جامعة الملك فهد للبترول والمعادن
King Fahd University of Petroleum & Minerals

GEOMETRIC PORT-HAMILTONIAN MODELING AND CONTROL OF A FLYING END-EFFECTOR

RAMY RASHAD
ASSISTANT PROFESSOR, CIE DEPARTMENT

OUTLINE

- Recap last lecture
- Importance of Passivity
- Energy Aware Control
- Energy Aware Impedance Control

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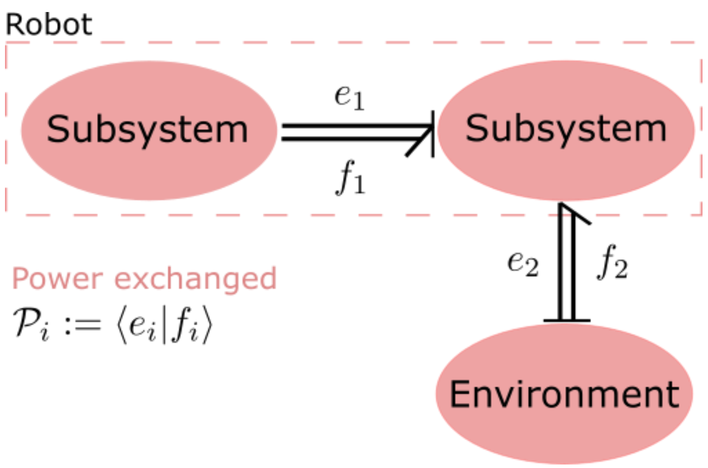
GEOMETRIC PORT-HAMILTONIAN FRAMEWORK

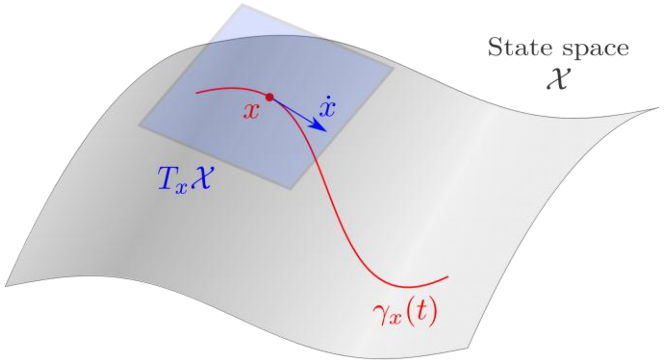
Port-based Modeling + **Geometric Mechanics** + **Systems & Control Theory**

- *Energy* instead of *Signals*
- Multi-physics & open systems

- Coordinate-free descriptions
- Finite & infinite-dimension systems

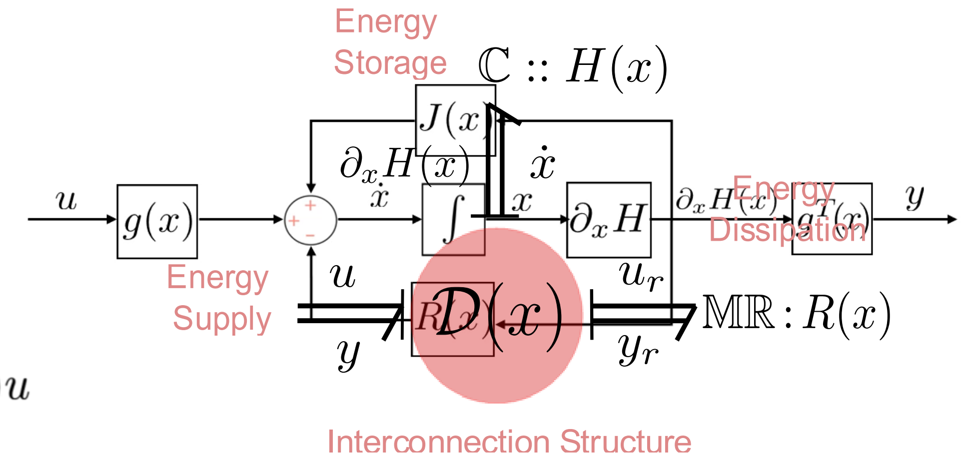
- Exploit underlying structure
- Energy-based analysis & control





$$\dot{x} = [J(x) - R(x)]\partial_x H(x) + g(x)u$$

$$y = g^T(x)\partial_x H(x)$$



FLYING END-EFFECTOR MODEL

$T_B^{*,I}$: Generalized Velocity = Twist
W_{src}^{*}: Generalized Forces = Wrench

