جامعة الملك فهد للبترول والمعادن
King Fahd University of Petroleum & Minerals

GEOMETRIC PORT-HAMILTONIAN MODELING AND CONTROL OF A FLYING END-EFFECTOR

RAMY RASHAD
ASSISTANT PROFESSOR, CIE DEPARTMENT

OUTLINE

- Recap last lecture
- Analysis of mass-damper dynamics
- Analysis of flying-effector dynamics
- Impedance control in the pH framework

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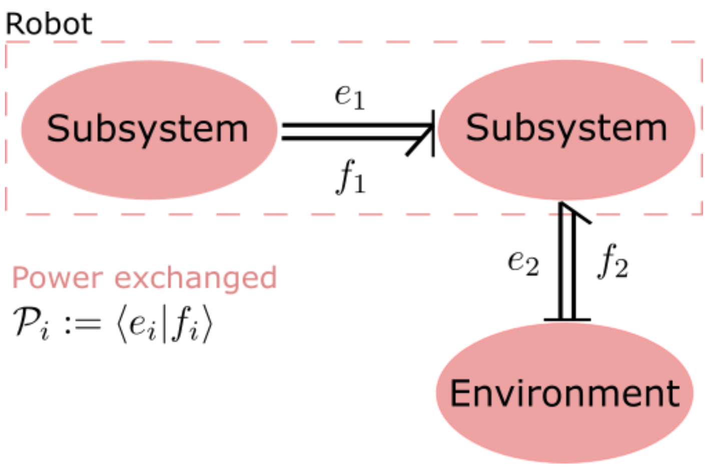
GEOMETRIC PORT-HAMILTONIAN FRAMEWORK

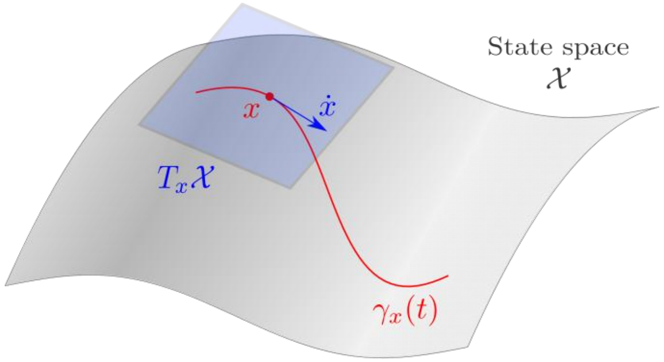
Port-based Modeling + **Geometric Mechanics** + **Systems & Control Theory**

- *Energy* instead of *Signals*
- Multi-physics & open systems

- Coordinate-free descriptions
- Finite & infinite-dimension systems

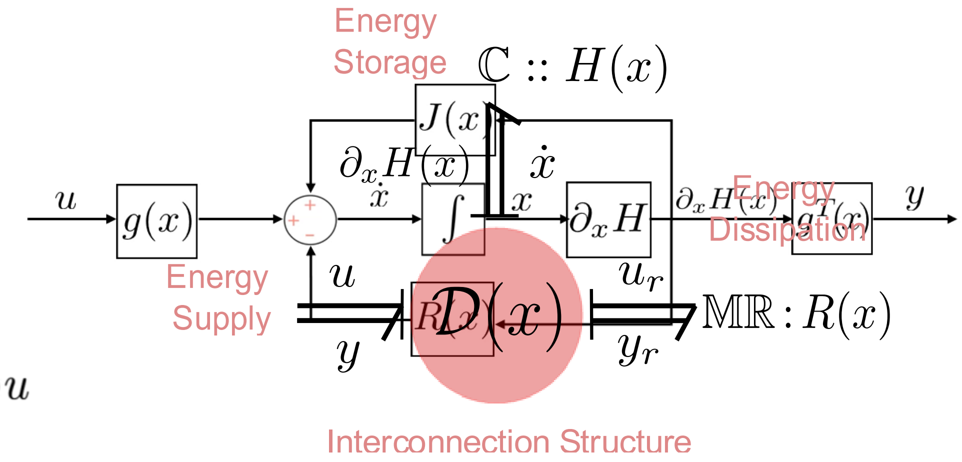
- Exploit underlying structure
- Energy-based analysis & control





$$\dot{x} = [J(x) + R(x)]\partial_x H(x) + g(x)u$$

$$y = g^T(x)\partial_x H(x)$$



INTERCONNECTION OF SUBSYSTEMS

- A physical system composed of interconnected physical subsystems can be seen as a set of subsystems interacting with each other
- Each *i-th* subsystem has its own

State	$x_i \in \mathcal{X}_i$
Energy function	$\mathcal{H}_i : \mathcal{X} \rightarrow \mathbb{R}$ $x_i \longmapsto \mathcal{H}_i(x_i)$
Flow	$f_i \in \mathcal{F}_i$
Effort	$e_i \in \mathcal{F}_i^*$
Power Balance	$\dot{\mathcal{H}}_i = \langle e_i \vert f_i \rangle_{\mathcal{F}_i}$

Example: Point mass moving in 3D

State	**Momentum** $\boldsymbol{p} = m\boldsymbol{v}$
Energy function	Kinetic energy $\mathcal{H}(\boldsymbol{p}) = \frac{1}{2\,m}\,\boldsymbol{p}^{\boldsymbol{T}}\boldsymbol{p}$
Flow	Rate of Change of Momentum $\dot{\boldsymbol{p}}$
Effort	Gradient of Kinetic Energy $\frac{\boldsymbol{\partial}\mathcal{H}}{\boldsymbol{\partial p}} = \frac{\boldsymbol{p}}{m} = \boldsymbol{v}$
Power Balance	$\dot{\mathcal{H}}_i = \left(\frac{\boldsymbol{\partial}\mathcal{H}}{\boldsymbol{\partial p}}\right)^{\top} \dot{p}$

MODELING COMPLEX SYSTEMS BY INTERCONNECTION

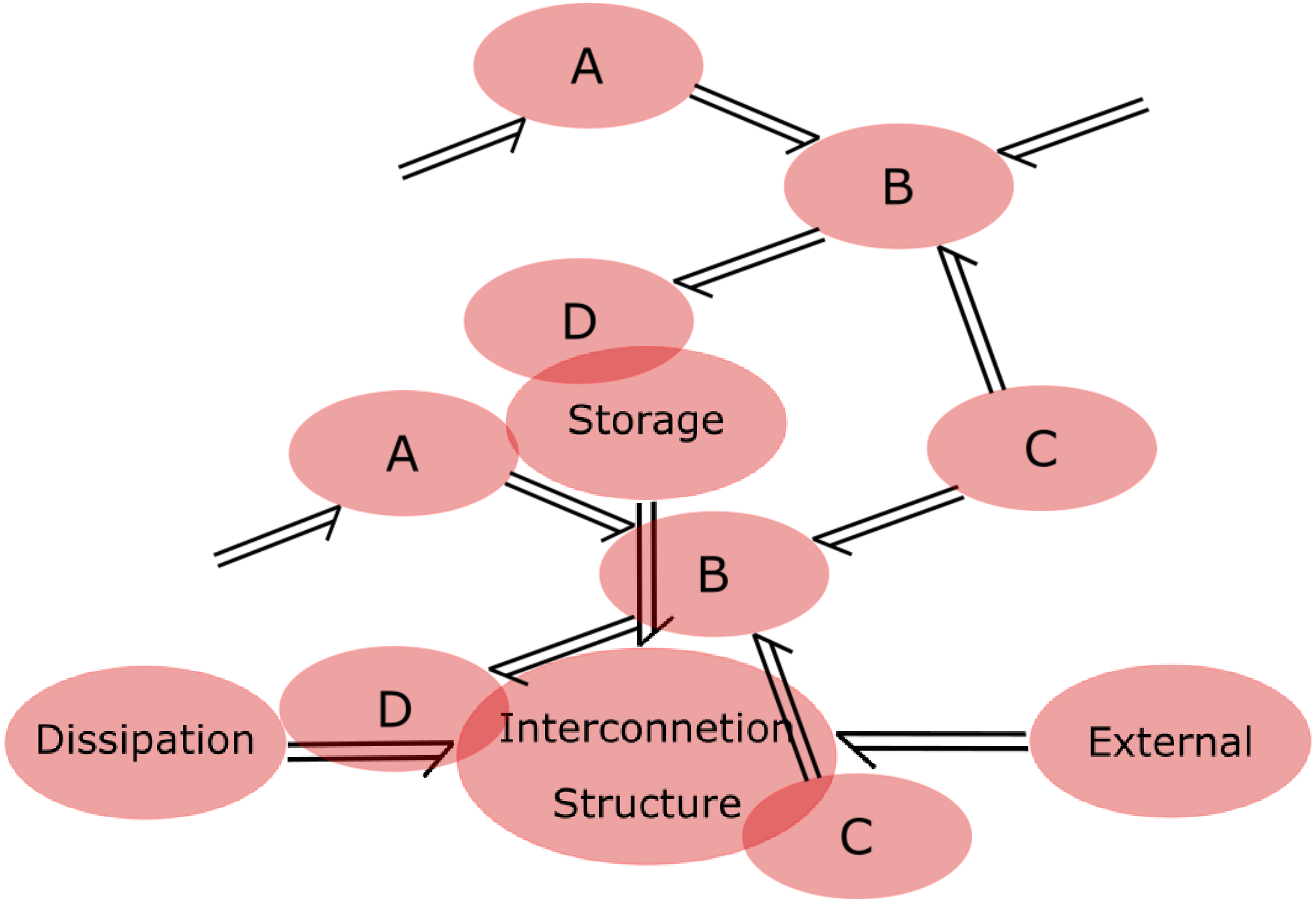
- Total State: $x = (x_1, \dots, x_n)$
- Total Energy: $\mathcal{H}(x) = \sum_i \mathcal{H}_i(x_i)$

Port-Hamiltonian Representation

$$\dot{x} = \big(J(x) - R(x)\big)\left(\frac{\partial \mathcal{H}}{\partial x}\right) + G\,u$$
$$y = G^T\left(\frac{\partial \mathcal{H}}{\partial x}\right)$$

Standard State-Space Representation

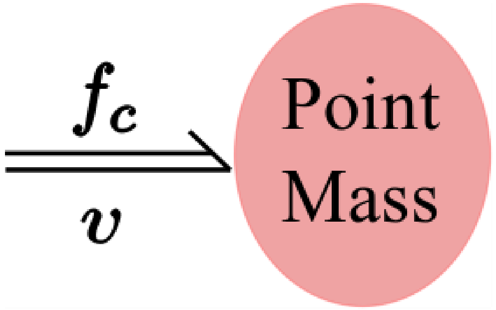
$$\dot{x} = f(x) + G\,u$$
$$y = h(x)$$



PORT-HAMILTONIAN MODEL MODELING OF POINT MASS

Point Mass
Model on $\mathbb{R}^3$:

$$\begin{aligned} \dot{\xi} &= v \\ m\dot{v} &= -mg\hat{z}_I + f_c \end{aligned}$$



Hamiltonian
Energy:

$$\mathcal{H} = \tfrac{1}{2m}p^\top p + mg\xi_z$$

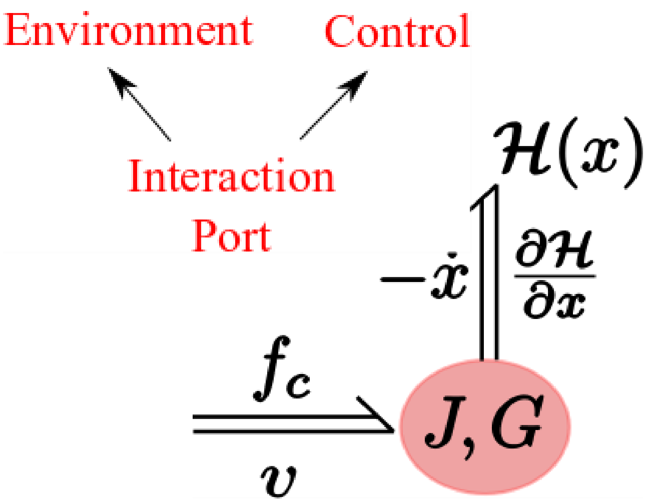
$$\boxed{\begin{pmatrix} \partial\mathcal{H}/\partial\xi \\ \partial\mathcal{H}/\partial p \end{pmatrix} = \begin{pmatrix} mg\hat{z}_I \\ p/m \end{pmatrix}}$$

Port-Hamiltonian Model:

$$\boxed{\begin{aligned} \begin{pmatrix} \dot{\xi} \\ \dot{p} \end{pmatrix} &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \partial\mathcal{H}/\partial\xi \\ \partial\mathcal{H}/\partial p \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} f_c \\ v &= \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} \partial\mathcal{H}/\partial\xi \\ \partial\mathcal{H}/\partial p \end{pmatrix} \end{aligned}}$$

$$\boxed{\begin{aligned} \dot{x} &= J\frac{\partial\mathcal{H}}{\partial x} + Gf_c \\ v &= G^\top\frac{\partial\mathcal{H}}{\partial x} \end{aligned}}$$

Skew
Symmetric



FLYING END-EFFECTOR MODEL

- **Nonlinear Equations of Motion**
  - Kinematics

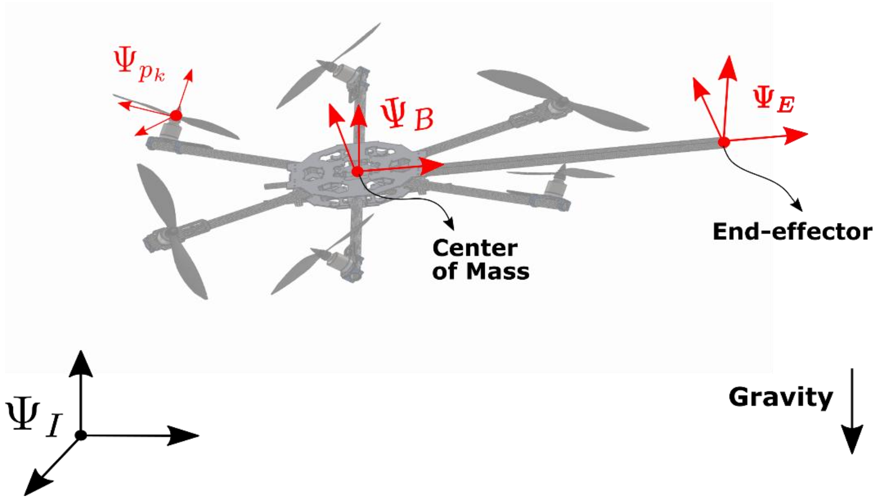
$$\dot{\boldsymbol{\xi}}_B^I = \boldsymbol{R}_B^I \boldsymbol{v}_B^{B,I}$$

$$\dot{\boldsymbol{R}}_B^I = \boldsymbol{R}_B^I \tilde{\boldsymbol{\omega}}_B^{B,I}$$

  - Dynamics

$$\boldsymbol{J}\dot{\boldsymbol{\omega}}_B^{B,I} = -\tilde{\boldsymbol{\omega}}_B^{B,I}\boldsymbol{J}\boldsymbol{\omega}_B^{B,I} + \overbrace{\boldsymbol{\tau}_{\mathrm{c}}^B}^{\text{Control Torques}} + \overbrace{\boldsymbol{\tau}_{\mathrm{int}}^B}^{\text{Interaction Torques}}$$

$$m\dot{\boldsymbol{v}}_B^{B,I} = -m\tilde{\boldsymbol{\omega}}_B^{B,I}\boldsymbol{v}_B^{B,I} + \boldsymbol{f}_{\mathrm{g}}^B + \underbrace{\boldsymbol{f}_{\mathrm{c}}^B}_{\text{Control Forces}} + \underbrace{\boldsymbol{f}_{\mathrm{int}}^B}_{\text{Interaction Forces}}$$



FLYING END-EFFECTOR MODEL

$T_B^{*,I}$: Generalized Velocity = Twist
W_{src}^{*}: Generalized Forces = Wrench

