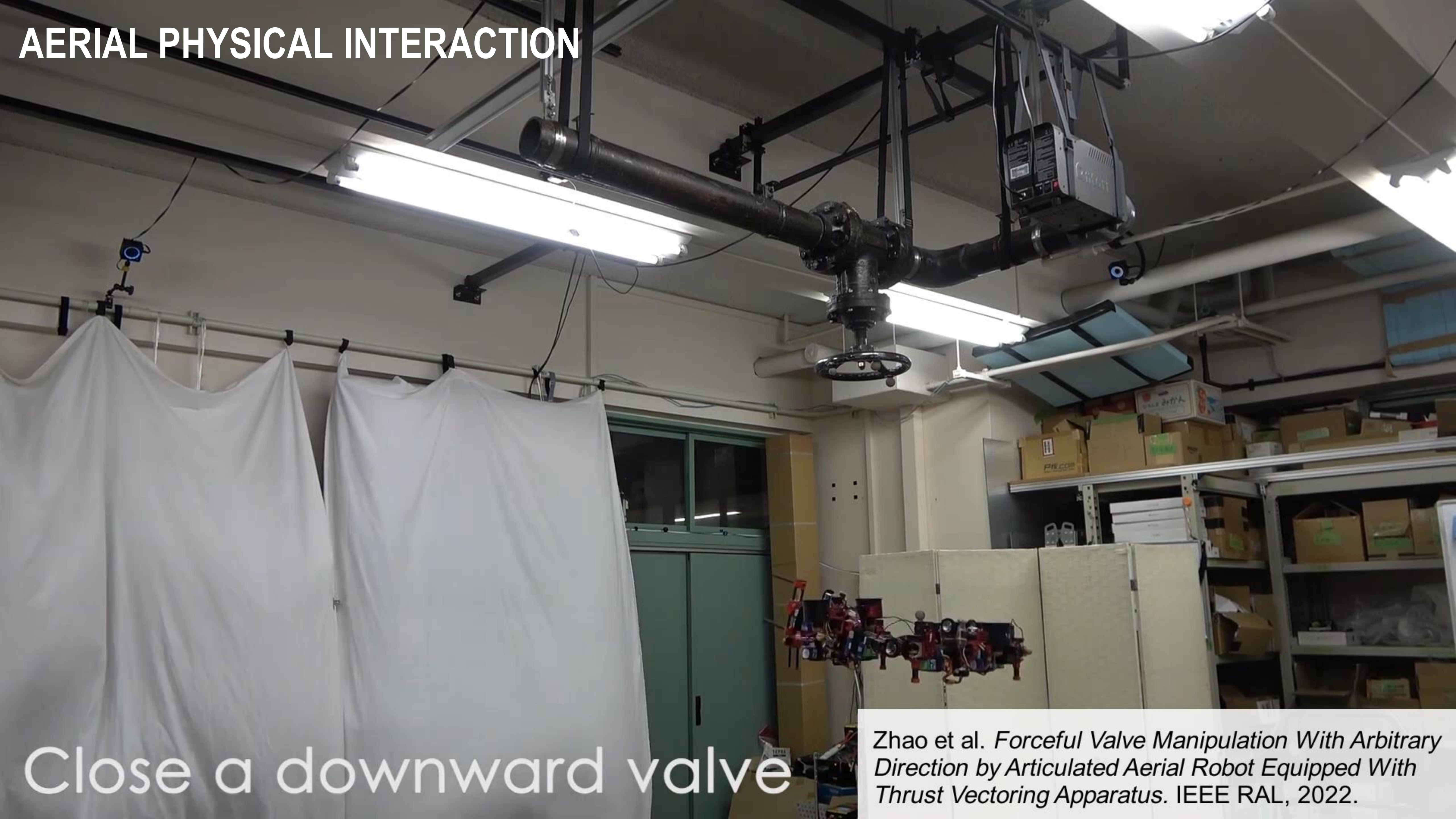
جامعة الملك فهد للبترول والمعادن
King Fahd University of Petroleum & Minerals

GEOMETRIC PORT-HAMILTONIAN MODELING AND CONTROL OF A FLYING END-EFFECTOR

RAMY RASHAD
ASSISTANT PROFESSOR, CIE DEPARTMENT

AERIAL PHYSICAL INTERACTION



Zhao et al. *Forceful Valve Manipulation With Arbitrary Direction by Articulated Aerial Robot Equipped With Thrust Vectoring Apparatus*. IEEE RAL, 2022.

UNIVERSITY OF TWENTE.

ICRA
LONDON · 2023

IEEE TRO 2022

ENERGY AWARE IMPEDANCE CONTROL OF A FLYING END-EFFECTOR IN THE PORT-HAMILTONIAN FRAMEWORK

Ramy Rashad*, Davide Bicego, Jelle Zult, Santiago Sanchez-Escalonilla, Ran Jiao, Antonio Franchi, Stefano Stramigioli