- **Nonlinear Equations of Motion on SE(3)**
  - Kinematics

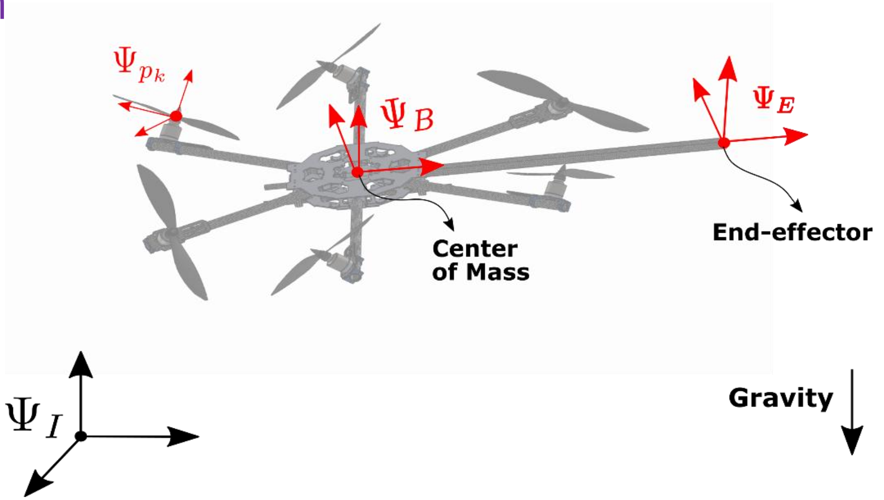
$$\dot{H}_B^I = H_B^I \tilde{T}_B^{B,I}$$

  - Dynamics

$$\dot{P}^B = J_{\mathrm{k}}(P^B)\mathcal{I}^{-1}P^B + W_{\mathrm{g}}^B + W_{\mathrm{c}}^B + W_{\mathrm{int}}^B$$

Control Wrench

Interaction Wrench



- $H_B^I \in SE(3)$- Pose of Ψ_B w.r.t. Ψ_I
- $T_B^{*,I} \in \mathbb{R}^6$- Generalized velocity of Ψ_B w.r.t. Ψ_I, expressed in Ψ_*
- $W_{\mathrm{src}}^{*} \in (\mathbb{R}^6)^*$- Generalized forces by src acting on body expressed in Ψ_*
- $P^* \in (\mathbb{R}^6)^*$- Generalized momentum of body expressed in Ψ_*

