) - (\mathfrak{B}\dot{q} + \mathfrak{K}q)]$$

Recap: Comparison

Impedance Control 3

- Control Law:

$$\tau_{\mathrm{con}} = J^{\top}(\theta)[\gamma(\theta) - (\mathfrak{B}\dot{q} + \mathfrak{K}q)]$$

- Task-space controller
- Uses Geometric Jacobian
- Goal: Regulate end-effector to x_d with compliant behavior
- End-effector behaves like Cartesian spring-damper

PD + Gravity Compensation

- Control Law:

$$\tau = g(\theta) - K_p\,(\theta - \theta_d) - K_d\dot{\theta}$$

- Joint-space controller
- Does not Use Geometric Jacobian
- Goal: Regulate joints to θ_d
- Each joint behaves like spring-damper

Outline

- Recap last lecture
- Desired Impedance in 6D
- Admittance Control

Desired Impedance in 6D

- Let's take a closer look at the desired impedance behavior

$$\mathfrak{M}\,\ddot{q} + \mathfrak{B}\dot{q} + \mathfrak{K}q = \mathcal{W}_{\text{int}}^{e,e}$$

- Caution must be considered in defining this impedance in a geometrically consistent manner.

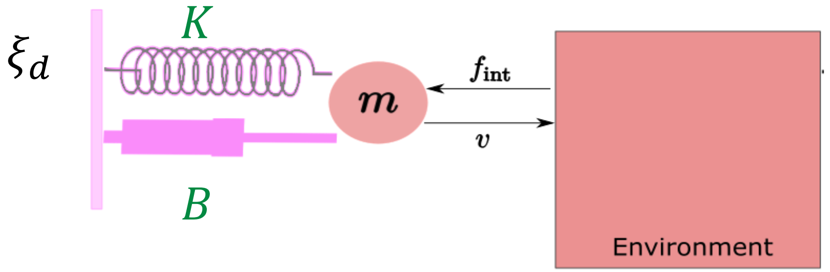
Desired Impedance in 6D

- Let's take a closer look at the desired impedance behavior

$$\mathfrak{M}\,\ddot{q} + \mathfrak{B}\dot{q} + \mathfrak{K}q = \mathcal{W}_{\text{int}}^{e,e}$$

- For the point-mass case, we had that $q := \xi - \xi_d \in \mathbb{R}^3$, and the desired impedance behavior:

$$\boxed{m\,\ddot{q} + B\dot{q} + Kq = f_{\text{int}}}$$

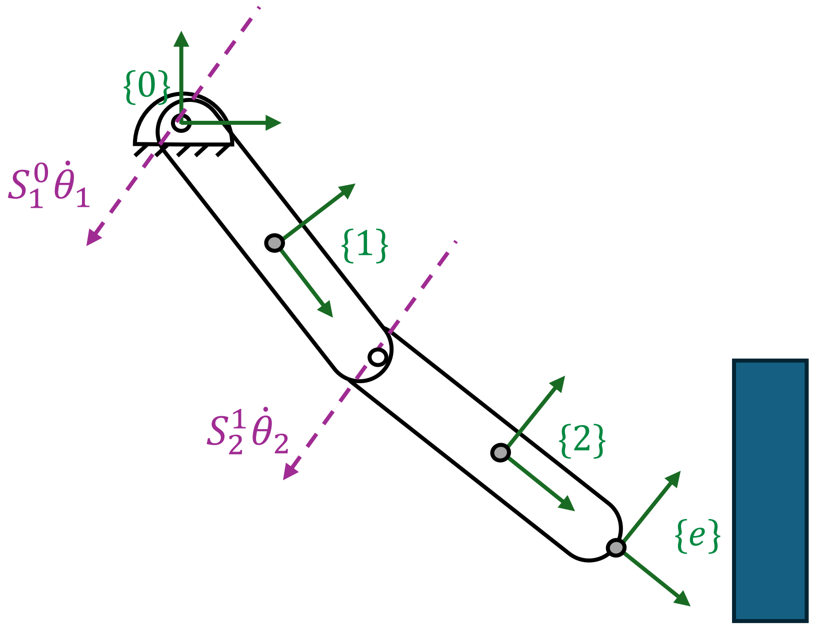


Desired Impedance in 6D

- Let’s take a closer look at the desired impedance behavior

$$\mathfrak{M}\,\ddot{q} + \mathfrak{B}\dot{q} + \mathfrak{K}q = \mathcal{W}_{\text{int}}^{e,e}$$