RAM ROBOTICS AND MECHATRONICS

MINI-COURSE: GEOMETRIC PORT-HAMILTONIAN MODELING AND CONTROL OF A FLYING END-EFFECTOR

- **Research Question:**

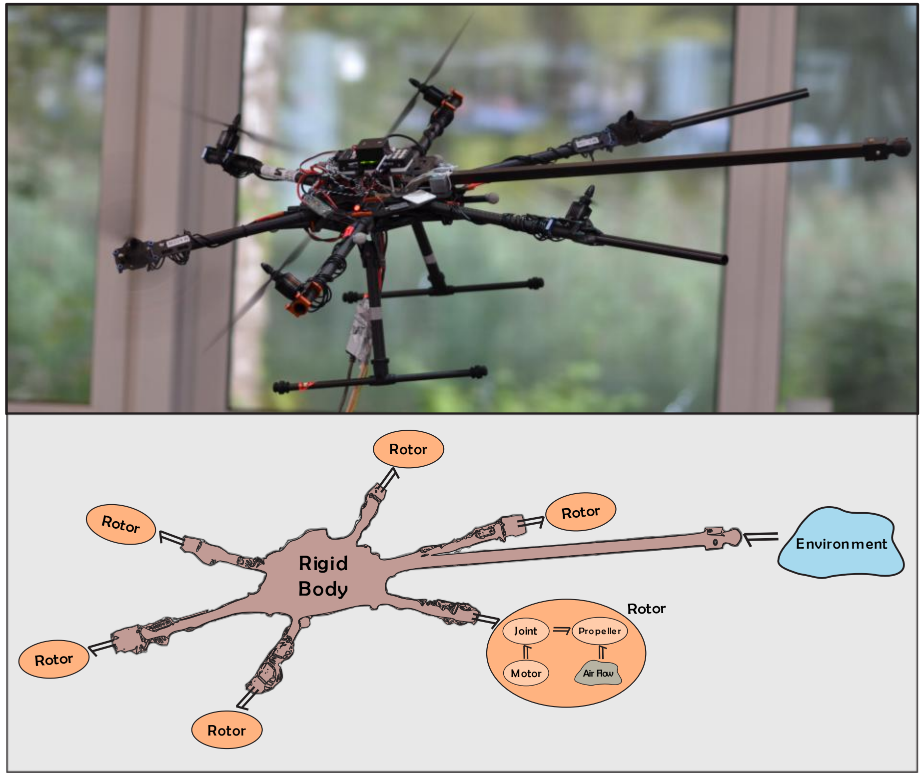
> How to design a **control** algorithm that can enable a fully-actuated multirotor-aerial-vehicle (i.e., a flying end-effector) to physically interact with an **unknown** environment in a **stable** and **safe** manner.

- **Lecture 1:** Oct. 16, 10:00–11:30 am — *Foundations: From Aerial Interaction to Geometric Impedance on SE(3)*
- **Lecture 2:** Oct. 19, 10:00–11:30 am — *Wrench Regulation via Variable Stiffness (and Why It Breaks Passivity)*
- **Lecture 3:** Oct. 21, 10:00–11:30 am — *Energy-Aware Control: Tanks, Routing, and Safety in Contact-Loss*

~1-2 hour Study Material before every lecture.

MINI-COURSE: GEOMETRIC PORT-HAMILTONIAN MODELING AND CONTROL OF A FLYING END-EFFECTOR

- **What you'll be exposed to Today:**
  - How to model a fully-actuated MAV?
  - What is the Geometric Port-Hamiltonian framework ?
  - How to represent rigid body dynamics in this framework ?



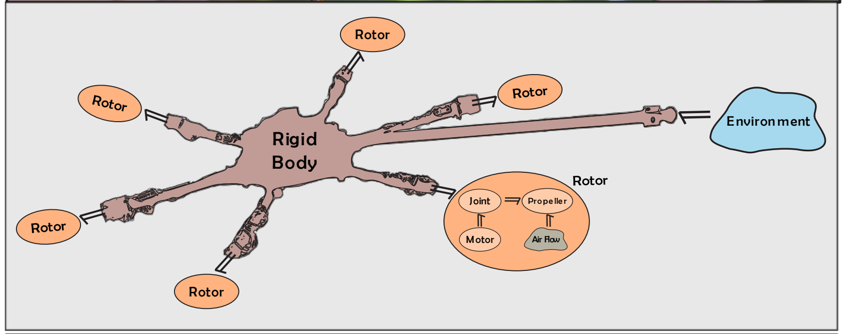
MATHEMATICAL MODELING OF A FLYING END- EFFECTOR

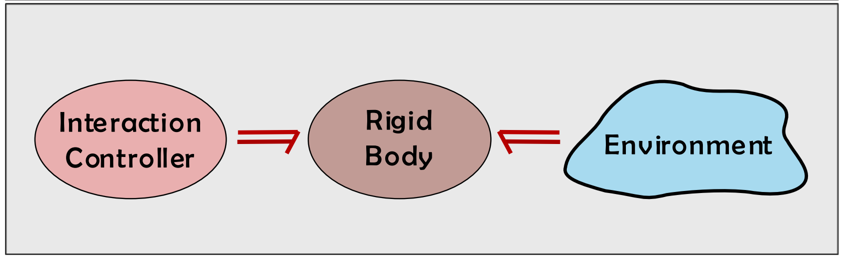
FLYING END-EFFECTOR

- We consider the flying end-effector to be a fully-actuated MAV consisting of a single rigid body that is connected to 6 tilted propellers via revolute joints activated by electric motors.

- For control purposes, it is common to neglect any dynamics of the MAV's propellers and consider the whole MAV as a single rigid body.

- This is a valid assumption because they change at a faster time scale compared to the dynamics of the MAV's body.



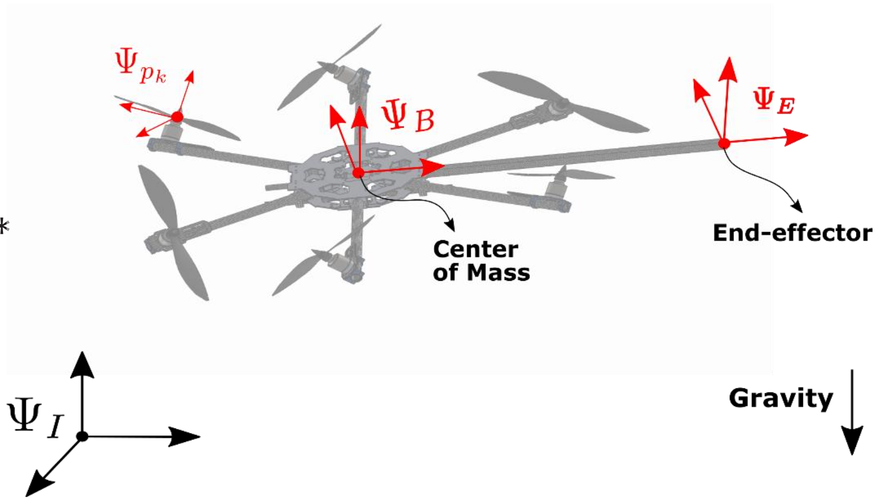


FLYING END-EFFECTOR MODEL

- We denote by
  - Ψ_I - Inertial reference frame
  - Ψ_B - Body-fixed reference frame attached to center-of-mass
  - Ψ_E - Body-fixed reference frame attached to tip (end-effector)