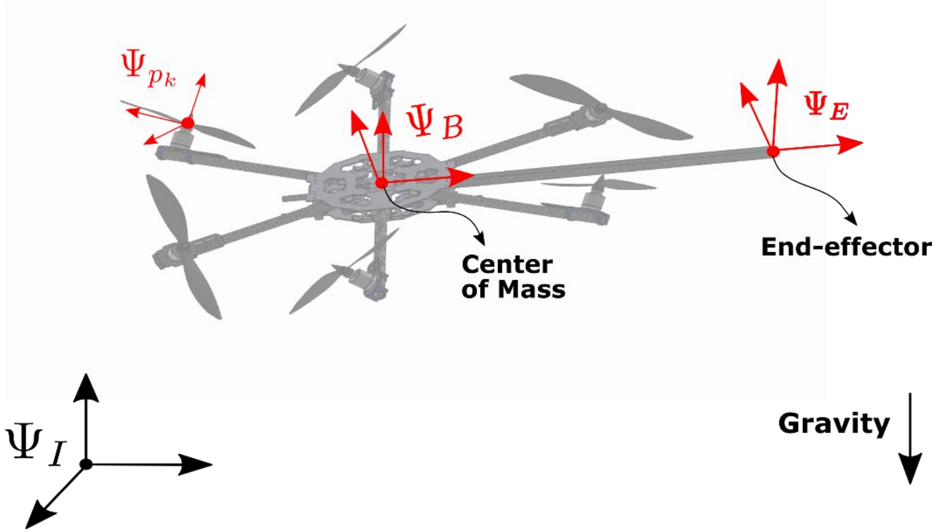
FLYING END-EFFECTOR DYNAMICS

- **Equations of motion**

$$\dot{H}^{I}_{B} = H^{I}_{B}\tilde{T}^{B,I}_{B}$$
$$\dot{P}^{B} = J_{\mathrm{k}}(P^{B})\mathcal{I}^{-1}P^{B} + W^{B}_{\mathrm{g}} + W^{B}_{\mathrm{c}} + W^{B}_{\mathrm{int}}$$

- **Port-Hamiltonian Representation**

$$\dot{H}^{I}_{B} = \chi_{H^{I}_{B}}(\partial_{P^{B}}\mathcal{H})$$
$$\dot{P}^{B} = -\chi^{*}_{H^{I}_{B}}(\partial_{H^{I}_{B}}\mathcal{H}) + J_{\mathrm{k}}(P^{B})\partial_{P^{B}}\mathcal{H} + W^{B}_{\mathrm{c}} + W^{B}_{\mathrm{int}}$$



Hamiltonian function

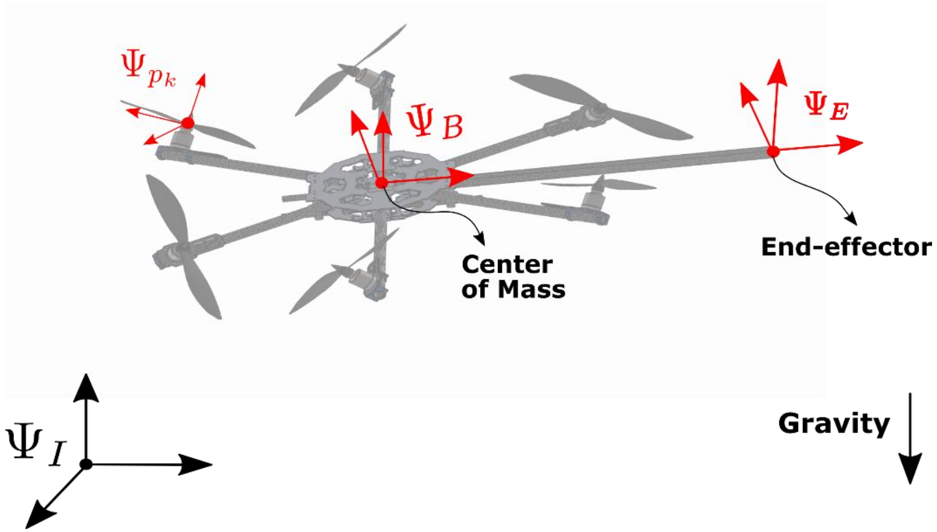
$$\boxed{\mathcal{H}(x) = \mathcal{H}_{\mathrm{k}}(P^{B}) + \mathcal{H}_{\mathrm{g}}(H^{I}_{B}) = \frac{1}{2}(P^{B})^{\top}\mathcal{I}^{-1}P^{B} + m(\xi^{I}_{B})^{\top}g}$$

FLYING END-EFFECTOR DYNAMICS

- **Port-Hamiltonian Representation**

$$\begin{pmatrix} \dot{\boldsymbol{H}}^{I}_{B} \\ \dot{\boldsymbol{P}}^{B} \end{pmatrix} = \begin{pmatrix} \mathbf{0} & \chi_{\boldsymbol{H}^{I}_{B}} \\ -\chi^{*}_{\boldsymbol{H}^{I}_{B}} & \boldsymbol{J}_{\mathrm{k}}(\boldsymbol{P}^{B}) \end{pmatrix} \begin{pmatrix} \partial_{\boldsymbol{H}^{I}_{B}}\mathcal{H} \\ \partial_{\boldsymbol{P}^{B}}\mathcal{H} \end{pmatrix} + \begin{pmatrix} \mathbf{0} \\ \boldsymbol{I}_{6} \end{pmatrix} (\boldsymbol{W}^{B}_{\mathrm{c}} + \boldsymbol{W}^{B}_{\mathrm{int}})$$