- Similarly for planar robots, the task space is $SE(2)$ such that
  - $H_e^0 = \begin{pmatrix} c_\phi & -s_\phi & \xi_x \\ s_\phi & c_\phi & \xi_y \\ 0 & 0 & 1 \end{pmatrix}, \quad \mathcal{W}_{\text{int}}^{e,e} = \begin{pmatrix} \tau_{\text{int}} \\ f_{x,\text{int}} \\ f_{y,\text{int}} \end{pmatrix}$
- Therefore, orientation is simply 1-dimensional
- We have that
  - $q := \begin{pmatrix} \phi - \phi_{\text{d}} \\ \xi_x - \xi_{x,\text{d}} \\ \xi_y - \xi_{y,\text{d}} \end{pmatrix} \in \mathbb{R}^3$

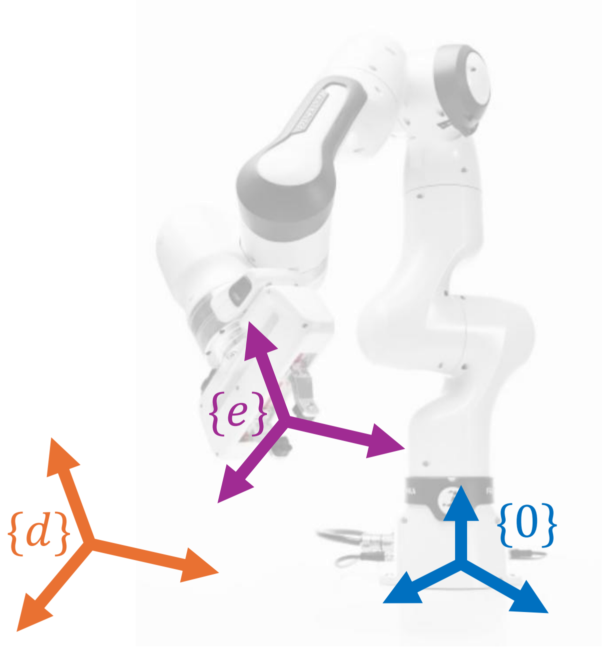


Desired Impedance in 6D

- Let's take a closer look at the desired impedance behavior

$$\mathfrak{M}\,\ddot{q} + \mathfrak{B}\dot{q} + \mathfrak{K}q = \mathcal{W}_{\text{int}}^{e,e}$$

- However, for a generic spatial robot, we cannot simply define the error $q \in \mathbb{R}^6$ as $q = H_e^0 - H_d^0 \notin SE(3)$.