- Configuration of a Rigid Body
  - $\xi_B^I \in \mathbb{R}^3$ - Position of the origin of Ψ_B w.r.t. Ψ_I
  - $R_B^I \in SO(3)$- Orientation of Ψ_B w.r.t. Ψ_I

- Velocity of a Rigid Body
  - $v_B^{*,I} \in \mathbb{R}^3$- Linear velocity of Ψ_B w.r.t. Ψ_I, expressed in Ψ_*
  - $\omega_B^{*,I} \in \mathbb{R}^3$- Angular velocity of Ψ_B w.r.t. Ψ_I, expressed in Ψ_*



FLYING END-EFFECTOR MODEL

- **Nonlinear Equations of Motion**
  - Kinematics

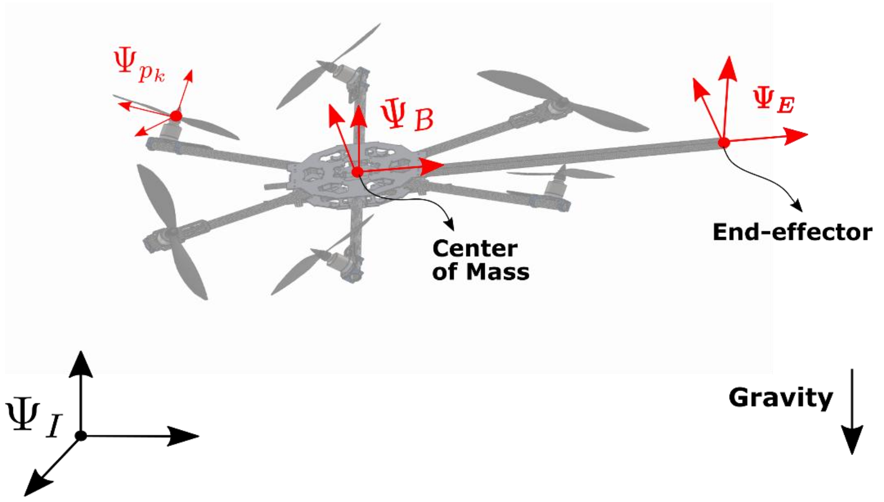
$$\dot{\boldsymbol{\xi}}_B^I = \boldsymbol{R}_B^I \boldsymbol{v}_B^{B,I}$$

$$\dot{\boldsymbol{R}}_B^I = \boldsymbol{R}_B^I \tilde{\boldsymbol{\omega}}_B^{B,I}$$

  - Dynamics

$$\boldsymbol{J}\dot{\boldsymbol{\omega}}_B^{B,I} = -\tilde{\boldsymbol{\omega}}_B^{B,I}\boldsymbol{J}\boldsymbol{\omega}_B^{B,I} + \boldsymbol{\tau}_{\mathrm{c}}^B + \boldsymbol{\tau}_{\mathrm{int}}^B$$

$$m\dot{\boldsymbol{v}}_B^{B,I} = -m\tilde{\boldsymbol{\omega}}_B^{B,I}\boldsymbol{v}_B^{B,I} + \boldsymbol{f}_{\mathrm{g}}^B + \boldsymbol{f}_{\mathrm{c}}^B + \boldsymbol{f}_{\mathrm{int}}^B$$



FLYING END-EFFECTOR MODEL

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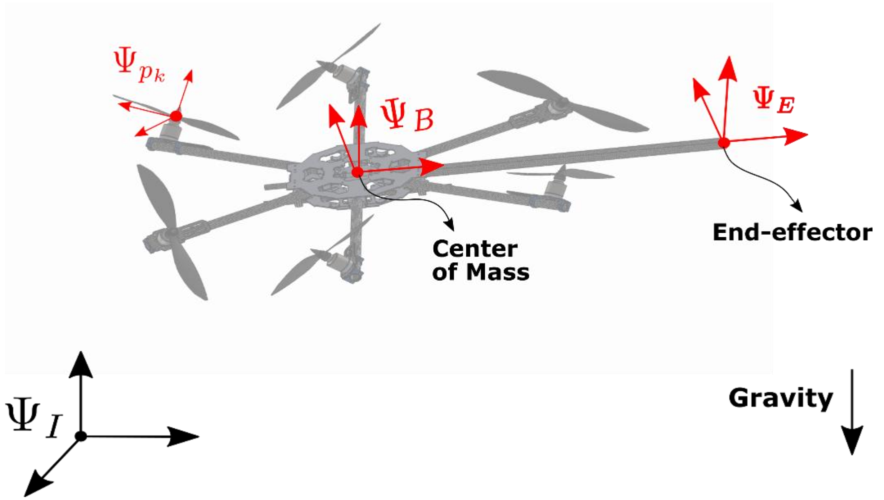
$$\dot{\boldsymbol{\xi}}_B^I = \boldsymbol{R}_B^I \boldsymbol{v}_B^{B,I}$$

$$\dot{\boldsymbol{R}}_B^I = \boldsymbol{R}_B^I \tilde{\boldsymbol{\omega}}_B^{B,I}$$

  - Dynamics

$$\boldsymbol{J}\dot{\boldsymbol{\omega}}_B^{B,I} = -\tilde{\boldsymbol{\omega}}_B^{B,I}\boldsymbol{J}\boldsymbol{\omega}_B^{B,I} + \overbrace{\boldsymbol{\tau}_{\mathrm{c}}^B}^{\text{Control Torques}} + \overbrace{\boldsymbol{\tau}_{\mathrm{int}}^B}^{\text{Interaction Torques}}$$

$$m\dot{\boldsymbol{v}}_B^{B,I} = -m\tilde{\boldsymbol{\omega}}_B^{B,I}\boldsymbol{v}_B^{B,I} + \boldsymbol{f}_{\mathrm{g}}^B + \underbrace{\boldsymbol{f}_{\mathrm{c}}^B}_{\text{Control Forces}} + \underbrace{\boldsymbol{f}_{\mathrm{int}}^B}_{\text{Interaction Forces}}$$