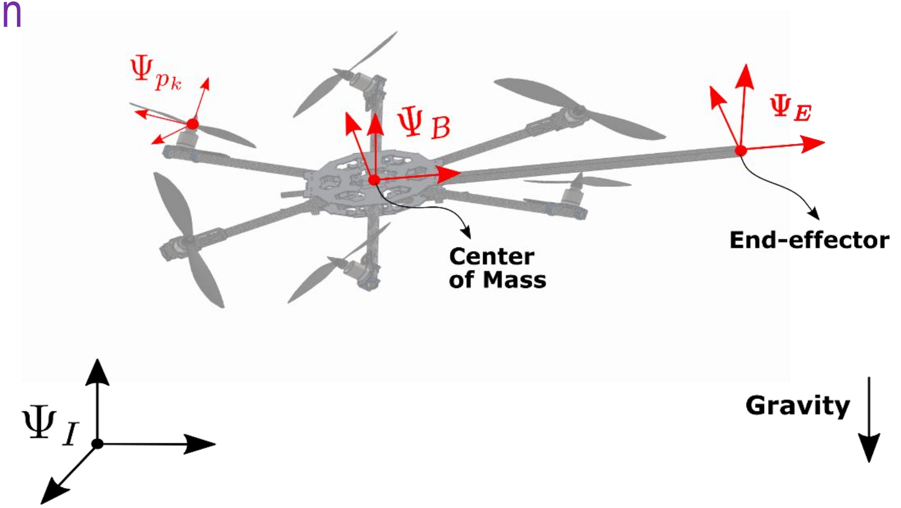
- **Nonlinear Equations of Motion on SE(3)**
  - Kinematics

$$\dot{H}_B^I = H_B^I \tilde{T}_B^{B,I}$$

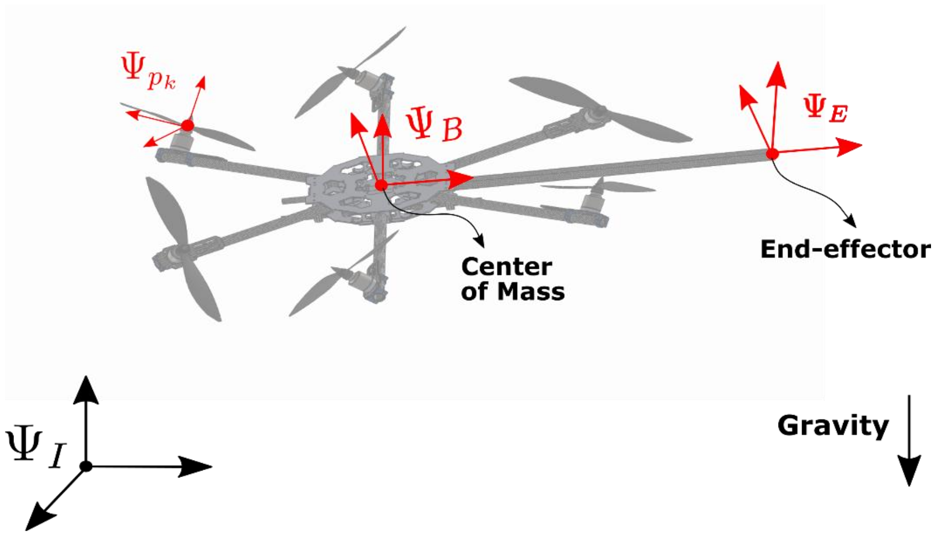
  - Dynamics

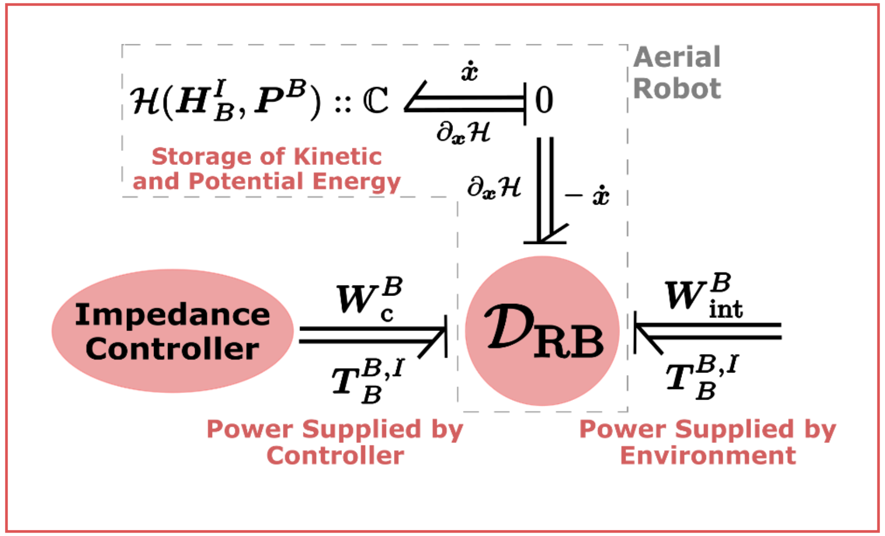
$$\dot{P}^B = J_{\mathrm{k}}(P^B)\mathcal{I}^{-1}P^B + W_{\mathrm{g}}^B + \underbrace{W_{\mathrm{c}}^B}_{\text{Control Wrench}} + \underbrace{W_{\mathrm{int}}^B}_{\text{Interaction Wrench}}$$

- $H_B^I \in SE(3)$- Pose of Ψ_B w.r.t. Ψ_I
- $T_B^{*,I} \in \mathbb{R}^6$- Generalized velocity of Ψ_B w.r.t. Ψ_I, expressed in Ψ_*
- $W_{\mathrm{src}}^{*} \in (\mathbb{R}^6)^*$- Generalized forces by src acting on body expressed in Ψ_*
- $P^* \in (\mathbb{R}^6)^*$- Generalized momentum of body expressed in Ψ_*



PORT-HAMILTONIAN MODEL OF A FLYING END-EFFECTOR





Port-Hamiltonian dynamics

$$\begin{pmatrix}\dot{\boldsymbol{H}}^I_B\\ \dot{\boldsymbol{P}}^B\end{pmatrix} = \begin{pmatrix}\boldsymbol{0} & \chi_{\boldsymbol{H}^I_B}\\ -\chi^*_{\boldsymbol{H}^I_B} & \boldsymbol{J}_{\mathrm{k}}(\boldsymbol{P}^B)\end{pmatrix}\begin{pmatrix}\partial_{\boldsymbol{H}^I_B}\mathcal{H}\\ \partial_{\boldsymbol{P}^B}\mathcal{H}\end{pmatrix} + \begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{I}_6\end{pmatrix}(\boldsymbol{W}^B_{\mathrm{c}} + \boldsymbol{W}^B_{\mathrm{int}})$$

$$\boldsymbol{T}^{B,I}_B = (\boldsymbol{0} \quad \boldsymbol{I}_6)\begin{pmatrix}\partial_{\boldsymbol{H}^I_B}\mathcal{H}\\ \partial_{\boldsymbol{P}^B}\mathcal{H}\end{pmatrix}$$

Energy of aerial robot

$$\mathcal{H}(\boldsymbol{H}^I_B, \boldsymbol{P}^B) = \underbrace{\frac{1}{2}(\boldsymbol{P}^B)^\top \boldsymbol{\mathcal{I}}^{-1}\boldsymbol{P}^B}_{\text{Kinetic}} + \underbrace{m(\boldsymbol{\xi}^I_B)^\top \boldsymbol{g}}_{\text{Potential}}$$

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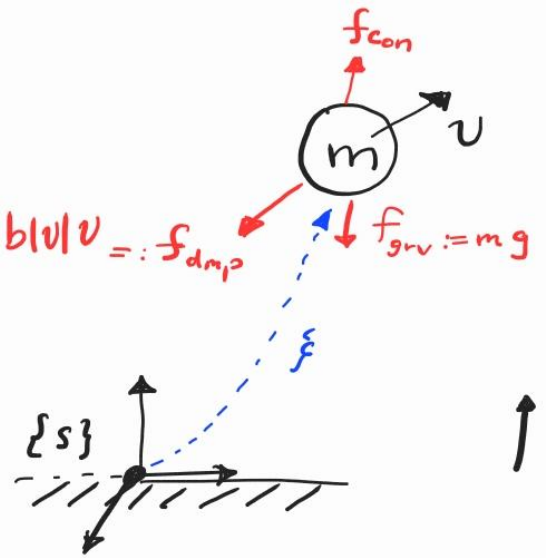
MASS DAMPER DYNAMICS

- **Equations of motion**
  - $\dot{\xi} = v$
  - $m\dot{v} = -b\,|v|v\; - mg + f_{\mathrm{con}}$

- **State space representation**
  - $x \;\coloneqq (x_1, x_2) = (\xi, v)$
  - $\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ -\frac{b}{m}\,|x_2|\; x_2 - g \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{m} I_3 \end{pmatrix} f_{\mathrm{con}}$



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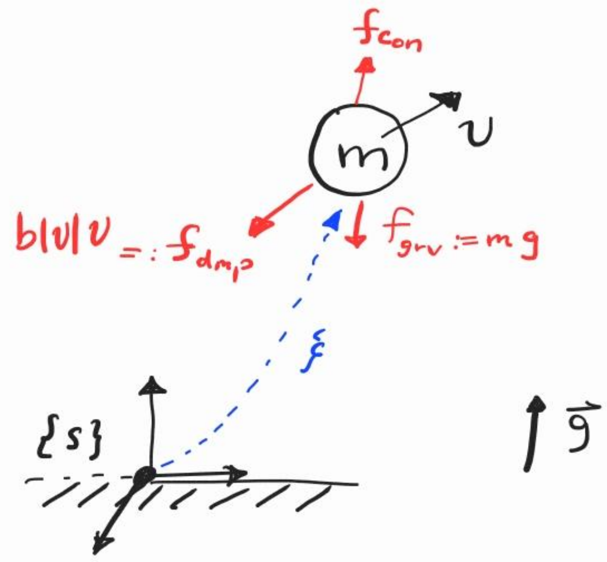
- **Stored energy analysis**
  - $H(x_1, x_2) = \frac{1}{2}\, m\, x_2^\top x_2 + m g^\top x_1$



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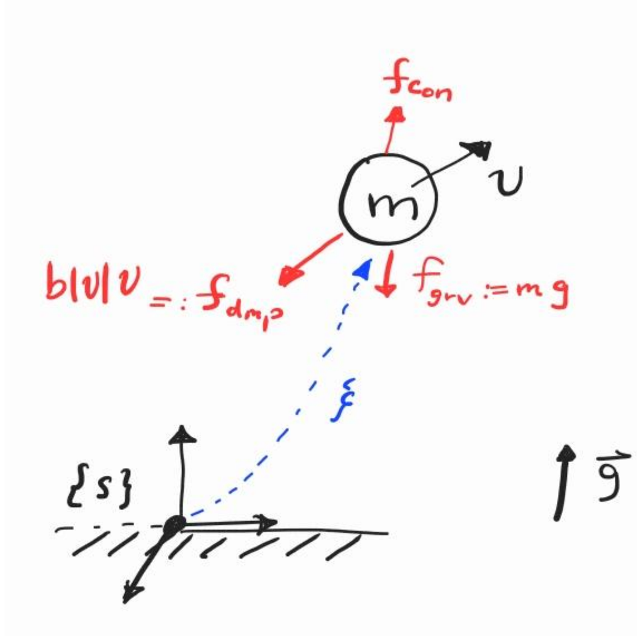
  - $\dot{H}(x) = \left(\frac{\partial H}{\partial x_1}\right)^\top \dot{x}_1 + \left(\frac{\partial H}{\partial x_2}\right)^\top \dot{x}_2$



Inner product between 2 vectors in $\mathbb{R}^n$

MASS DAMPER DYNAMICS

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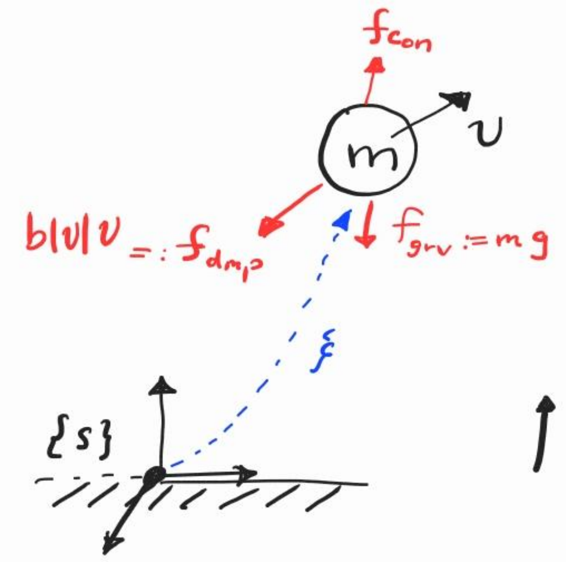
$$= m g^\top \dot{x}_1 + m\, x_2^\top\, \dot{x}_2$$

$$= m g^\top x_2 + x_2^\top \left(-b\, |x_2| x_2 \; - m g + f_{\text{con}}\right)$$



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  - $H(x_1, x_2) = \frac{1}{2} \, m \, x_2^\top x_2 + m g^\top x_1$
  - $\dot{H}(x) = \left(\frac{\partial H}{\partial x_1} \right)^\top \dot{x}_1 + \left(\frac{\partial H}{\partial x_2} \right)^\top \dot{x}_2$
    $= m g^\top \dot{x}_1 + m \, x_2^\top \dot{x}_2$
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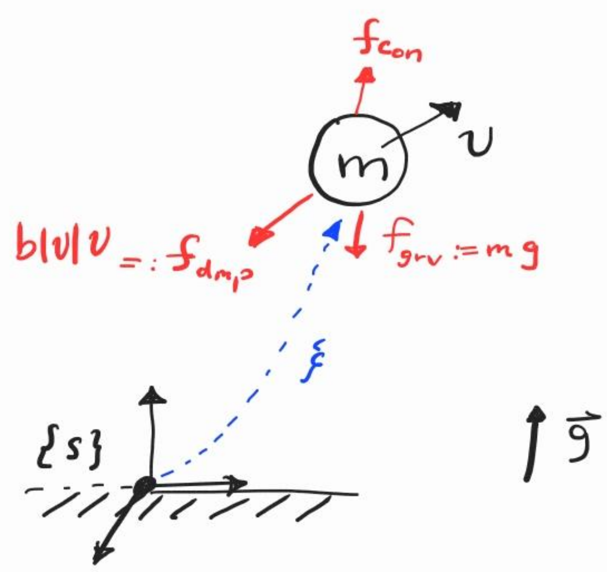

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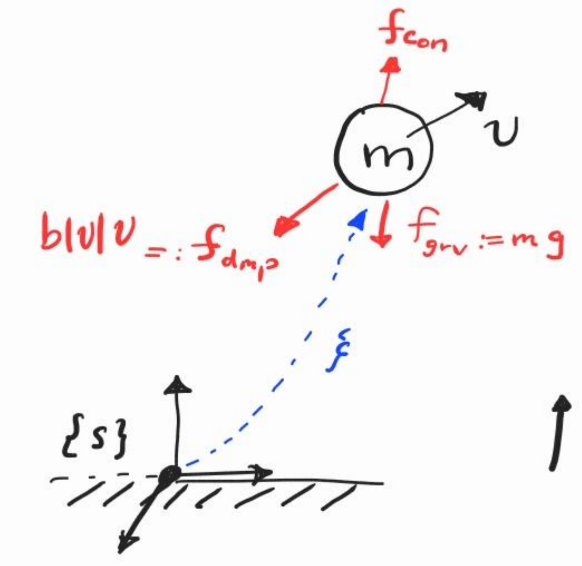
$$= -x_2^\top B(x_2) x_2 + v^\top f_{\text{con}}$$

$$\boxed{B(x_2) \coloneqq b\, |x_2| I_3 \in \mathbb{R}^{3\times 3} \succcurlyeq 0}$$