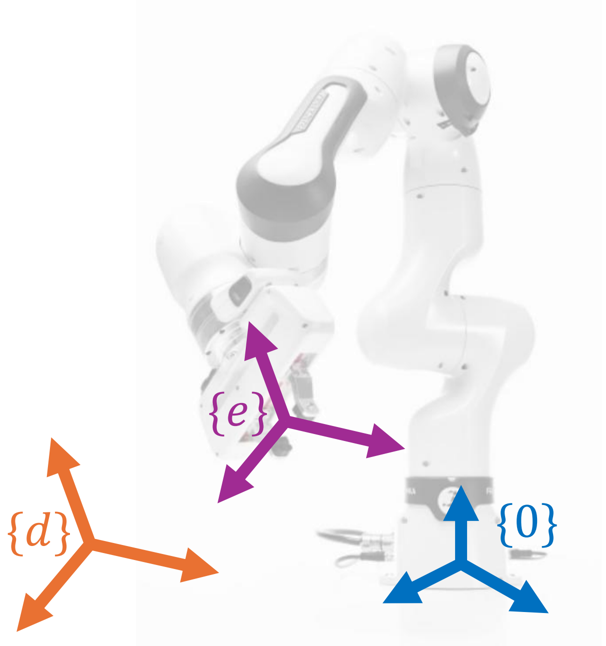


Common approaches

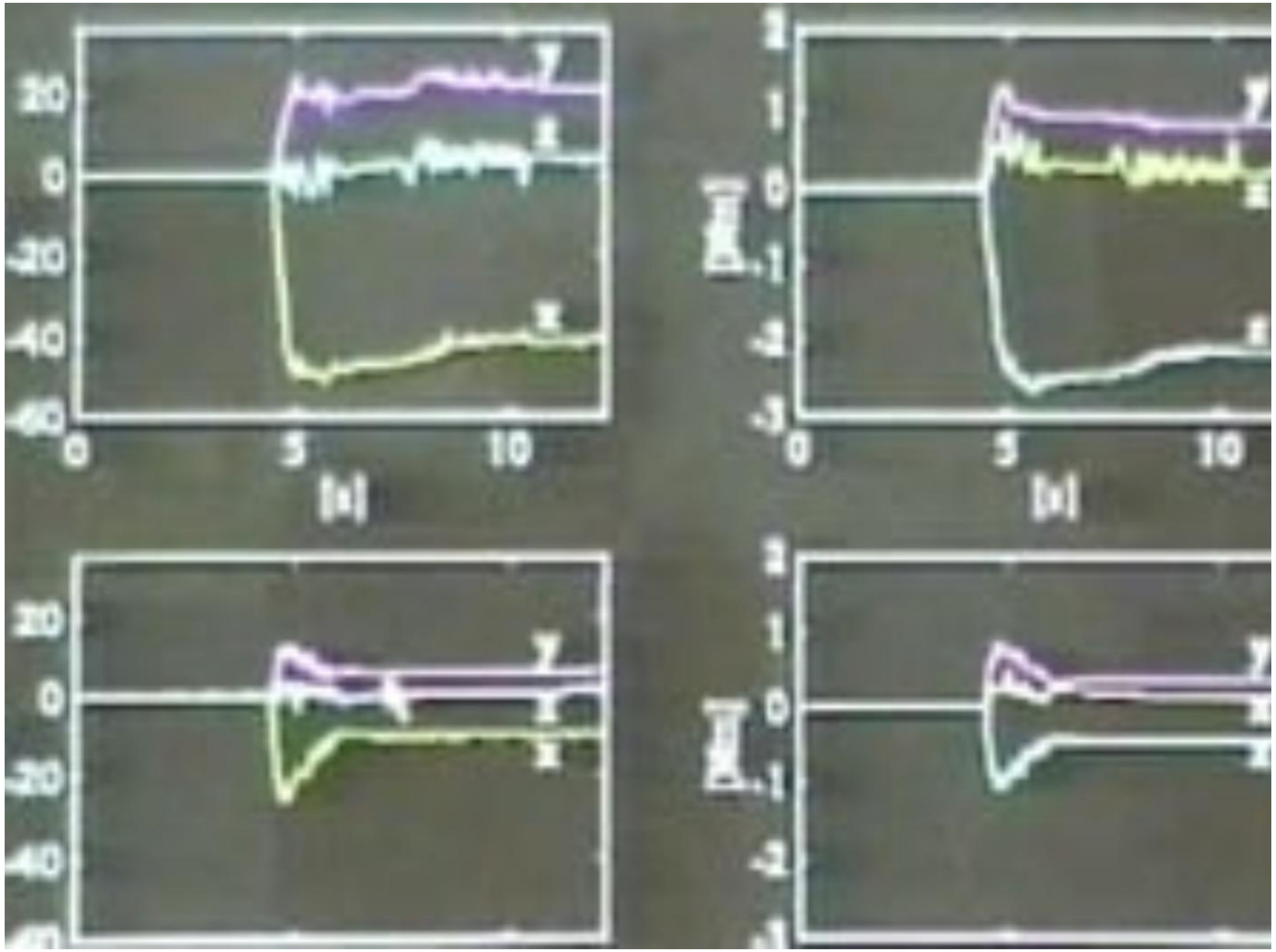
1. Split translation and rotation
   - $q = \begin{pmatrix} e_R \\ e_\xi \end{pmatrix} \in \mathbb{R}^6$
   - Relative displacement $e_\xi := \xi - \xi_d \in \mathbb{R}^3$

$$\mathfrak{M}\,\ddot{q} + \mathfrak{B}\dot{q} + \mathfrak{K}q = \mathcal{W}_{\text{int}}^{e,e}$$