FLYING END-EFFECTOR MODEL

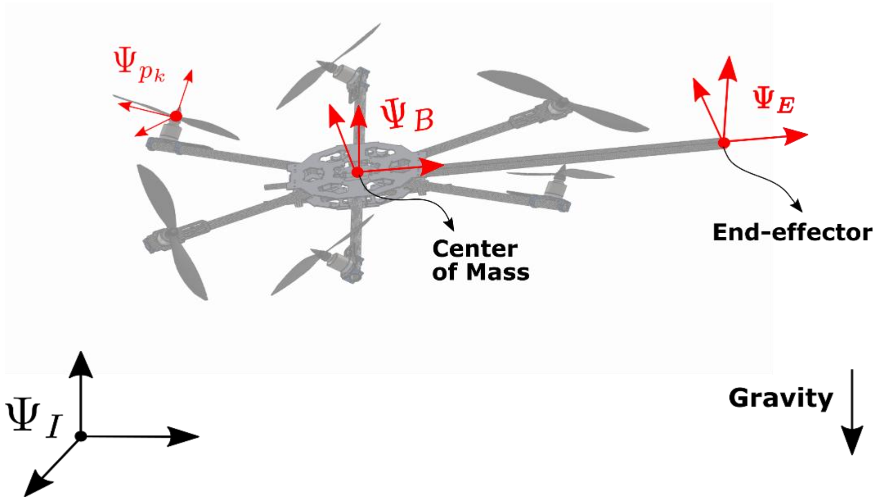
- **Nonlinear Equations of Motion on SE(3)**
  - Kinematics

$$\dot{H}^I_B = H^I_B \tilde{T}^{B,I}_B$$

  - Dynamics

$$\dot{P}^B = J_{\mathrm{k}}(P^B)\mathcal{I}^{-1}P^B + W^B_{\mathrm{g}} + W^B_{\mathrm{c}} + W^B_{\mathrm{int}}$$

- $H^I_B \in SE(3)$- Pose of Ψ_B w.r.t. Ψ_I
- $T^{*,I}_B \in \mathbb{R}^6$- Generalized velocity of Ψ_B w.r.t. Ψ_I, expressed in Ψ_*
- $W^*_{\mathrm{src}} \in (\mathbb{R}^6)^*$- Generalized forces by src acting on body expressed in Ψ_*
- $P^* \in (\mathbb{R}^6)^*$- Generalized momentum of body expressed in Ψ_*



FLYING END-EFFECTOR MODEL

- **Nonlinear Equations of Motion on SE(3)**
  - Kinematics

$$\dot{H}_B^I = H_B^I \tilde{T}_B^{B,I}$$

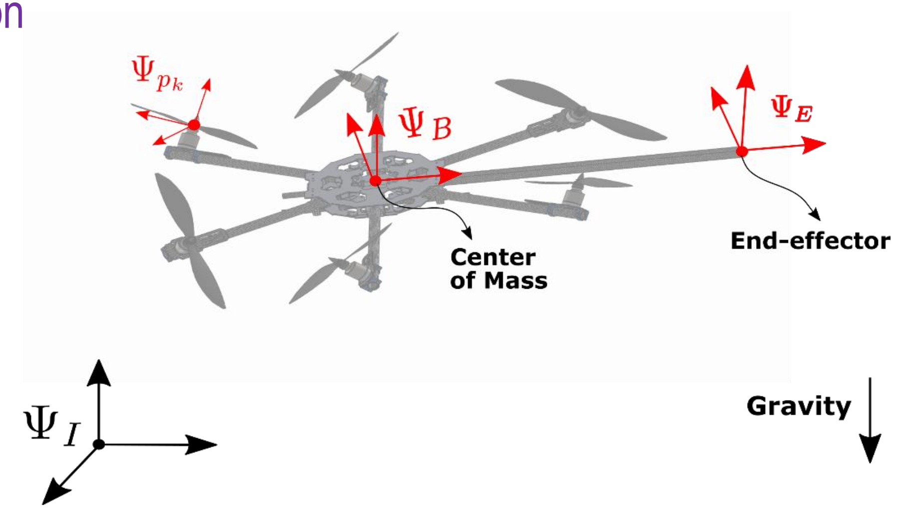
  - Dynamics

$$\dot{P}^B = J_k(P^B)\mathcal{I}^{-1}P^B + W_g^B + W_c^B + W_{int}^B$$

Control Wrench

Interaction Wrench

$T_B^{*,I}$: Generalized Velocity = Twist
W_{src}^*: Generalized Forces = Wrench



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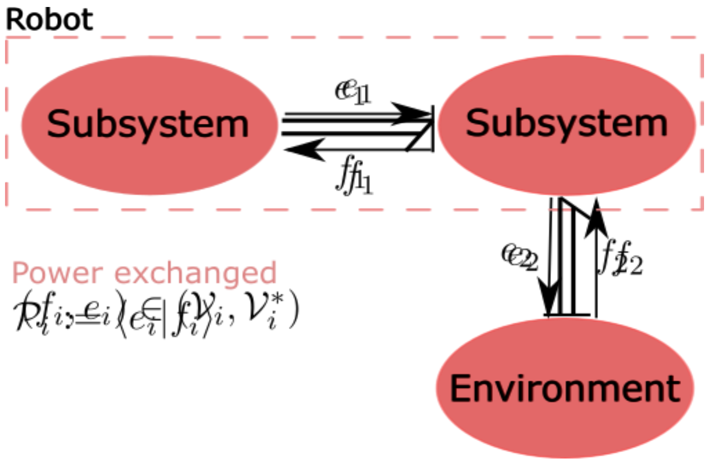
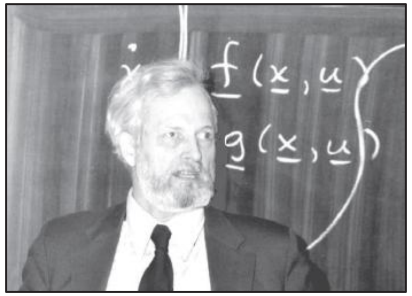
GEOMETRIC PORT-HAMILTONIAN FRAMEWORK