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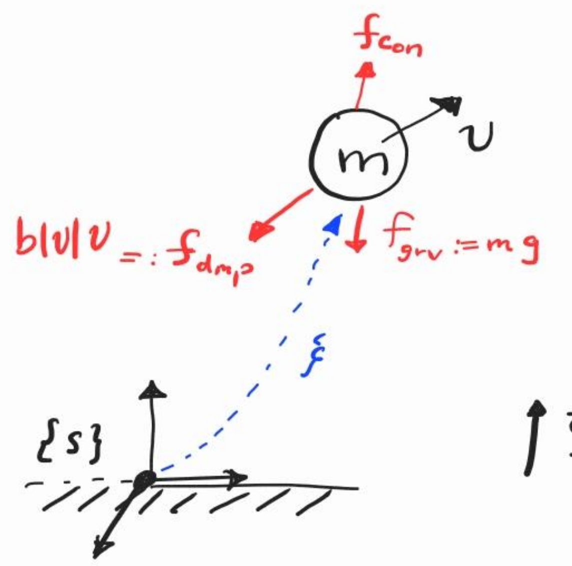
$$= \underbrace{-x_2^\top\, B(x_2) x_2}_{\text{Power dissipated}} + \underbrace{v^\top f_{\text{con}}}_{\text{Power supplied by control}}$$

$$\boxed{B(x_2) := b\,|x_2| I_3 \in \mathbb{R}^{3\times 3} \succcurlyeq 0}$$



PASSIVITY PROPERTY

- **Stored energy analysis**
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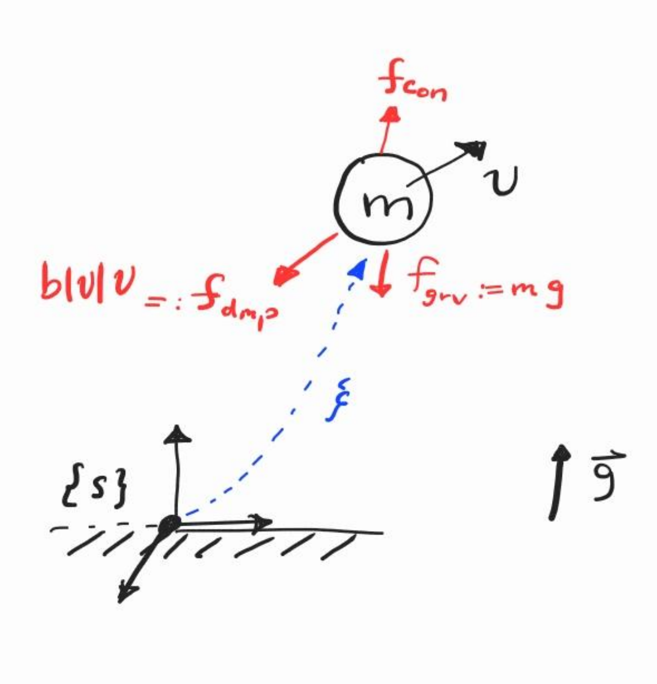

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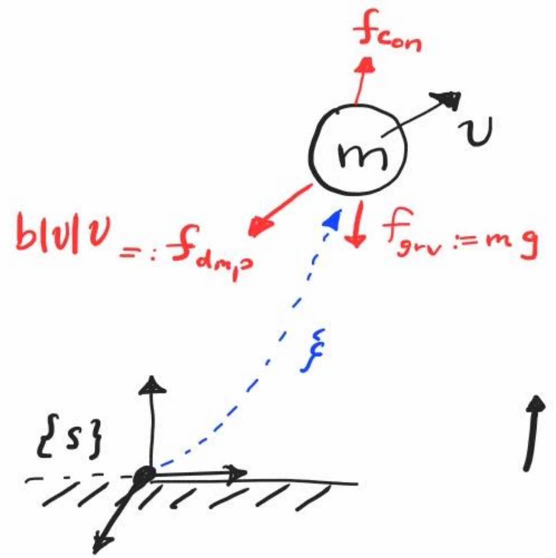
- **Stored energy analysis**
  - $H(x_1, x_2) = \frac{1}{2}\, m\, x_2^\top x_2 + m g^\top x_1$
  - $$\begin{aligned}\dot{H}(x) &= \left(\frac{\partial H}{\partial x_1}\right)^\top \dot{x}_1 + \left(\frac{\partial H}{\partial x_2}\right)^\top \dot{x}_2 \\ &= m g^\top \dot{x}_1 + m\, x_2^\top\, \dot{x}_2 \\ &= -x_2^\top\, B(x_2) x_2 \; + v^\top f_{\mathrm{con}} \\ &\leq v^\top f_{\mathrm{con}}\end{aligned}$$



- We say that the mass-damper system is passive with respect to the input output pair (f_{con}, v)

PASSIVITY PROPERTY

- In dynamical systems and control theory, **passivity** describes systems that **do not generate energy** — they can only **store or dissipate** it.
- Formally, a system with input $u(t)$and output $y(t)$is **passive** if there exists a nonnegative **storage function** $V(x) \geq 0$ such that for all time $t \geq 0$:

$$V\big(x(t)\big) - V\big(x(0)\big) \leq \int_0^t u^\top(\tau) y(\tau)\, d\tau$$

- If $V(x)$ is differentiable, we can write the inequality **differentially** as:

$$\dot{V}(x) \leq u^\top y$$

- **Stored energy analysis**
  - $H(x_1, x_2) = \frac{1}{2}\, m\, x_2^\top x_2 + m g^\top x_1$
  - $$\begin{aligned} \dot{H}(x) &= \left(\frac{\partial H}{\partial x_1}\right)^\top \dot{x}_1 + \left(\frac{\partial H}{\partial x_2}\right)^\top \dot{x}_2 \\ &= m g^\top \dot{x}_1 + m\, x_2^\top\, \dot{x}_2 \\ &= -x_2^\top\, B(x_2) x_2 \; + v^\top f_{\text{con}} \\ &\leq v^\top f_{\text{con}} \end{aligned}$$

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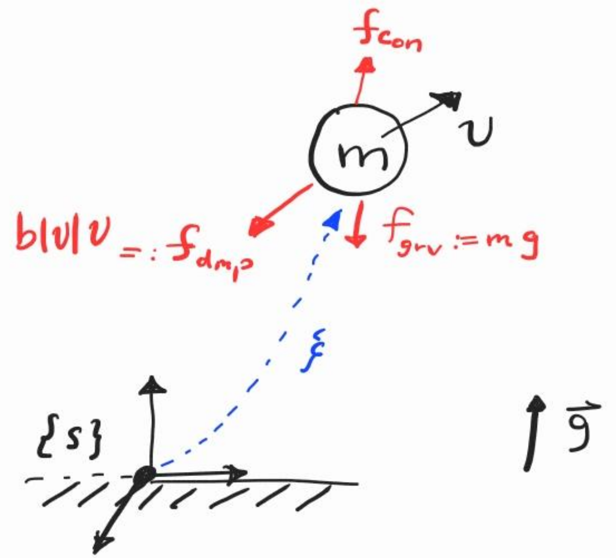
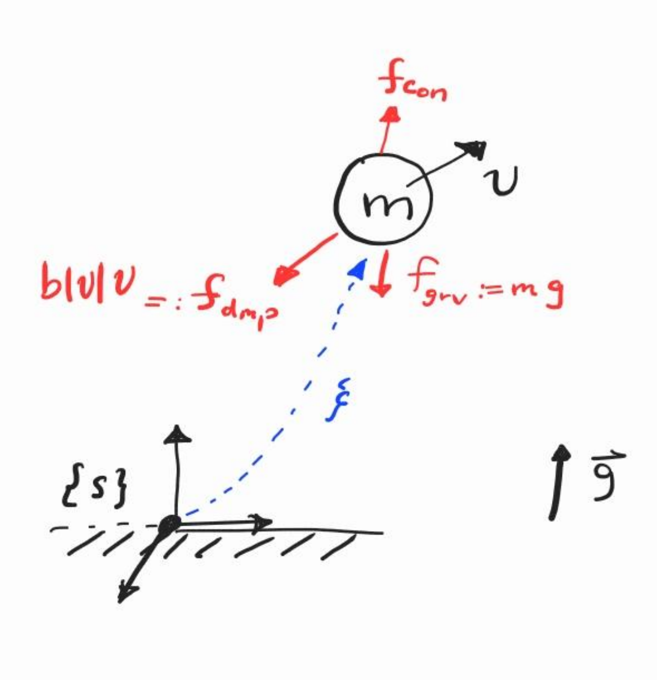


PORT-HAMILTONIAN REPRESENTATION

- **State variables** $x := (\xi, p)$
- **Hamiltonian function** $H(\xi, p) = \frac{1}{2m} p^\top p + m g^\top \xi$
- **Co-state variables** $\frac{\partial H}{\partial x}(x) = \left(\frac{\partial H}{\partial \xi}(x), \frac{\partial H}{\partial p}(x) \right) = (mg, \frac{p}{m})$



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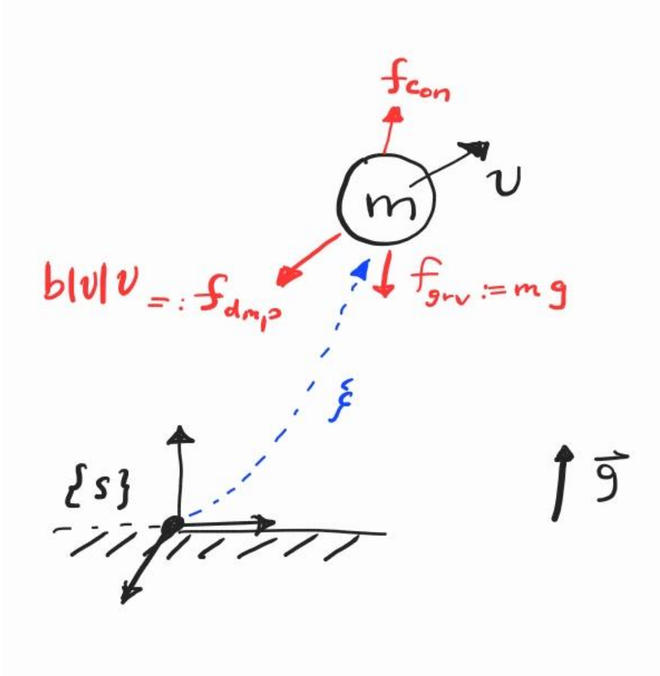
- **Dynamics**
  - $\dot{\xi} = \frac{\partial H}{\partial p}(x)$
  - $\dot{p} = -\frac{\partial H}{\partial \xi}(x) - \frac{b}{m}|p|\frac{\partial H}{\partial p}(x) + f_{\text{con}}$
  - $v = \frac{\partial H}{\partial p}(x)$

Standard form

$$\boxed{\begin{aligned} &\dot{\xi} = v \\ &m\dot{v} = -b\,|v|v - mg + f_{\text{con}} \end{aligned}}$$



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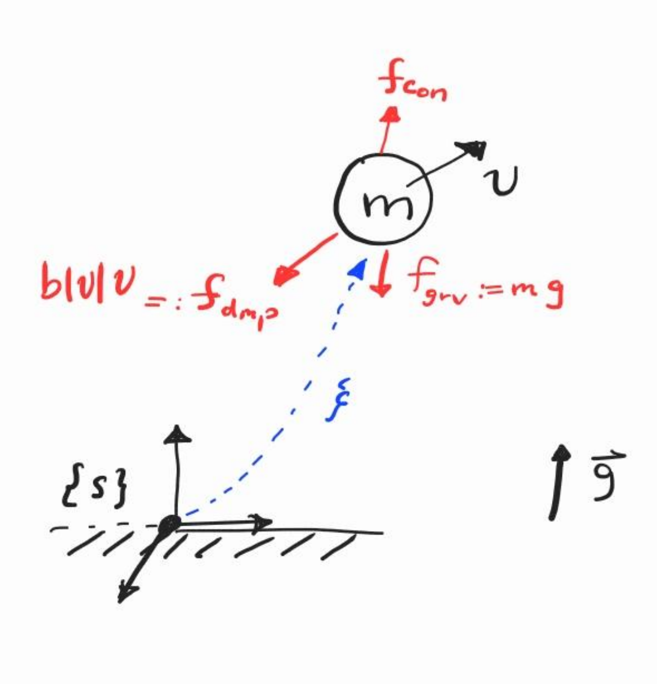
- **Dynamics**

$$\begin{pmatrix} \dot{\xi} \\ \dot{p} \end{pmatrix} = \left[\begin{pmatrix} 0 & I_3 \\ -I_3 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & \frac{b}{m}|p| \end{pmatrix} \right] \begin{pmatrix} \frac{\partial H}{\partial \xi}(x) \\ \frac{\partial H}{\partial p}(x) \end{pmatrix} + \begin{pmatrix} 0 \\ I_3 \end{pmatrix} f_{\text{con}}$$

$$v = \begin{pmatrix} 0 & I_3 \end{pmatrix} \begin{pmatrix} \frac{\partial H}{\partial \xi}(x) \\ \frac{\partial H}{\partial p}(x) \end{pmatrix}$$



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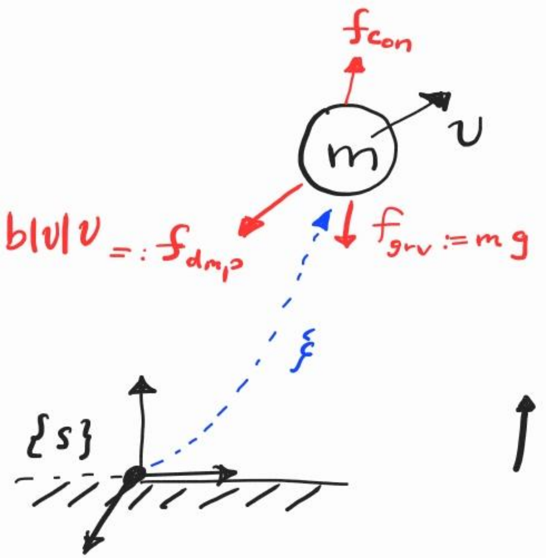
- **Co-state variables** $\frac{\partial H}{\partial x}(x) = \left(\frac{\partial H}{\partial \xi}(x), \frac{\partial H}{\partial p}(x) \right) = (mg, \frac{p}{m})$

- **Dynamics**

$$\begin{pmatrix} \dot{\xi} \\ \dot{p} \end{pmatrix} = \left[\begin{pmatrix} 0 & I_3 \\ -I_3 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & \frac{b}{m} \, |p| \end{pmatrix} \right] \begin{pmatrix} \frac{\partial H}{\partial \xi}(x) \\ \frac{\partial H}{\partial p}(x) \end{pmatrix} + \begin{pmatrix} 0 \\ I_3 \end{pmatrix} f_{\text{con}}$$

$$v = \begin{pmatrix} 0 & I_3 \end{pmatrix} \begin{pmatrix} \frac{\partial H}{\partial \xi}(x) \\ \frac{\partial H}{\partial p}(x) \end{pmatrix}$$



$$\boxed{J = -J^\top, \quad R(x) \succcurlyeq 0, \quad G}$$

PORT-HAMILTONIAN REPRESENTATION

- **Stored energy analysis**
  - $H(\xi, p) = \frac{1}{2m} p^\top p + m g^\top \xi$
  - $\dot{H}(x) = \left(\frac{\partial H}{\partial x}\right)^\top \dot{x} = \left(\frac{\partial H}{\partial x}\right)^\top J \left(\frac{\partial H}{\partial x}\right) - \left(\frac{\partial H}{\partial x}\right)^\top R(x) \left(\frac{\partial H}{\partial x}\right) + \left(\frac{\partial H}{\partial x}\right)^\top G f_{\mathrm{con}}$

pH form

$$\boxed{\begin{aligned} \dot{x} &= [J - R(x)] \left(\frac{\partial H}{\partial x}\right) + G f_{\mathrm{con}} \\ v &= G^\top \left(\frac{\partial H}{\partial x}\right) \end{aligned}}$$

PORT-HAMILTONIAN REPRESENTATION

- **Stored energy analysis**
  - $H(\xi, p) = \frac{1}{2m} p^\top p + m g^\top \xi$
  - $\dot{H}(x) = \left(\frac{\partial H}{\partial x}\right)^\top \dot{x} = \left(\frac{\partial H}{\partial x}\right)^\top J \left(\frac{\partial H}{\partial x}\right) - \left(\frac{\partial H}{\partial x}\right)^\top R(x) \left(\frac{\partial H}{\partial x}\right) + \left(\frac{\partial H}{\partial x}\right)^\top G f_{\text{con}}$