Relative orientation	Method	Limitations
$e_R = \phi - \phi_d$	ϕ : Euler-angles	Stiffness behavior becomes coupled in the 3 axes.
$e_R = S^{-1}\left(\log R_d^\top R\right)$	$\log : \mathrm{SO}(3) \rightarrow \mathrm{so}(3)$ Exponential Coordinates	singular or ambiguous at $\theta = \pi$.
$e_R = \frac{1}{2} S^{-1}\left(R_d^\top R - R^\top R_d\right)$	R: Rotation matrix	Error magnitude decreases near 180°and becomes zero at exactly 180° ,which leads to slow convergence.



Importance of Geometric Consistency



Villani, L., & De Schutter, J. (2016). Force control. In *Springer handbook of robotics* (pp. 195-220). Cham: Springer International Publishing.

https://vimeo.com/123031416

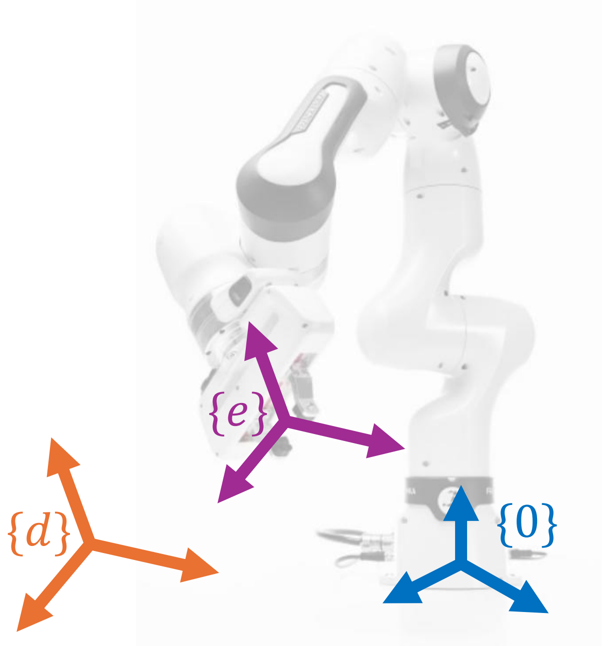
Common approaches

2. Energy-based Approach on SE(3)

$$\mathfrak{M}\dot{\mathcal{V}}_e^{e,d} + \mathfrak{B}\mathcal{V}_e^{e,d} + \beta_H^*\left(d\Psi\left(H_e^d\right)\right) = \mathcal{W}_{\text{int}}^{e,e}$$

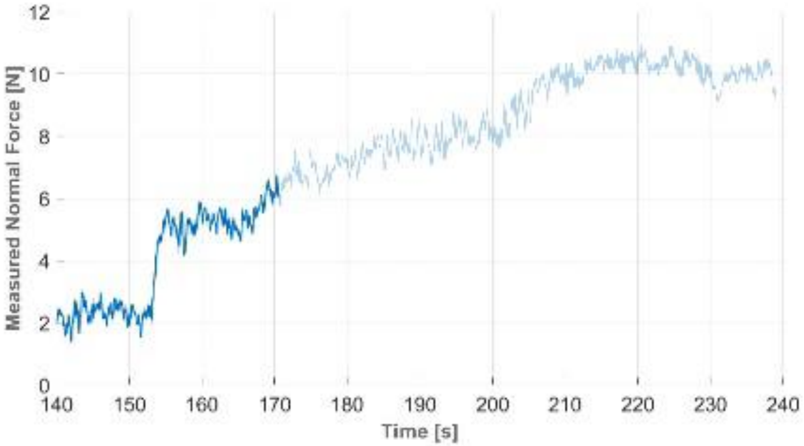
- Potential Function on SE(3) - $\Psi\left(H_e^d\right)$