GEOMETRIC PORT-HAMILTONIAN FRAMEWORK

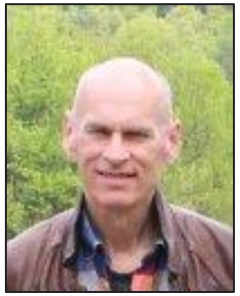
Port-based Modeling + **Geometric Mechanics** + **Systems & Control Theory**

- *Energy* instead of *Signals*
- Multi-physics & open systems





H. Paynter



A. van der Schaft

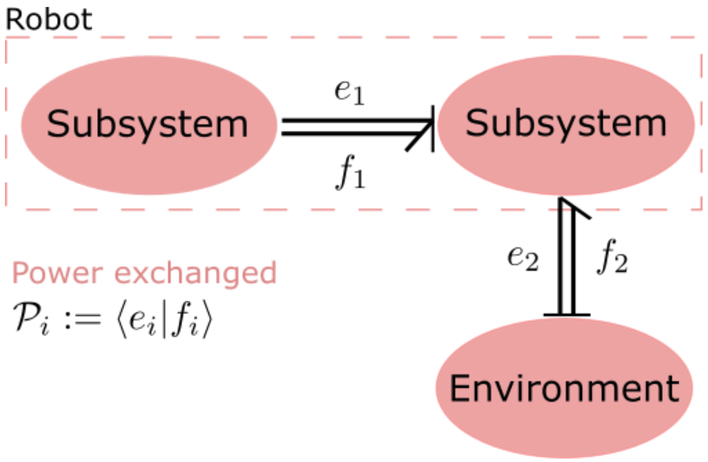
B. Maschke

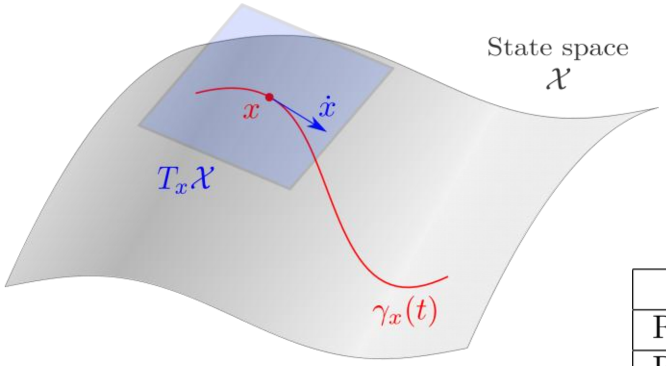
GEOMETRIC PORT-HAMILTONIAN FRAMEWORK

Port-based Modeling + **Geometric Mechanics** + **Systems & Control Theory**

- *Energy* instead of *Signals*
- Multi-physics & open systems

- Coordinate-free descriptions
- Finite & infinite-dimension systems





$$\dot{x} = f(x) + g(x)u$$
$$y = h(x)$$

Dynamics	Configuration space	$\mathcal{X}$
Rigid body rotation	Special Orthogonal Group	$SO(3)$
Rigid body motion	Special Eucldiean Group	$SE(3)$
Fluid flow	Diffeomorphism Group	$\mathrm{Diff}(M)$
Nonlinear elasticity	Space of Embeddings	$\mathrm{Emb}(B, A)$

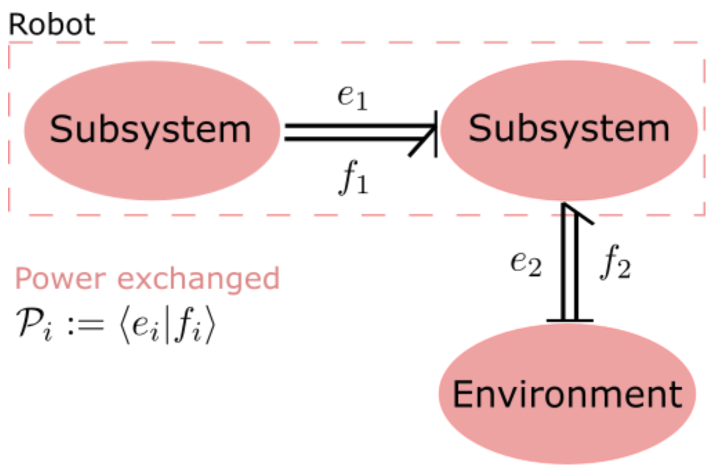
GEOMETRIC PORT-HAMILTONIAN FRAMEWORK

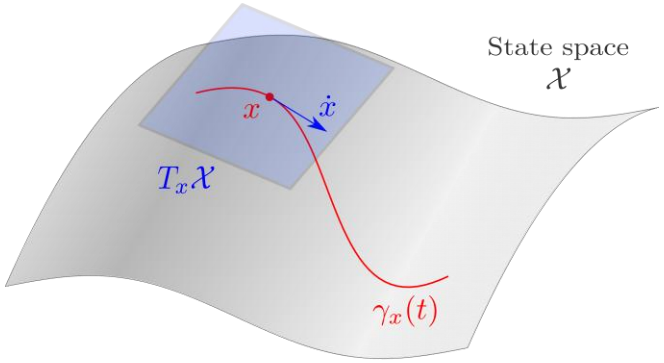
Port-based Modeling + **Geometric Mechanics** + **Systems & Control Theory**