Skew-symmetry property

$$\boxed{y^\top J(x) y = 0\,, \qquad \forall\, y \in \mathbb{R}^n}$$

pH form

$$\boxed{\begin{aligned} \dot{x} &= [J - R(x)] \left(\frac{\partial H}{\partial x}\right) + G f_{\text{con}} \\ v &= G^\top \left(\frac{\partial H}{\partial x}\right) \end{aligned}}$$

PORT-HAMILTONIAN REPRESENTATION

- **Stored energy analysis**
  - $H(\xi, p) = \frac{1}{2m} p^\top p + m g^\top \xi$
  - $\dot{H}(x) = \left(\frac{\partial H}{\partial x}\right)^\top \dot{x} = -\left(\frac{\partial H}{\partial x}\right)^\top R(x) \left(\frac{\partial H}{\partial x}\right) + \left(\frac{\partial H}{\partial x}\right)^\top G f_{\mathrm{con}}$

$$= \underbrace{-v^\top B(p) v}_{\text{Power dissipated}} + \underbrace{v^\top f_{\mathrm{con}}}_{\text{Power supplied by control}}$$

Nonlinear Damping

$$\boxed{B(p) \coloneqq \frac{b}{m} |p| I_3 \in \mathbb{R}^{3\times 3} \succcurlyeq 0}$$

$$\boxed{\begin{aligned} \dot{x} &= [J - R(x)] \left(\frac{\partial H}{\partial x}\right) + G f_{\mathrm{con}} \\ v &= G^\top \left(\frac{\partial H}{\partial x}\right) \end{aligned}}$$

OUTLINE

- Recap last lecture
- Analysis of mass-damper dynamics
- Analysis of flying-effector dynamics
- Impedance control in the pH framework

FLYING END-EFFECTOR DYNAMICS

- **Equations of motion**

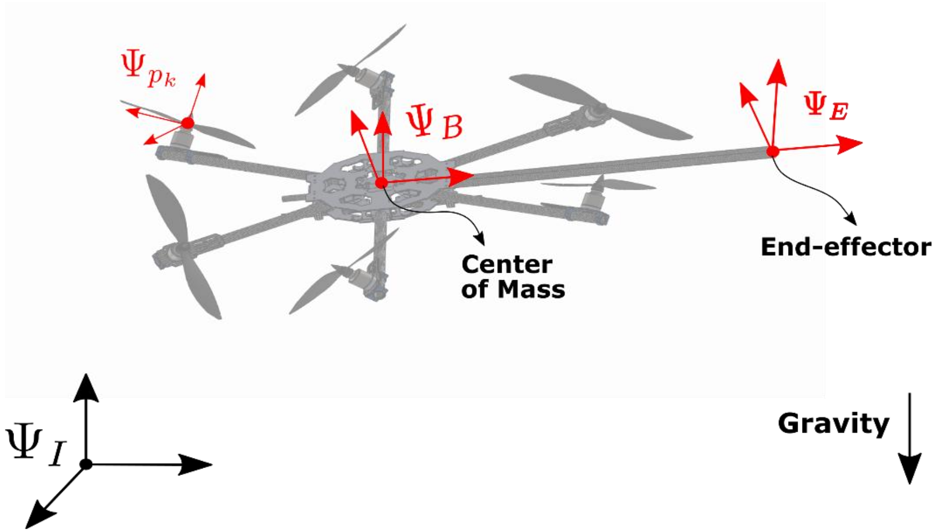
$$\dot{\boldsymbol{H}}{}^{I}_{B} = \boldsymbol{H}{}^{I}_{B}\tilde{\boldsymbol{T}}{}^{B,I}_{B}$$

$$\dot{\boldsymbol{P}}{}^{B} = J_{\mathrm{k}}(\boldsymbol{P}^{B})\boldsymbol{\mathcal{I}}^{-1}\boldsymbol{P}^{B} + \boldsymbol{W}{}^{B}_{\mathrm{g}} + \boldsymbol{W}{}^{B}_{\mathrm{c}} + \boldsymbol{W}{}^{B}_{\mathrm{int}}$$

- **Port-Hamiltonian Representation**

$$\dot{\boldsymbol{H}}{}^{I}_{B} = \chi_{\boldsymbol{H}^{I}_{B}}(\partial_{\boldsymbol{P}^{B}}\mathcal{H})$$

$$\dot{\boldsymbol{P}}{}^{B} = -\chi^{*}_{\boldsymbol{H}^{I}_{B}}(\partial_{\boldsymbol{H}^{I}_{B}}\mathcal{H}) + J_{\mathrm{k}}(\boldsymbol{P}^{B})\partial_{\boldsymbol{P}^{B}}\mathcal{H} + \boldsymbol{W}{}^{B}_{\mathrm{c}} + \boldsymbol{W}{}^{B}_{\mathrm{int}}$$



Hamiltonian function

$$\boxed{\mathcal{H}(x) = \mathcal{H}_{\mathrm{k}}(\boldsymbol{P}^{B}) + \mathcal{H}_{\mathrm{g}}(\boldsymbol{H}{}^{I}_{B}) \;= \frac{1}{2}(\boldsymbol{P}^{B})^{\top}\boldsymbol{\mathcal{I}}^{-1}\boldsymbol{P}^{B} + m(\boldsymbol{\xi}{}^{I}_{B})^{\top}\boldsymbol{g}}$$

FLYING END-EFFECTOR DYNAMICS

- **Equations of motion**

$$\dot{\boldsymbol{H}}^{I}_{B} = \boldsymbol{H}^{I}_{B}\tilde{\boldsymbol{T}}^{B,I}_{B}$$

$$\dot{\boldsymbol{P}}^{B} = \boldsymbol{J}_{\mathrm{k}}(\boldsymbol{P}^{B})\boldsymbol{\mathcal{I}}^{-1}\boldsymbol{P}^{B} + \boldsymbol{W}^{B}_{\mathrm{g}} + \boldsymbol{W}^{B}_{\mathrm{c}} + \boldsymbol{W}^{B}_{\mathrm{int}}$$

- **Port-Hamiltonian Representation**

$$\dot{\boldsymbol{H}}^{I}_{B} = \chi_{\boldsymbol{H}^{I}_{B}}(\partial_{\boldsymbol{P}^{B}}\mathcal{H})$$

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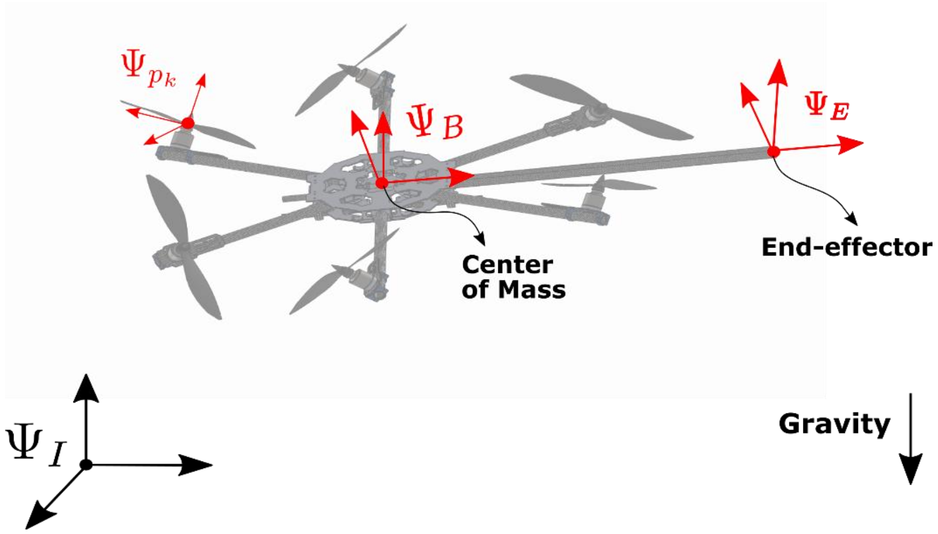


Dual maps

$$\chi_{\boldsymbol{H}} : \mathbb{R}^6 \to T_{\boldsymbol{H}}SE(3)$$

$$\chi^{*}_{\boldsymbol{H}} : T^{*}_{\boldsymbol{H}}SE(3) \to (\mathbb{R}^6)^{*}$$

Hamiltonian function

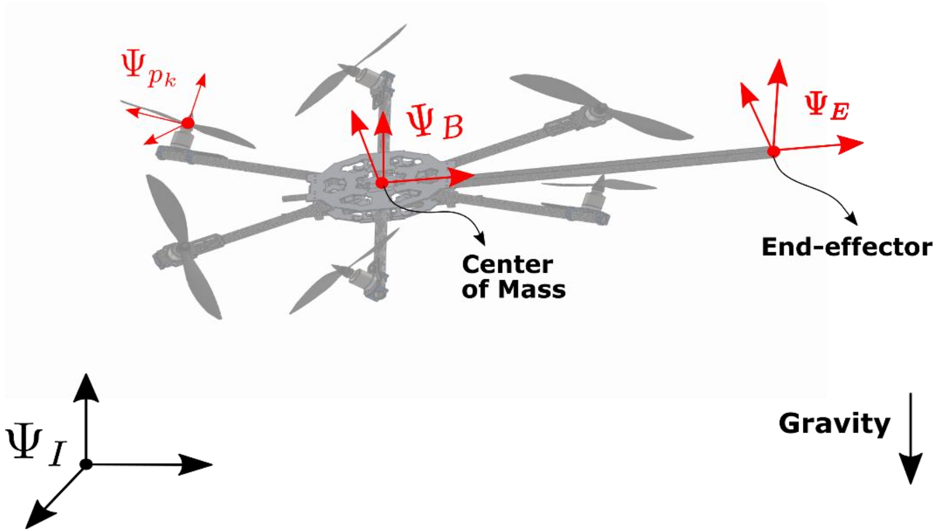
$$\mathcal{H}(x) = \mathcal{H}_{\mathrm{k}}(\boldsymbol{P}^{B}) + \mathcal{H}_{\mathrm{g}}(\boldsymbol{H}^{I}_{B}) = \frac{1}{2}(\boldsymbol{P}^{B})^{\top}\boldsymbol{\mathcal{I}}^{-1}\boldsymbol{P}^{B} + m(\boldsymbol{\xi}^{I}_{B})^{\top}\boldsymbol{g}$$

FLYING END-EFFECTOR DYNAMICS

- **Port-Hamiltonian Representation**

$$\begin{pmatrix} \dot{\boldsymbol{H}}^{I}_{B} \\ \dot{\boldsymbol{P}}^{B} \end{pmatrix} = \begin{pmatrix} \mathbf{0} & \chi_{\boldsymbol{H}^{I}_{B}} \\ -\chi^{*}_{\boldsymbol{H}^{I}_{B}} & \boldsymbol{J}_{\mathrm{k}}(\boldsymbol{P}^{B}) \end{pmatrix} \begin{pmatrix} \partial_{\boldsymbol{H}^{I}_{B}} \mathcal{H} \\ \partial_{\boldsymbol{P}^{B}} \mathcal{H} \end{pmatrix} + \begin{pmatrix} \mathbf{0} \\ \boldsymbol{I}_{6} \end{pmatrix} (\boldsymbol{W}^{B}_{\mathrm{c}} + \boldsymbol{W}^{B}_{\mathrm{int}})$$

$$\boldsymbol{T}^{B,I}_{B} = (\mathbf{0} \quad \boldsymbol{I}_{6}) \begin{pmatrix} \partial_{\boldsymbol{H}^{I}_{B}} \mathcal{H} \\ \partial_{\boldsymbol{P}^{B}} \mathcal{H} \end{pmatrix}$$



Hamiltonian function

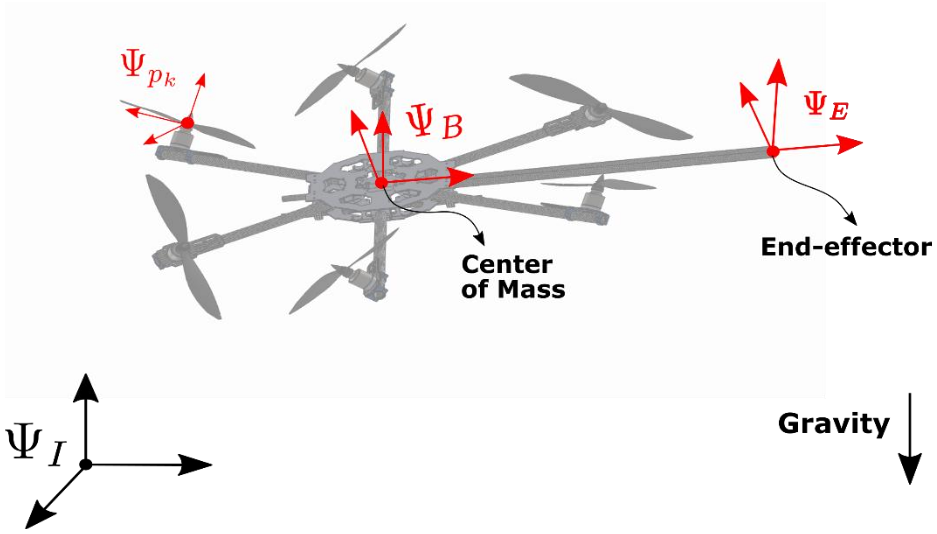
$$\mathcal{H}(x) = \mathcal{H}_{\mathrm{k}}(\boldsymbol{P}^{B}) + \mathcal{H}_{\mathrm{g}}(\boldsymbol{H}^{I}_{B}) = \frac{1}{2}(\boldsymbol{P}^{B})^{\top}\boldsymbol{\mathcal{I}}^{-1}\boldsymbol{P}^{B} + m(\boldsymbol{\xi}^{I}_{B})^{\top}\boldsymbol{g}$$

FLYING END-EFFECTOR DYNAMICS

- **Port-Hamiltonian Representation**

$$\begin{pmatrix}\dot{\boldsymbol{H}}^I_B\\ \dot{\boldsymbol{P}}^B\end{pmatrix} = \begin{pmatrix}\mathbf{0} & \chi_{\boldsymbol{H}^I_B}\\ -\chi^*_{\boldsymbol{H}^I_B} & \boldsymbol{J}_{\mathrm{k}}(\boldsymbol{P}^B)\end{pmatrix}\begin{pmatrix}\partial_{\boldsymbol{H}^I_B}\mathcal{H}\\ \partial_{\boldsymbol{P}^B}\mathcal{H}\end{pmatrix} + \begin{pmatrix}\mathbf{0}\\ \boldsymbol{I}_6\end{pmatrix}(\boldsymbol{W}^B_{\mathrm{c}} + \boldsymbol{W}^B_{\mathrm{int}})$$

$$\boldsymbol{T}^{B,I}_B = \begin{pmatrix}\mathbf{0} & \boldsymbol{I}_6\end{pmatrix}\begin{pmatrix}\partial_{\boldsymbol{H}^I_B}\mathcal{H}\\ \partial_{\boldsymbol{P}^B}\mathcal{H}\end{pmatrix}$$



- **Stored energy analysis**
  - $\dot{\mathcal{H}}(x) = \left(\frac{\partial \mathcal{H}}{\partial x}\right)^{\top} \dot{x}$

We cannot take the standard inner product between 2 vectors in $\mathbb{R}^n$, because $\dot{x} := \left(\dot{H}^I_B, \dot{P}^B\right) \notin \mathbb{R}^n$