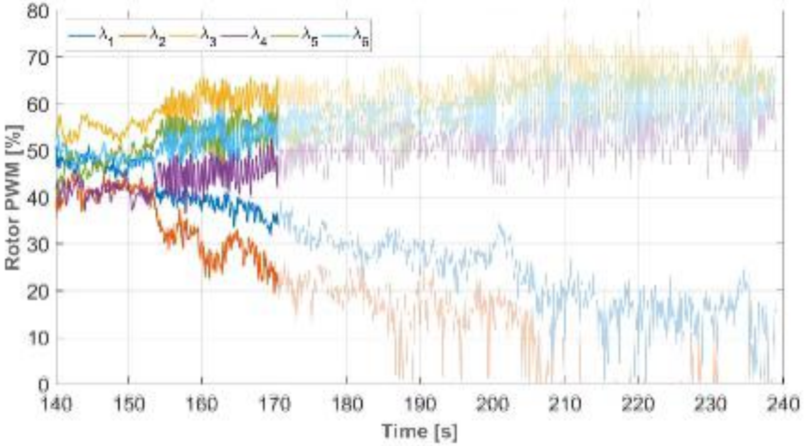
$$\phi_1(R_d^T R) + \tfrac{1}{2}\|p - p_d\|_{K_2}^2$$
$$\phi_1(R_d^T R) + \tfrac{1}{2}\|R_d^T(p - p_d)\|_{K_2}^2$$
$$\phi_1(R_d^T R) + \tfrac{1}{2}\|R^T(p - p_d)\|_{K_2}^2$$
$$\phi_1(R_d^T R) + \tfrac{1}{2}\|(R^T + R_d^T)(p - p_d)\|_{K_2}^2$$
$$\phi_1(R R_d^T) + \tfrac{1}{2}\|(R + R_d)(p - p_d)\|_{K_2}^2$$
$$\phi_1(R R_d^T) + \tfrac{1}{2}\|R^T p - R_d^T p_d\|_{K_2}^2$$



Bullo, F., & Murray, R. M. (1999). Tracking for fully actuated mechanical systems: a geometric framework. *Automatica*, *35*(1), 17-34.

Impedance Control of Fully-actuated Hexarotor





Rashad, R., Califano, F., & Stramigioli, S. (2019). Port-Hamiltonian passivity-based control on SE(3) of a fully actuated UAV for aerial physical interaction near-hovering. *IEEE Robotics and automation letters*, *4*(4), 4378-4385.

https://youtu.be/7T5WdH0gaew?si=oLVCVKXJatbhLOxb

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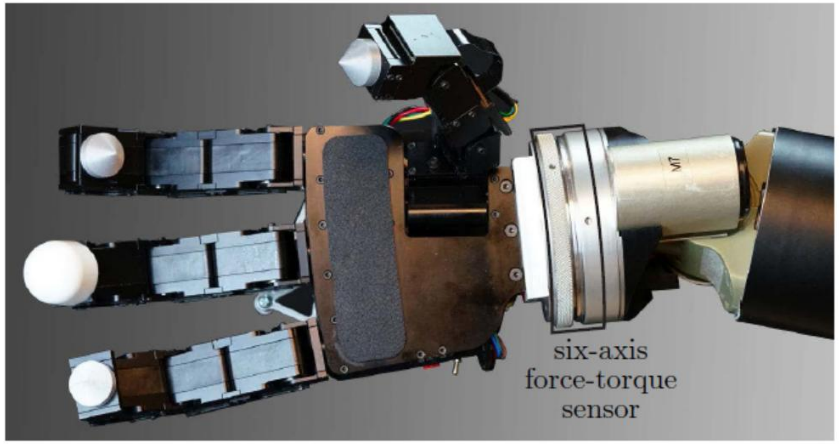
Admittance Control

- In an admittance-control algorithm, the wrench applied by the user at the end-effector is sensed using a force/torque sensor.
- The robot then should respond with the desired end-effector acceleration $\ddot{q}_d$ satisfying

$$\ddot{q}_d = \mathfrak{M}^{-1}\left(-\mathfrak{B}\dot{q} - \mathfrak{K}q + \mathcal{W}_{\text{int}}^{e,e}\right)$$

- Using the inverse kinematics, one can transform this to desired joint accelerations $\ddot{\theta}_d$ and then use the inverse dynamics to calculate the control torques.



Impedance: Motion input, Force output
Admittance: Force input, Motion output

Admittance Control of UR5e

https://youtu.be/OZGieHzbpAg?si=gf4W2h0xHJwbRoc9

Admittance Control of Quadrotor

https://www.youtube.com/watch?v=w2itwFJCgFQ