- *Energy* instead of *Signals*
- Multi-physics & open systems

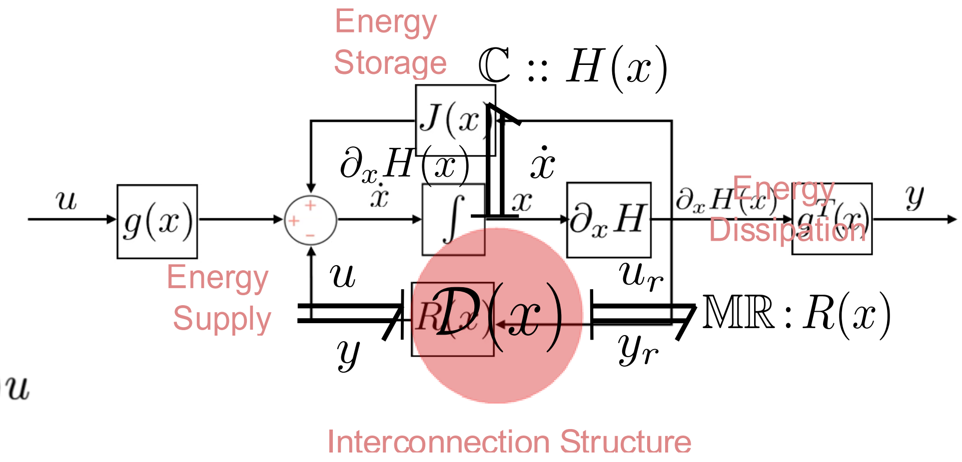
- Coordinate-free descriptions
- Finite & infinite-dimension systems

- Exploit underlying structure
- Energy-based analysis & control



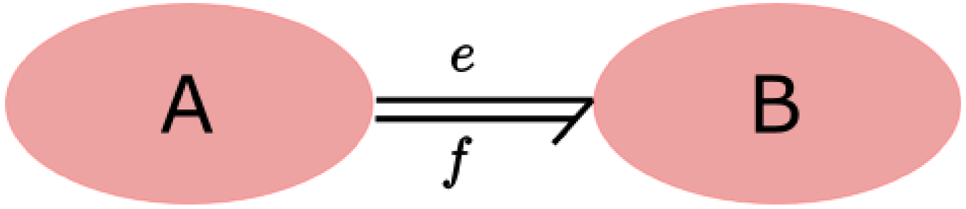


$$\dot{x} = [J(x) - R(x)]\partial_x H(x) + g(x)u$$
$$y = g^T(x)\partial_x H(x)$$

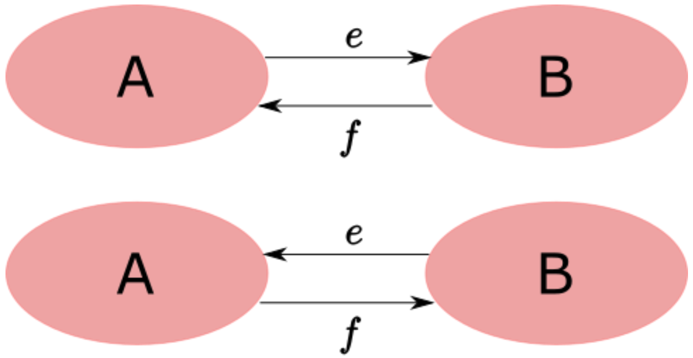


BASICS OF PORT-BASED PARADIGM

- The most important physical concept on which physics is based on is "**Energy**".
- **Properties:**
  - Invariance in time: 1st principle of Thermodynamics
  - Continuity in space: Heaviside Principle
- Two subsystems (A & B) **interact** with each other by flow of energy.
- Exchange of energy is characterized by "**Power Ports**"



$$(f, e) \in (\mathcal{F}, \mathcal{F}^*)$$

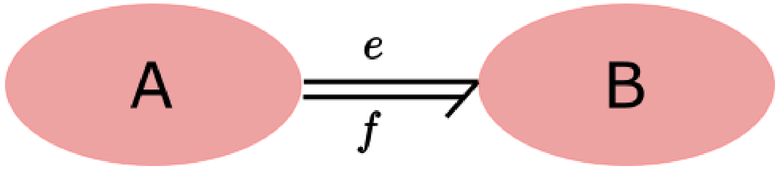


$$\text{Power} = \langle e|f\rangle_{\mathcal{F}} \in \mathbb{R}$$

BASICS OF PORT-BASED PARADIGM

- Efforts and Flows Examples

Physical Domain	Effort	Flow
Translation Mechanical	Force (F)	Velocity (v)
Rotational Mechanical	Torque (τ)	Angular velocity (ω)
Rigid Body Mechanics	Wrench (W)	Twist (T)
Electromagnetism	Voltage (V)	Current (i)
Hydraulic	Pressure (p)	Flow rate (Q)
Thermal	Temperature (T)	Entropy flow (f_s)
Chemical	Chemical Potential (μ)	Molar flow (f_N)



$$(f, e) \in (\mathcal{F}, \mathcal{F}^*)$$

INTERCONNECTION OF SUBSYSTEMS

- A physical system composed of interconnected physical subsystems can be seen as a set of subsystems interacting with each other
- Each *i-th* subsystem has its own

State	$x_i \in \mathcal{X}_i$
Energy function	$\mathcal{H}_i : \mathcal{X} \to \mathbb{R}$ $x_i \longmapsto \mathcal{H}_i(x_i)$
Flow	$f_i \in \mathcal{F}_i$
Effort	$e_i \in \mathcal{F}_i^*$
Power Balance	$\dot{\mathcal{H}}_i = \langle e_i \vert f_i \rangle_{\mathcal{F}_i}$

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Power Balance	$\dot{\mathcal{H}}_i = \langle e_i \vert f_i \rangle_{\mathcal{F}_i}$

Example: Point mass moving in 3D

State	**Momentum** $\boldsymbol{p} = m\boldsymbol{v}$
Energy function	Kinetic energy $\mathcal{H}(\boldsymbol{p}) = \frac{1}{2\,m}\,\boldsymbol{p}^{\boldsymbol{T}}\boldsymbol{p}$
Flow	Rate of Change of Momentum $\dot{\boldsymbol{p}}$
Effort	Gradient of Kinetic Energy $\frac{\partial\mathcal{H}}{\partial\boldsymbol{p}} = \frac{\boldsymbol{p}}{m} = \boldsymbol{v}$
Power Balance	$\dot{\mathcal{H}}_i = \left(\frac{\partial\mathcal{H}}{\partial\boldsymbol{p}}\right)^{\top} \dot{p}$