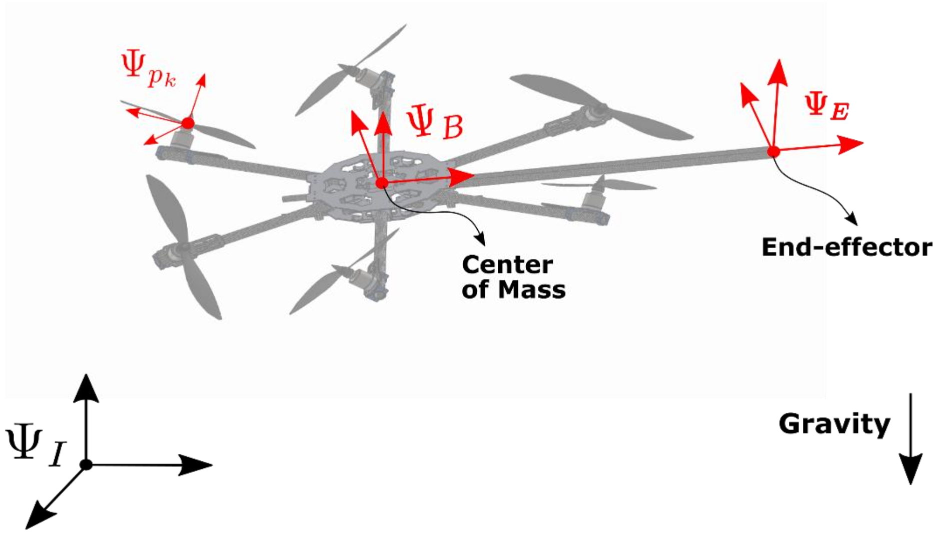
$$\dot{\mathcal{H}}(x) = \langle d\mathcal{H}(x) | \dot{x} \rangle_{T_x\mathcal{X}}$$

FLYING END-EFFECTOR DYNAMICS

- **Port-Hamiltonian Representation**

$$\begin{pmatrix}\dot{\boldsymbol{H}}_B^I \\ \dot{\boldsymbol{P}}^B\end{pmatrix} = \begin{pmatrix}\mathbf{0} & \chi_{\boldsymbol{H}_B^I} \\ -\chi^*_{\boldsymbol{H}_B^I} & \boldsymbol{J}_\mathrm{k}(\boldsymbol{P}^B)\end{pmatrix}\begin{pmatrix}\partial_{\boldsymbol{H}_B^I}\mathcal{H} \\ \partial_{\boldsymbol{P}^B}\mathcal{H}\end{pmatrix} + \begin{pmatrix}\mathbf{0} \\ \boldsymbol{I}_6\end{pmatrix}(\boldsymbol{W}_\mathrm{c}^B + \boldsymbol{W}_\mathrm{int}^B)$$

$$\boldsymbol{T}_B^{B,I} = (\mathbf{0} \quad \boldsymbol{I}_6)\begin{pmatrix}\partial_{\boldsymbol{H}_B^I}\mathcal{H} \\ \partial_{\boldsymbol{P}^B}\mathcal{H}\end{pmatrix}$$



- **Stored energy analysis**
  - $\dot{\mathcal{H}}(x) = \left(\frac{\partial \mathcal{H}}{\partial x}\right)^\top \dot{x} = \left(\frac{\partial H}{\partial x}\right)^\top J(x)\left(\frac{\partial H}{\partial x}\right) + \left(\frac{\partial H}{\partial x}\right)^\top G\left(W_\mathrm{c}^B + W_\mathrm{int}^B\right)$

FLYING END-EFFECTOR DYNAMICS

- **Port-Hamiltonian Representation**

$$\begin{pmatrix}\dot{\boldsymbol{H}}^{I}_{B}\\ \dot{\boldsymbol{P}}^{B}\end{pmatrix} = \begin{pmatrix}\boldsymbol{0} & \chi_{\boldsymbol{H}^{I}_{B}}\\ -\chi^{*}_{\boldsymbol{H}^{I}_{B}} & \boldsymbol{J}_{\mathrm{k}}(\boldsymbol{P}^{B})\end{pmatrix}\begin{pmatrix}\partial_{\boldsymbol{H}^{I}_{B}}\mathcal{H}\\ \partial_{\boldsymbol{P}^{B}}\mathcal{H}\end{pmatrix} + \begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{I}_{6}\end{pmatrix}(\boldsymbol{W}^{B}_{\mathrm{c}} + \boldsymbol{W}^{B}_{\mathrm{int}})$$

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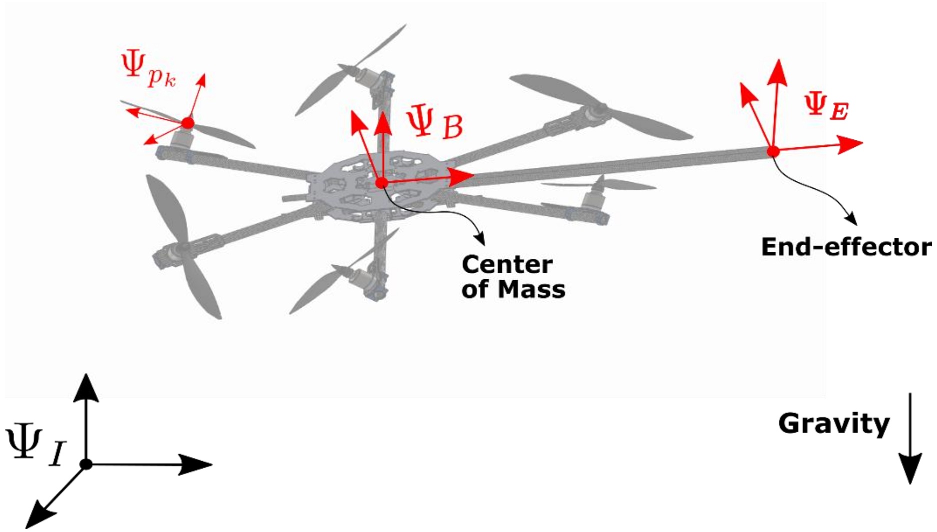


- **Stored energy analysis**
  - $\dot{\mathcal{H}}(x) = \left(\frac{\partial \mathcal{H}}{\partial x}\right)^{\top} \dot{x} = \left(\frac{\partial H}{\partial x}\right)^{\top} J(x) \left(\frac{\partial H}{\partial x}\right) + \left(\frac{\partial H}{\partial x}\right)^{\top} G\left(W_{\mathrm{c}}^{B} + W_{\mathrm{int}}^{B}\right)$

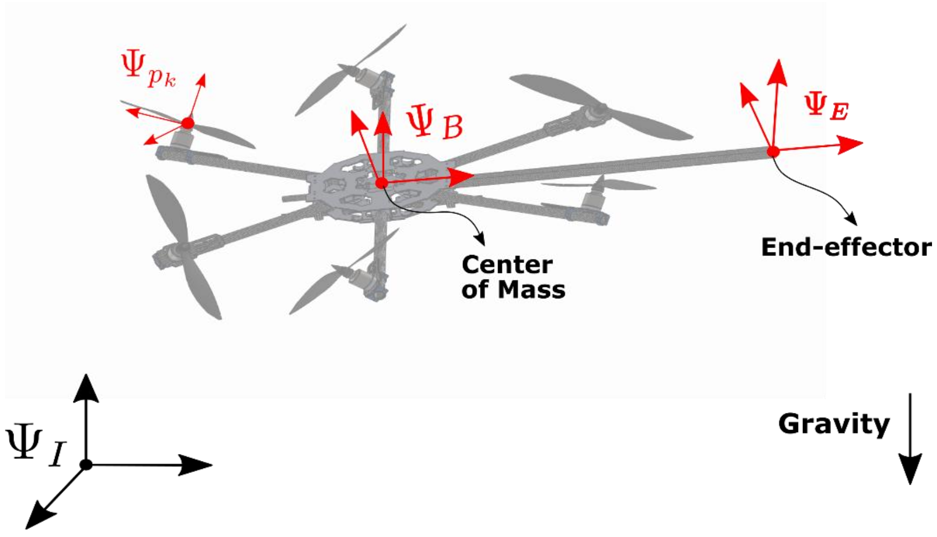
FLYING END-EFFECTOR DYNAMICS

- **Port-Hamiltonian Representation**

$$\begin{pmatrix} \dot{\boldsymbol{H}}^I_B \\ \dot{\boldsymbol{P}}^B \end{pmatrix} = \begin{pmatrix} \mathbf{0} & \chi_{\boldsymbol{H}^I_B} \\ -\chi^*_{\boldsymbol{H}^I_B} & \boldsymbol{J}_\mathrm{k}(\boldsymbol{P}^B) \end{pmatrix} \begin{pmatrix} \partial_{\boldsymbol{H}^I_B}\mathcal{H} \\ \partial_{\boldsymbol{P}^B}\mathcal{H} \end{pmatrix} + \begin{pmatrix} \mathbf{0} \\ \boldsymbol{I}_6 \end{pmatrix} (\boldsymbol{W}^B_\mathrm{c} + \boldsymbol{W}^B_\mathrm{int})$$

$$\boldsymbol{T}^{B,I}_B = (\mathbf{0} \quad \boldsymbol{I}_6) \begin{pmatrix} \partial_{\boldsymbol{H}^I_B}\mathcal{H} \\ \partial_{\boldsymbol{P}^B}\mathcal{H} \end{pmatrix}$$

- **Stored energy analysis**
  - $$\dot{\mathcal{H}}(x) = \left(\frac{\partial \mathcal{H}}{\partial x}\right)^\top \dot{x} = \left(\frac{\partial H}{\partial x}\right)^\top G\left(W^B_\mathrm{c} + W^B_\mathrm{int}\right) = \left(W^B_\mathrm{c} + W^B_\mathrm{int}\right)^\top G^\top \left(\frac{\partial H}{\partial x}\right)$$



FLYING END-EFFECTOR DYNAMICS

- **Port-Hamiltonian Representation**

$$\begin{pmatrix}\dot{\boldsymbol{H}}^{I}_{B}\\ \dot{\boldsymbol{P}}^{B}\end{pmatrix} = \begin{pmatrix}\mathbf{0} & \chi_{\boldsymbol{H}^{I}_{B}}\\ -\chi^{*}_{\boldsymbol{H}^{I}_{B}} & \boldsymbol{J}_{\mathrm{k}}(\boldsymbol{P}^{B})\end{pmatrix}\begin{pmatrix}\partial_{\boldsymbol{H}^{I}_{B}}\mathcal{H}\\ \partial_{\boldsymbol{P}^{B}}\mathcal{H}\end{pmatrix} + \begin{pmatrix}\mathbf{0}\\ \boldsymbol{I}_{6}\end{pmatrix}(\boldsymbol{W}^{B}_{\mathrm{c}} + \boldsymbol{W}^{B}_{\mathrm{int}})$$

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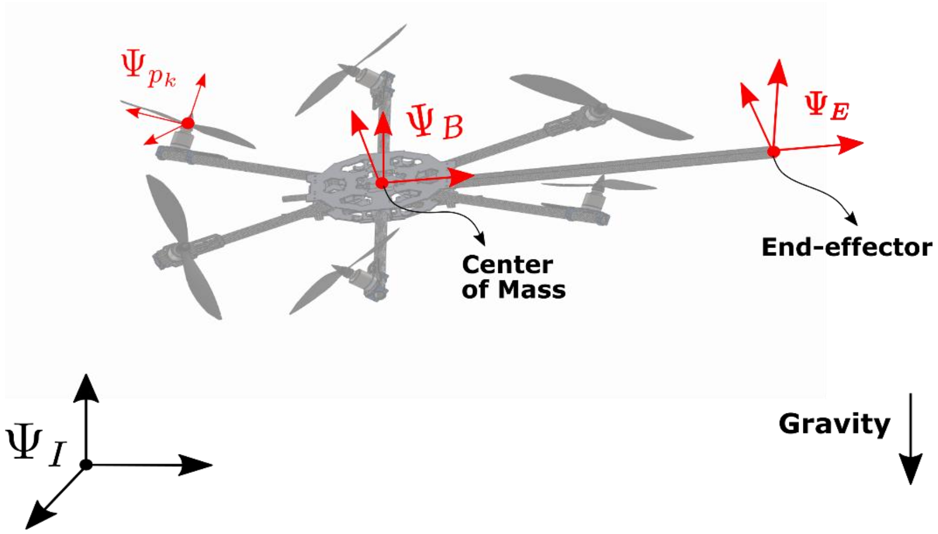
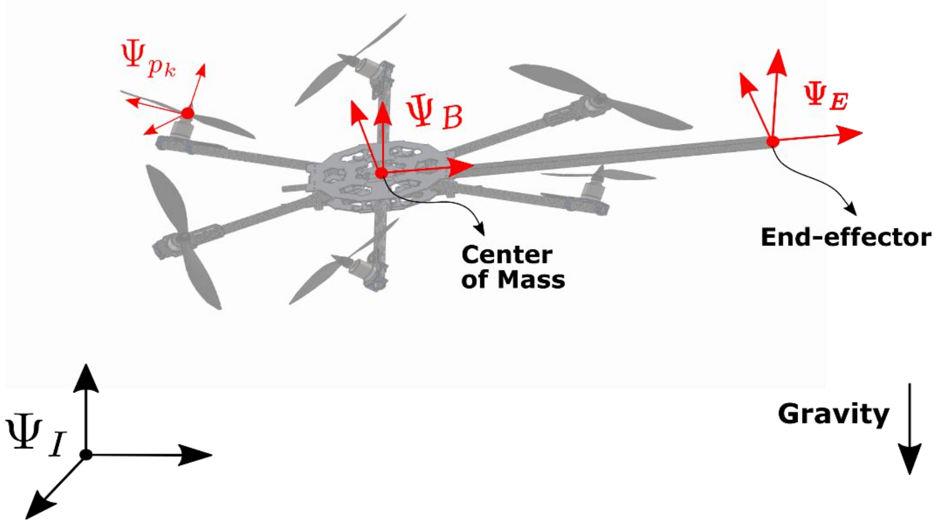


- **Stored energy analysis**
  - $\dot{\mathcal{H}}(x) = \left(\frac{\partial \mathcal{H}}{\partial x}\right)^{\top} \dot{x} = \left(\frac{\partial H}{\partial x}\right)^{\top} G\left(W^{B}_{\mathrm{c}} + W^{B}_{\mathrm{int}}\right)$

$$= \left(W^{B}_{\mathrm{c}} + W^{B}_{\mathrm{int}}\right)^{\top} G^{\top}\left(\frac{\partial H}{\partial x}\right) = (W^{B}_{\mathrm{c}})^{\top} T^{B,I}_{B} + \left(W^{B}_{\mathrm{int}}\right)^{\top} T^{B,I}_{B}$$

FLYING END-EFFECTOR DYNAMICS



- **Port-Hamiltonian Representation**

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$$= \left(W_\mathrm{c}^B + W_\mathrm{int}^B\right)^\top G^\top\left(\frac{\partial H}{\partial x}\right) = \underbrace{(W_\mathrm{c}^B)^\top T_B^{B,I}}_{\substack{\text{Power supplied}\\ \text{by control}}} + \underbrace{\left(W_\mathrm{int}^B\right)^\top T_B^{B,I}}_{\substack{\text{Power supplied}\\ \text{by environment}}}$$

GRAPHICAL REPRESENTATION

- **Port-Hamiltonian Representation**

$$\begin{pmatrix}\dot{\boldsymbol{H}}^I_B\\ \dot{\boldsymbol{P}}^B\end{pmatrix} = \begin{pmatrix}\mathbf{0} & \chi_{\boldsymbol{H}^I_B}\\ -\chi^*_{\boldsymbol{H}^I_B} & \boldsymbol{J}_\mathrm{k}(\boldsymbol{P}^B)\end{pmatrix}\begin{pmatrix}\partial_{\boldsymbol{H}^I_B}\mathcal{H}\\ \partial_{\boldsymbol{P}^B}\mathcal{H}\end{pmatrix} + \begin{pmatrix}\mathbf{0}\\ \boldsymbol{I}_6\end{pmatrix}(\boldsymbol{W}^B_\mathrm{c} + \boldsymbol{W}^B_\mathrm{int})$$