MODELING COMPLEX SYSTEMS BY INTERCONNECTION

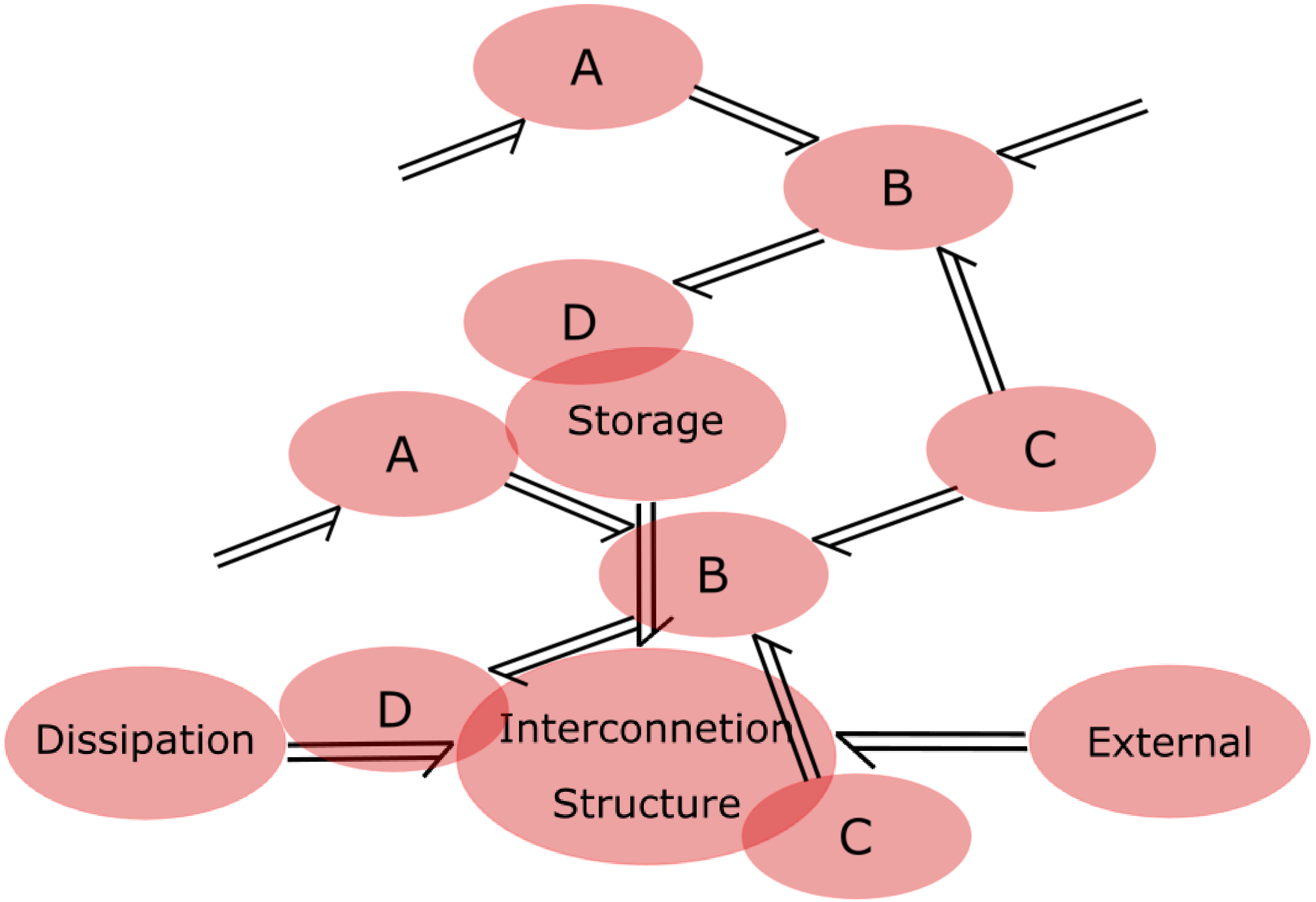
- <u>Total State</u>: $x = (x_1, \dots, x_n)$
- <u>Total Energy</u>: $\mathcal{H}(x) = \sum_i \mathcal{H}_i(x_i)$

Port-Hamiltonian Representation

$$\dot{x} = \big(J(x) - R(x)\big)\left(\frac{\partial \mathcal{H}}{\partial x}\right) + G\,u$$
$$y = G^T\left(\frac{\partial \mathcal{H}}{\partial x}\right)$$

Standard State-Space Representation

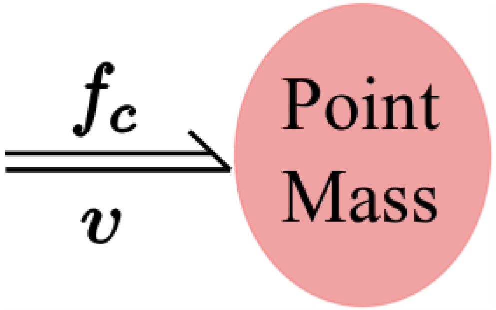
$$\dot{x} = f(x) + G\,u$$
$$y = h(x)$$



PORT-HAMILTONIAN MODEL MODELING OF POINT MASS

Point Mass Model on $\mathbb{R}^3$:

$$\dot{\xi} = v$$
$$m\dot{v} = -mg\hat{z}_I + f_c$$



Hamiltonian Energy:

$$\mathcal{H} = \frac{1}{2m}p^\top p + mg\xi_z$$

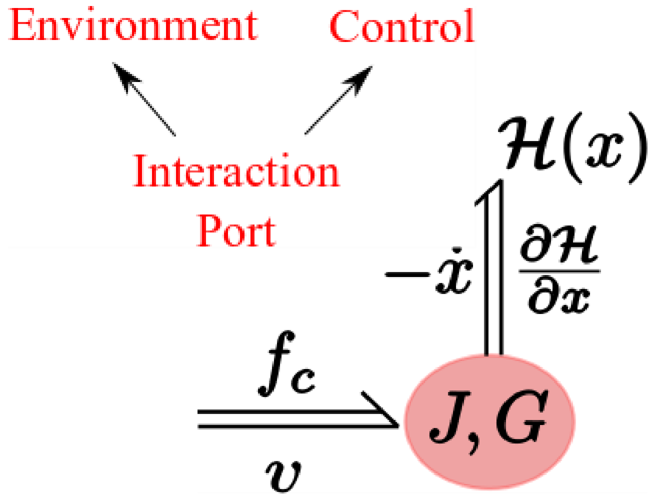
$$\boxed{\begin{pmatrix} \partial\mathcal{H}/\partial\xi \\ \partial\mathcal{H}/\partial p \end{pmatrix} = \begin{pmatrix} mg\hat{z}_I \\ p/m \end{pmatrix}}$$