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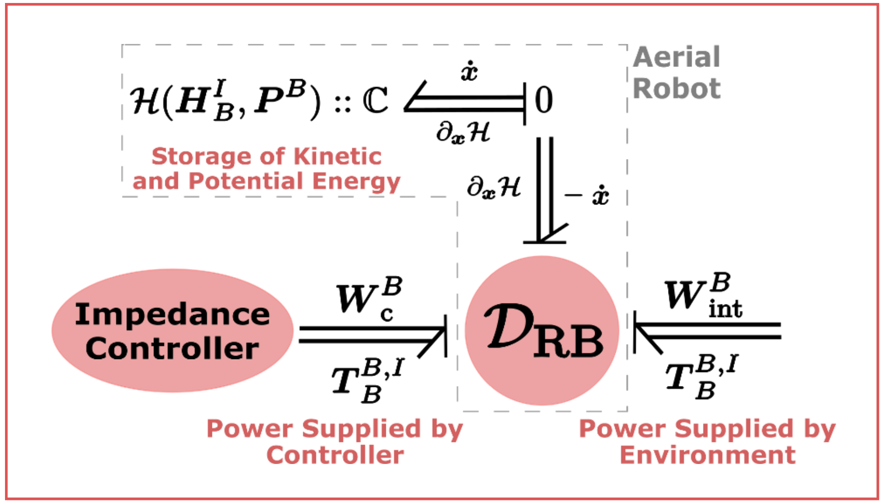

- **Stored energy analysis**
  - $\dot{\mathcal{H}}(x) = \underbrace{(W_\mathrm{c}^B)^\top T_B^{B,I}}_{\text{Power supplied by control}} + \underbrace{\left(W_\mathrm{int}^B\right)^\top T_B^{B,I}}_{\text{Power supplied by environment}}$

GRAPHICAL REPRESENTATION

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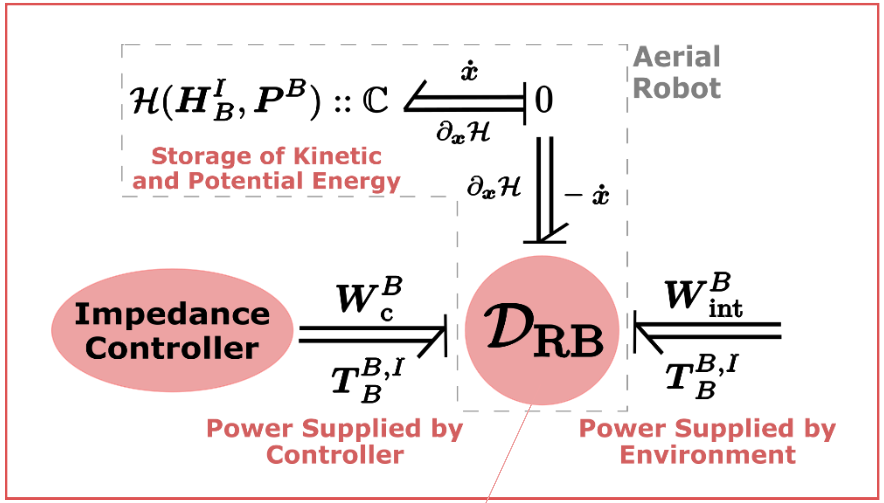
- **Stored energy analysis**
  - $\dot{\mathcal{H}}(x) = \underbrace{(W_\mathrm{c}^B)^\top T_B^{B,I}}_{\text{Power supplied by control}} + \underbrace{(W_\mathrm{int}^B)^\top T_B^{B,I}}_{\text{Power supplied by environment}}$



$$\begin{pmatrix}-\dot{\boldsymbol{H}}^I_B\\ -\dot{\boldsymbol{P}}^B\\ \boldsymbol{T}^{B,I}_B\\ \boldsymbol{T}^{B,I}_B\end{pmatrix} = \begin{pmatrix}0 & -\chi_{\boldsymbol{H}^I_B} & 0 & 0\\ \chi^*_{\boldsymbol{H}^I_B} & -\boldsymbol{J}_\mathrm{k}(\boldsymbol{P}^B) & -\boldsymbol{I}_6 & -\boldsymbol{I}_6\\ 0 & \boldsymbol{I}_6 & 0 & 0\\ 0 & \boldsymbol{I}_6 & 0 & 0\end{pmatrix}\begin{pmatrix}\partial_{\boldsymbol{H}^I_B}\mathcal{H}\\ \partial_{\boldsymbol{P}^B}\mathcal{H}\\ \boldsymbol{W}^B_\mathrm{c}\\ \boldsymbol{W}^B_\mathrm{int}\end{pmatrix}$$

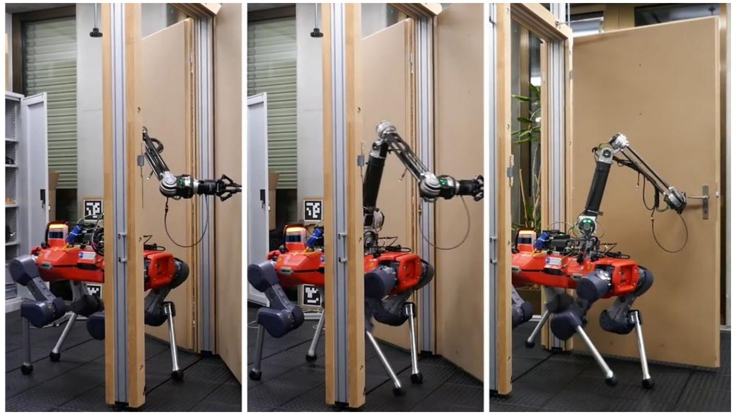
OUTLINE

- Recap last lecture
- Analysis of mass-damper dynamics
- Analysis of flying-effector dynamics
- Impedance control in the pH framework

Paradigm Shift: Motion Control vs. Interaction Control

- **Motion Control**
- Main objective is to achieve stability and reject external disturbances to maximize performance.

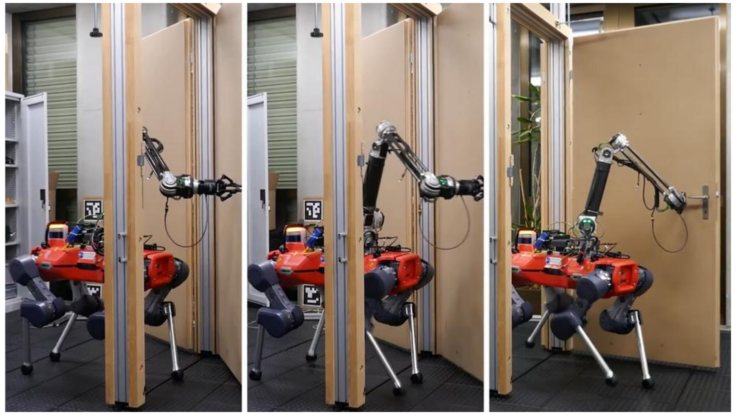
- **Interaction Control**
- Main objective is to be able to interact with an unknown environment in a stable & safe manner.



Paradigm Shift: Motion Control vs. Interaction Control

- **Motion Control**
- Closed dynamical system.
- Stability analysis requires closed loop model.

- **Interaction Control**
- Open dynamical system.
- Unknown environment needs to be incorporated in closed loop stability analysis.



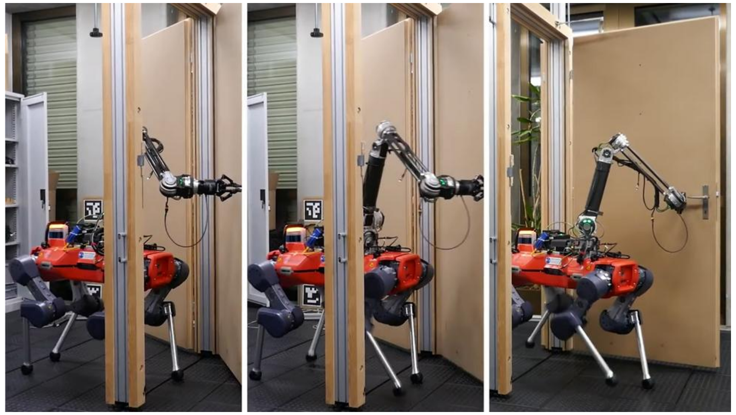
Paradigm Shift: Motion Control vs. Interaction Control

- **Motion Control**
- Based on unilateral signals.

e.g., control of position or velocity

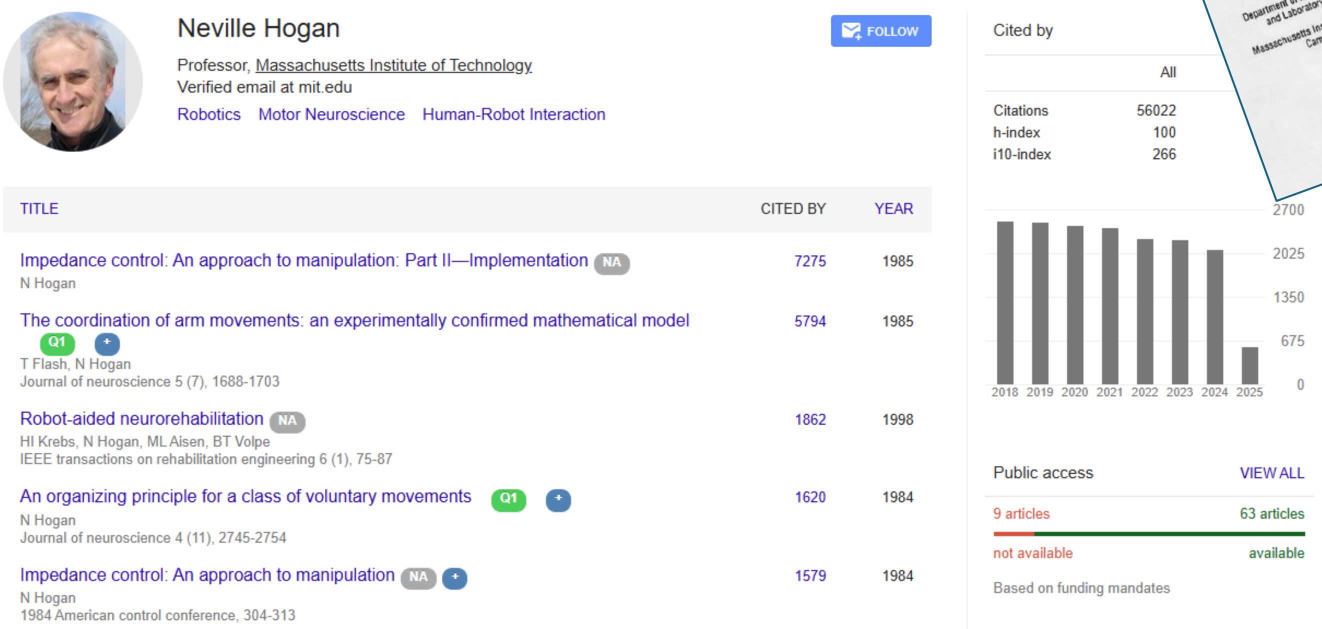
- **Interaction Control**
- Based on bilateral signals

e.g., control of relation between velocity and force.



Impedance Control

- The first one to address this issue in the field of robotics was Neville Hogan in his seminal trilogy.



Neville Hogan

Professor, Massachusetts Institute of Technology
Verified email at mit.edu
Robotics Motor Neuroscience Human-Robot Interaction

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Robot-aided neurorehabilitation NA HI Krebs, N Hogan, ML Aisen, BT Volpe IEEE transactions on rehabilitation engineering 6 (1), 75-87	1862	1998
An organizing principle for a class of voluntary movements Q1 + N Hogan Journal of neuroscience 4 (11), 2745-2754	1620	1984
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Impedance Control: An Approach to Manipulation: Part I—Theory

Neville Hogan
Associate Professor,
Department of Mechanical Engineering
and Laboratory for Manufacturing
and Productivity,
Massachusetts Institute of Technology,
Cambridge, Mass. 02139

Manipulation fundamentally requires the manipulator to be mechanically coupled to the object being manipulated; the manipulator may not be treated as an isolated system. This three-part paper presents an approach to the control of dynamic interaction between a manipulator and its environment. In Part I this approach is developed by considering the mechanics of interaction between physical systems. Control of position or force alone is inadequate; control of dynamic behavior is also required. It is shown that as manipulation is a fundamentally nonlinear problem, the distinction between impedance and admittance is essential, and given

Impedance Control: An Approach to Manipulation: Part II—Implementation

Neville Hogan
Associate Professor,
Department of Mechanical Engineering
and Laboratory for Manufacturing
and Productivity,
Massachusetts Institute of Technology,
Cambridge, Mass. 02139

This three-part paper presents an approach to the control of dynamic interaction between a manipulator and its environment. Part I presented the theoretical reasoning behind impedance control. In Part II the implementation of impedance control is considered. A feedback control algorithm for imposing a desired cartesian impedance on the end-point of a nonlinear manipulator is presented. This algorithm completely eliminates the need to solve the "inverse kinematics problem" in robot motion control. The modulation of end-point impedance without using feedback control is also considered, and it is shown that apparently "redundant"

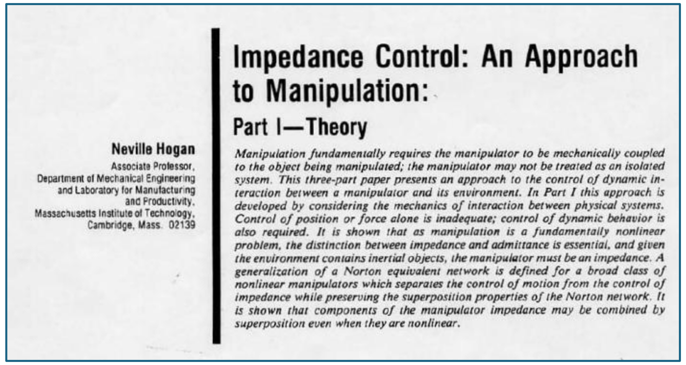
Impedance Control: An Approach to Manipulation: Part III—Applications

Neville Hogan
Associate Professor,
Department of Mechanical Engineering
and Laboratory for Manufacturing
and Productivity,
Massachusetts Institute of Technology,
Cambridge, Mass. 02139

This three-part paper presents a unified approach to the control of a manipulator applicable to free motions, kinematically constrained motions, and dynamic interaction between the manipulator and its environment. In Part I the approach was developed from a consideration of the fundamental mechanics of manipulation. Part II presented techniques for implementing a desired manipulator impedance. In Part III a technique for choosing the impedance appropriate to a given application using optimization theory is presented. Based on a simplified analysis it is shown that if the task objective is to tradeoff interface forces and motion errors, the manipulator impedance should be proportional to the environmental admittance. An application of impedance control to unconstrained motion is presented. The superposition properties of nonlinear impedances are used to develop a real-time feedback control algorithm which permits a manipulator to avoid unpredictably moving objects without explicit path planning.

Impedance Control

- Neville Hogan's idea of impedance control is that instead of commanding exact positions or forces, the robot is controlled to behave like a mechanical impedance:
  - It responds to external forces with a desired relationship between force, position, and velocity, like a spring-damper-mass system.
- The key is shaping the robot's dynamic behavior — its *stiffness*, *damping*, and *inertia* — to match the interaction task needs.



Impedance Control: An Approach to Manipulation:

Part I—Theory

Neville Hogan
Associate Professor,
Department of Mechanical Engineering
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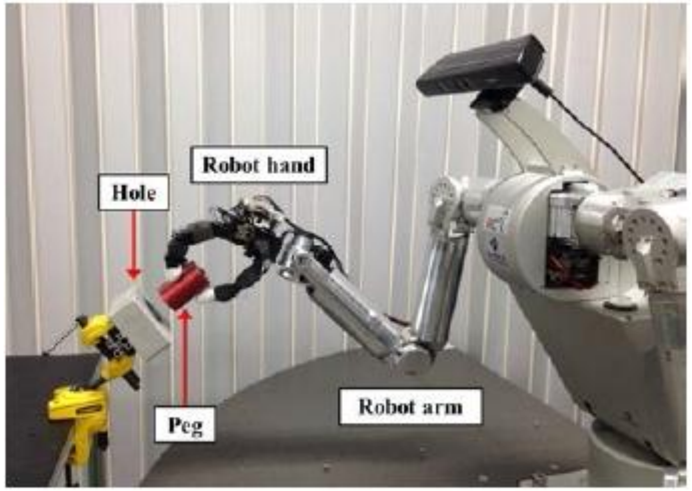
Massachusetts Institute of Technology,
Cambridge, Mass. 02139

Manipulation fundamentally requires the manipulator to be mechanically coupled to the object being manipulated; the manipulator may not be treated as an isolated system. This three-part paper presents an approach to the control of dynamic interaction between a manipulator and its environment. In Part I this approach is developed by considering the mechanics of interaction between physical systems. Control of position or force alone is inadequate; control of dynamic behavior is also required. It is shown that as manipulation is a fundamentally nonlinear problem, the distinction between impedance and admittance is essential, and given the environment contains inertial objects, the manipulator must be an impedance. A generalization of a Norton equivalent network is defined for a broad class of nonlinear manipulators which separates the control of motion from the control of impedance while preserving the superposition properties of the Norton network. It is shown that components of the manipulator impedance may be combined by superposition even when they are nonlinear.



Impedance Control of Mass Damper

- **Equations of motion**
  - $\dot{\xi} = v$
  - $m\dot{v} = -b\,|v|v\; - mg + f_{\text{con}} + f_{\text{int}}$
- **Impedance control law**
  - $f_{\text{con}} = mg\; - K_p\xi\; - K_d v$
- **Closed loop dynamics**

$$m\dot{v} = -b\,|v|v - K_p\xi\; - K_d v + f_{\text{int}}$$

Impedance Control of Mass Damper

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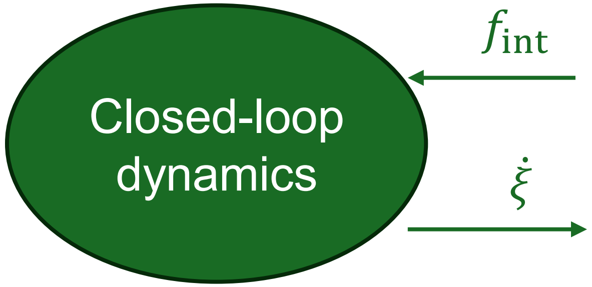
$$m\dot{v} = -b\,|v|v - K_p\xi\ - K_d v + f_{\text{int}}$$

$$\Downarrow$$

$$\boxed{m\ddot{\xi} + (B(v) + K_d)\dot{\xi} + K_p\xi = f_{\text{int}}}$$

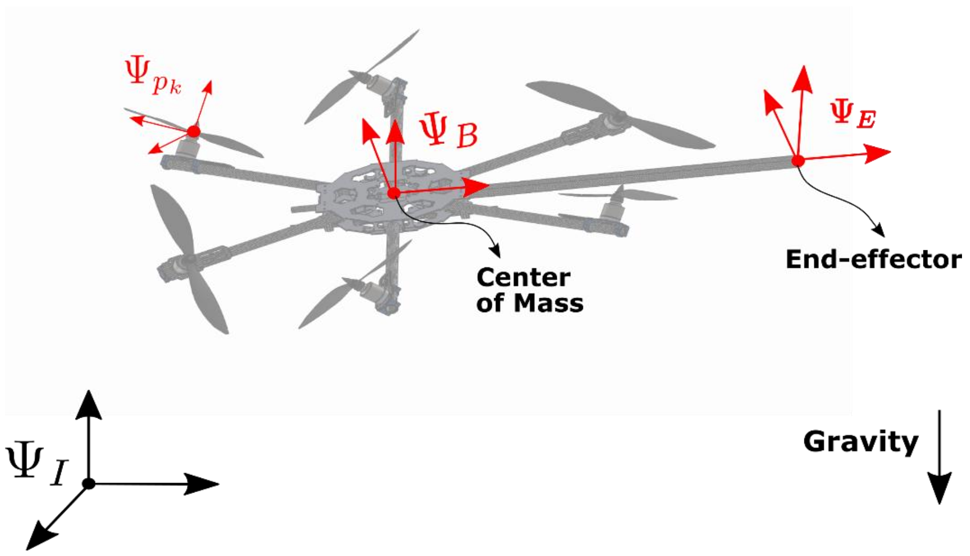
Equivalent mass-spring damper system with respect to $(f_{\text{int}}, \dot{\xi})$

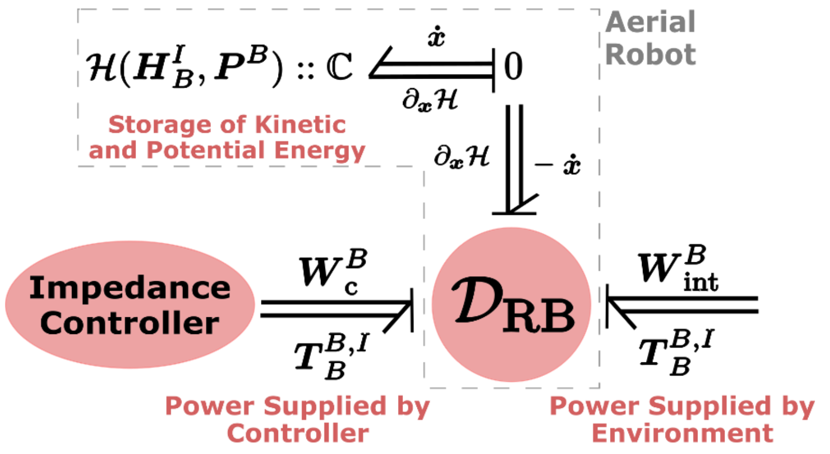


Next !

- Interpretation of impedance control law as another port-Hamiltonian System that does energy-balancing and damping-injection.
- Impedance control of flying end-effector
- Variable impedance control of flying end-effector

STANDARD IMPEDANCE CONTROL ON SE(3)





Port-Hamiltonian dynamics

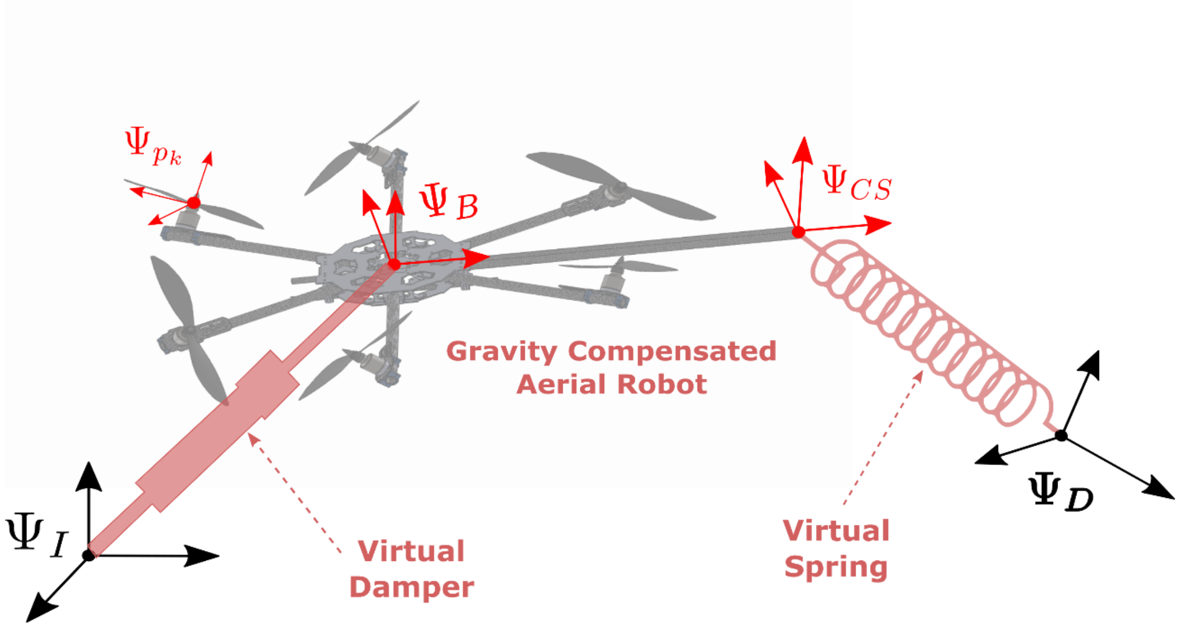
$$\begin{pmatrix}\dot{\boldsymbol{H}}^I_B\\ \dot{\boldsymbol{P}}^B\end{pmatrix} = \begin{pmatrix}\boldsymbol{0} & \chi_{\boldsymbol{H}^I_B}\\ -\chi^*_{\boldsymbol{H}^I_B} & \boldsymbol{J}_{\mathrm{k}}(\boldsymbol{P}^B)\end{pmatrix}\begin{pmatrix}\partial_{\boldsymbol{H}^I_B}\mathcal{H}\\ \partial_{\boldsymbol{P}^B}\mathcal{H}\end{pmatrix} + \begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{I}_6\end{pmatrix}(\boldsymbol{W}^B_{\mathrm{c}} + \boldsymbol{W}^B_{\mathrm{int}})$$

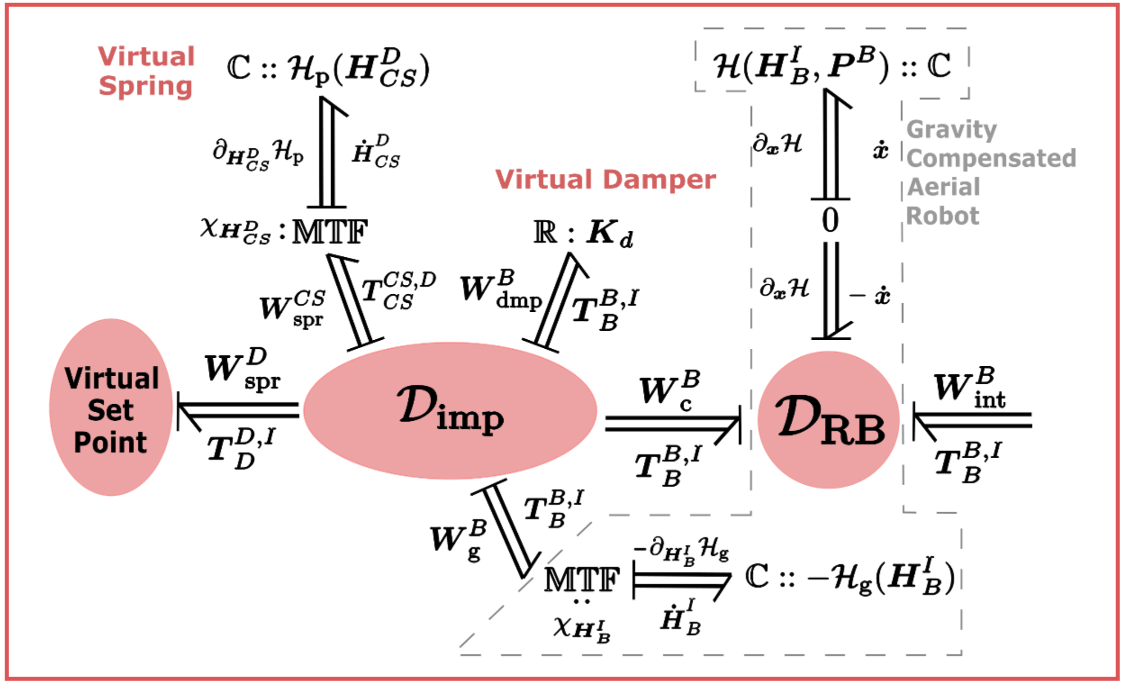
$$\boldsymbol{T}^{B,I}_B = \begin{pmatrix}\boldsymbol{0} & \boldsymbol{I}_6\end{pmatrix}\begin{pmatrix}\partial_{\boldsymbol{H}^I_B}\mathcal{H}\\ \partial_{\boldsymbol{P}^B}\mathcal{H}\end{pmatrix}$$

Energy of aerial robot

$$\mathcal{H}(\boldsymbol{H}^I_B, \boldsymbol{P}^B) = \underbrace{\frac{1}{2}(\boldsymbol{P}^B)^\top \boldsymbol{\mathcal{I}}^{-1}\boldsymbol{P}^B}_{\text{Kinetic}} + \underbrace{m(\boldsymbol{\xi}^I_B)^\top \boldsymbol{g}}_{\text{Potential}}$$

STANDARD IMPEDANCE CONTROL ON SE(3)





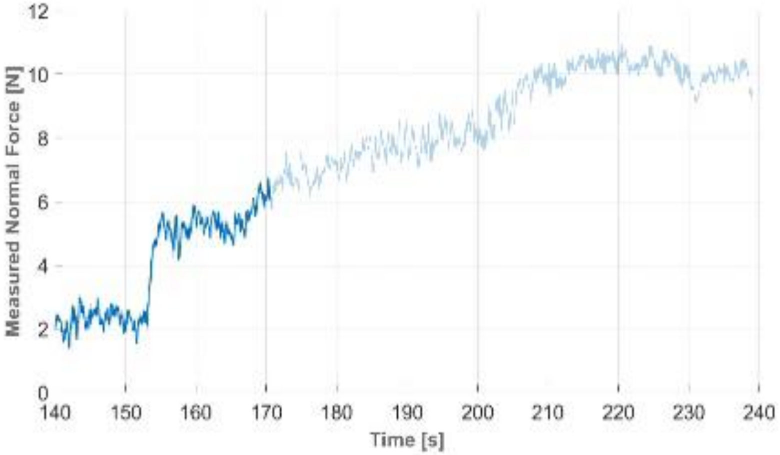
Impedance control law

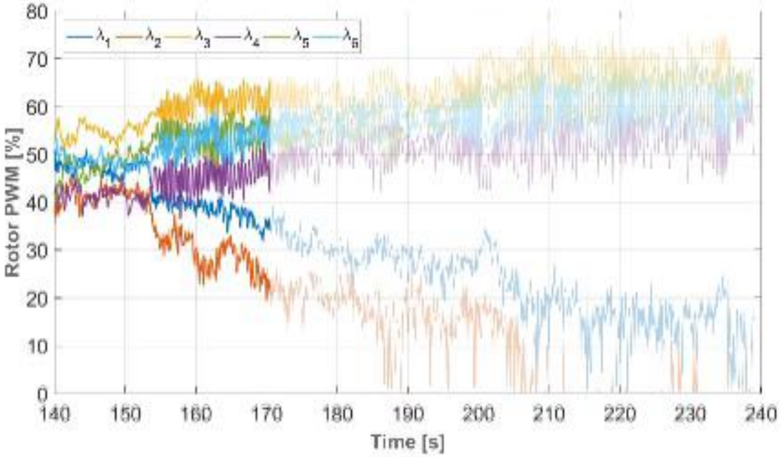
$$\boldsymbol{W}_c^B = -\underbrace{\boldsymbol{Ad}^\top_{\boldsymbol{H}_B^{CS}} \boldsymbol{W}_{spr}^{CS}}_{\text{Virtual Spring}} - \underbrace{\boldsymbol{W}_{dmp}^B}_{\text{Virtual Damper}} - \underbrace{\boldsymbol{W}_g^B}_{\text{Gravity Compensation}}$$