Port-Hamiltonian Model:

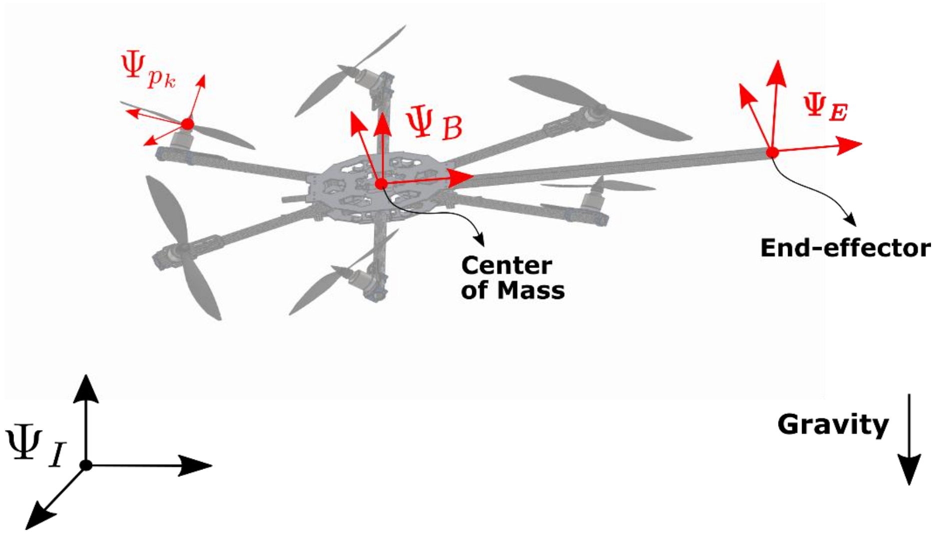
$$\boxed{\begin{aligned} \begin{pmatrix} \dot{\xi} \\ \dot{p} \end{pmatrix} &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \partial\mathcal{H}/\partial\xi \\ \partial\mathcal{H}/\partial p \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} f_c \\ v &= \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} \partial\mathcal{H}/\partial\xi \\ \partial\mathcal{H}/\partial p \end{pmatrix} \end{aligned}}$$

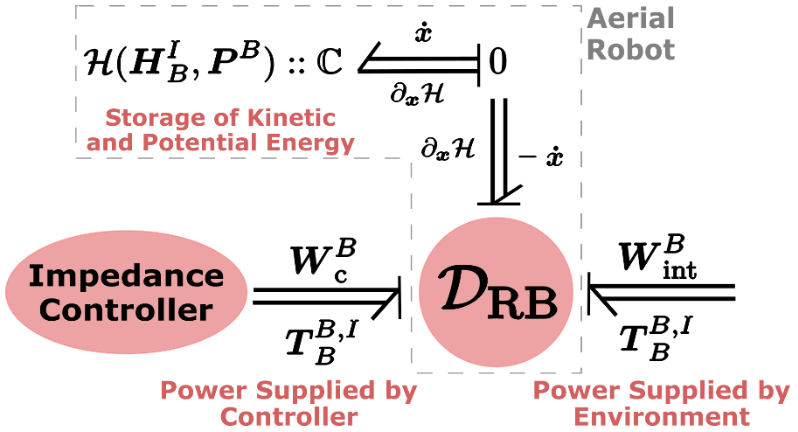
$$\boxed{\begin{aligned} \dot{x} &= J\frac{\partial\mathcal{H}}{\partial x} + Gf_c \\ v &= G^\top\frac{\partial\mathcal{H}}{\partial x} \end{aligned}}$$

Skew Symmetric



PORT-HAMILTONIAN MODEL OF A FLYING END-EFFECTOR





Port-Hamiltonian dynamics

$$\begin{pmatrix}\dot{\boldsymbol{H}}^I_B\\ \dot{\boldsymbol{P}}^B\end{pmatrix} = \begin{pmatrix}\boldsymbol{0} & \chi_{\boldsymbol{H}^I_B}\\ -\chi^*_{\boldsymbol{H}^I_B} & \boldsymbol{J}_{\mathrm{k}}(\boldsymbol{P}^B)\end{pmatrix}\begin{pmatrix}\partial_{\boldsymbol{H}^I_B}\mathcal{H}\\ \partial_{\boldsymbol{P}^B}\mathcal{H}\end{pmatrix} + \begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{I}_6\end{pmatrix}(\boldsymbol{W}^B_{\mathrm{c}} + \boldsymbol{W}^B_{\mathrm{int}})$$

$$\boldsymbol{T}^{B,I}_B = \begin{pmatrix}\boldsymbol{0} & \boldsymbol{I}_6\end{pmatrix}\begin{pmatrix}\partial_{\boldsymbol{H}^I_B}\mathcal{H}\\ \partial_{\boldsymbol{P}^B}\mathcal{H}\end{pmatrix}$$

Energy of aerial robot

$$\mathcal{H}(\boldsymbol{H}^I_B, \boldsymbol{P}^B) = \underbrace{\frac{1}{2}(\boldsymbol{P}^B)^\top \boldsymbol{\mathcal{I}}^{-1}\boldsymbol{P}^B}_{\text{Kinetic}} + \underbrace{m(\boldsymbol{\xi}^I_B)^\top \boldsymbol{g}}_{\text{Potential}}$$