Advantages

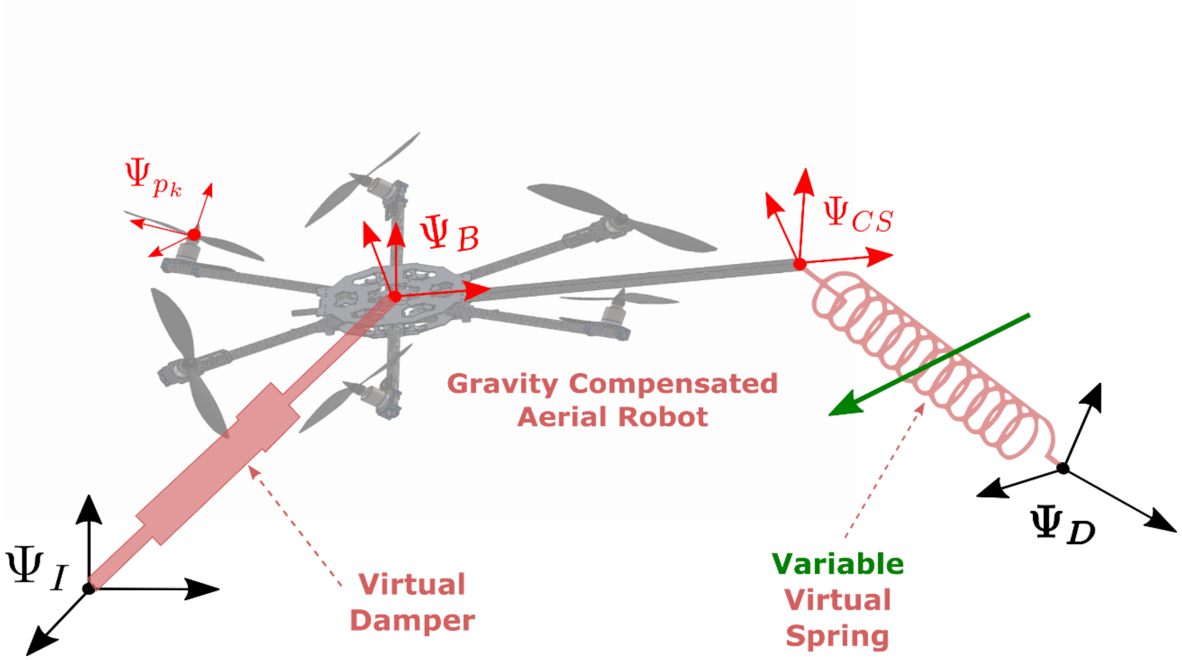
- Passive controller
- Contact-stability not guaranteed
- Robust to parametric uncertainties

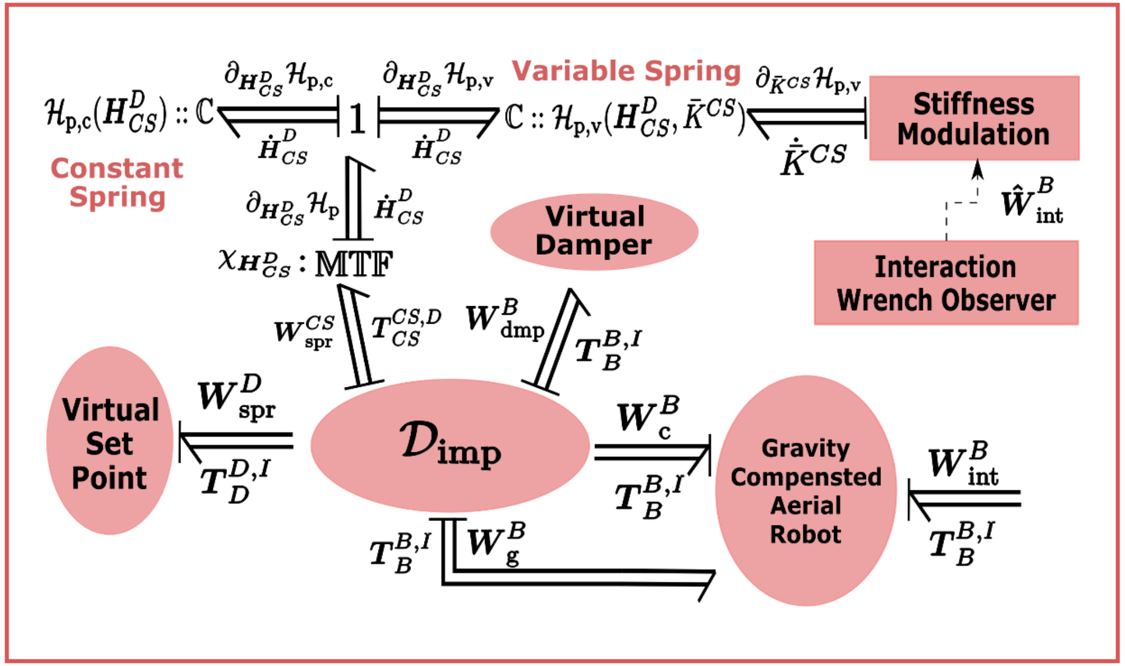
Robustness against uncertainties





VARIABLE IMPEDANCE CONTROL ON SE(3)





Variable impedance control law

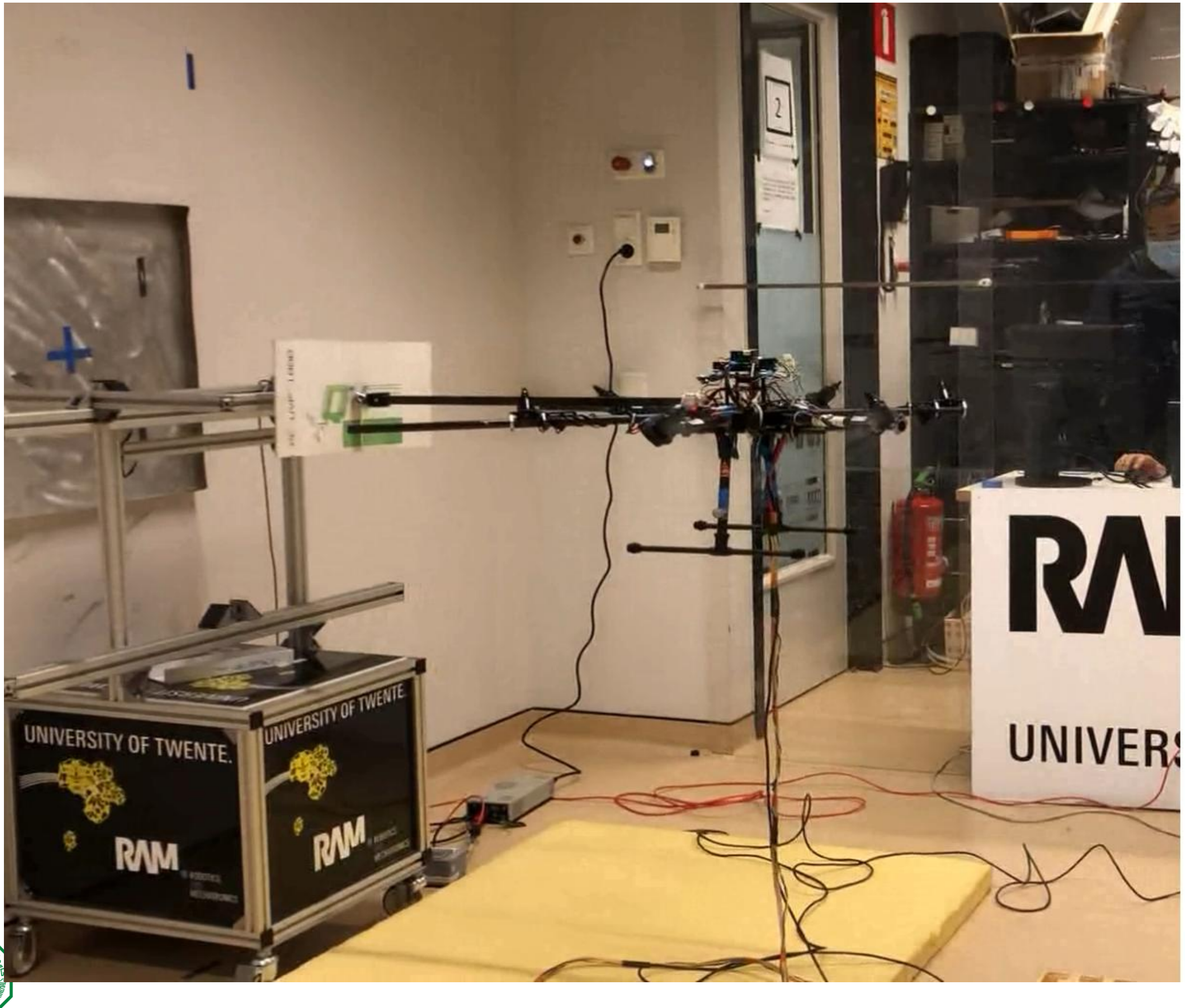
$$\boldsymbol{W}^B_{\mathrm{c}} = -\boldsymbol{Ad}^\top_{\boldsymbol{H}^{CS}_B} \left(\boldsymbol{W}^{CS}_{\mathrm{spr,c}} + \boldsymbol{W}^{CS}_{\mathrm{spr,v}}\right) - \boldsymbol{W}^B_{\mathrm{dmp}} - \boldsymbol{W}^B_{\mathrm{g}}$$

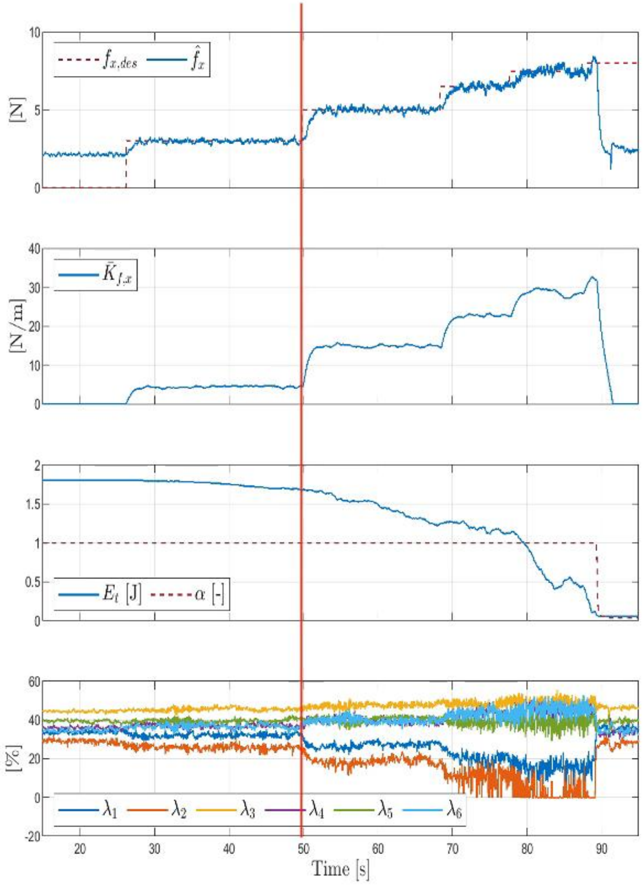
Used for regulating the interaction wrench

Drawbacks

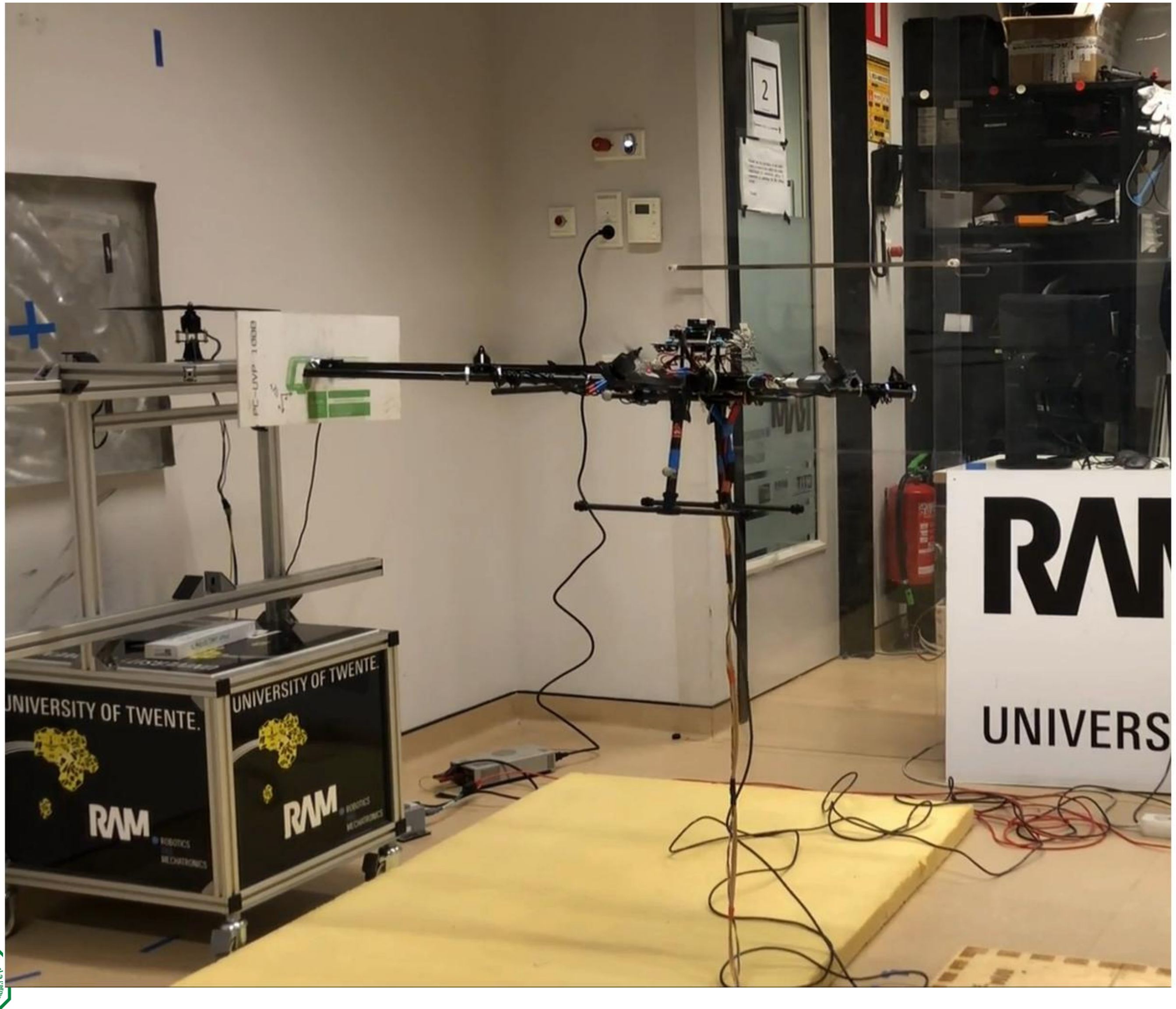
- Non-passive controller
- Contact-stability not guaranteed
- Not safe for generic interaction

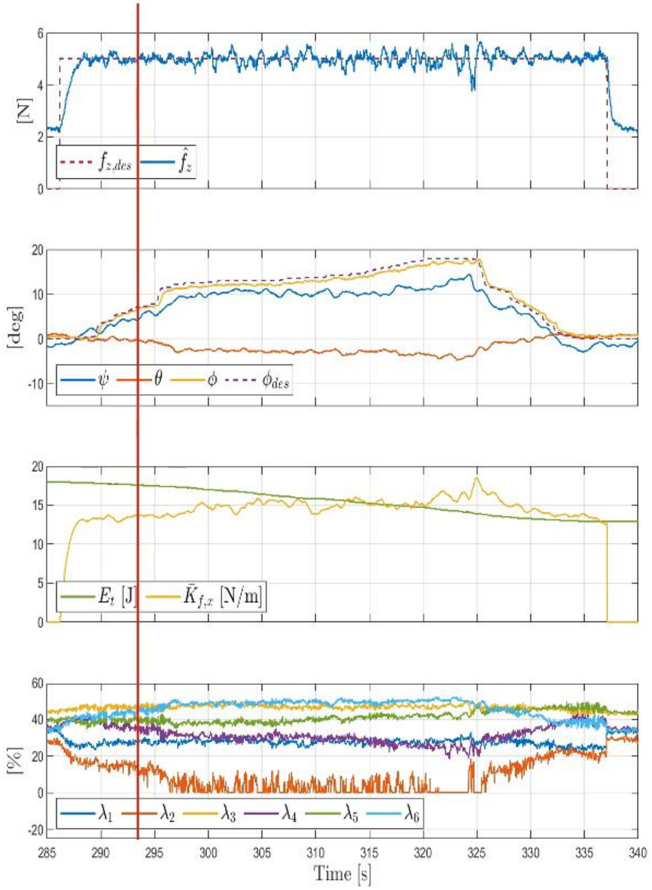
EXP.1: WRENCH REGULATION





EXP.3: FULL ACTUATION CAPABILITIES





EXP.4: IMPORTANCE OF ENERGY AWARENESS