$$\boldsymbol{T}^{B,I}_{B} = \begin{pmatrix} \mathbf{0} & \boldsymbol{I}_{6} \end{pmatrix} \begin{pmatrix} \partial_{\boldsymbol{H}^{I}_{B}}\mathcal{H} \\ \partial_{\boldsymbol{P}^{B}}\mathcal{H} \end{pmatrix}$$



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FLYING END-EFFECTOR DYNAMICS

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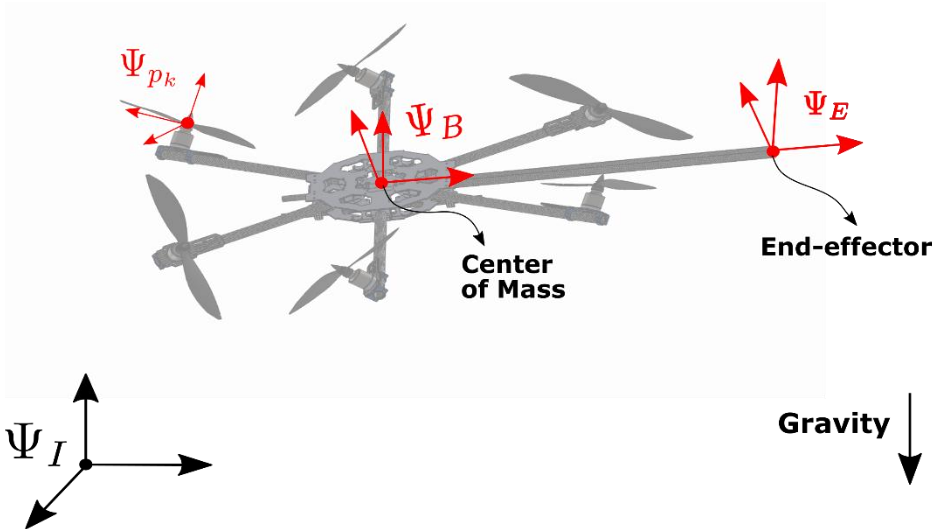
$$\begin{pmatrix} \dot{\boldsymbol{H}}_B^I \\ \dot{\boldsymbol{P}}^B \end{pmatrix} = \begin{pmatrix} \mathbf{0} & \chi_{\boldsymbol{H}_B^I} \\ -\chi^*_{\boldsymbol{H}_B^I} & \boldsymbol{J}_{\mathrm{k}}(\boldsymbol{P}^B) \end{pmatrix} \begin{pmatrix} \partial_{\boldsymbol{H}_B^I}\mathcal{H} \\ \partial_{\boldsymbol{P}^B}\mathcal{H} \end{pmatrix} + \begin{pmatrix} \mathbf{0} \\ \boldsymbol{I}_6 \end{pmatrix} (\boldsymbol{W}_{\mathrm{c}}^B + \boldsymbol{W}_{\mathrm{int}}^B)$$

$$\boldsymbol{T}_B^{B,I} = \begin{pmatrix} \mathbf{0} & \boldsymbol{I}_6 \end{pmatrix} \begin{pmatrix} \partial_{\boldsymbol{H}_B^I}\mathcal{H} \\ \partial_{\boldsymbol{P}^B}\mathcal{H} \end{pmatrix}$$

- **Stored energy analysis**
  - $$\dot{\mathcal{H}}(x) = \underbrace{(W_c^B)^\top T_B^{B,I}}_{\text{Power supplied by control}} + \underbrace{(W_{\mathrm{int}}^B)^\top T_B^{B,I}}_{\text{Power supplied by environment}}$$



GRAPHICAL REPRESENTATION

- **Port-Hamiltonian Representation**

$$\begin{pmatrix} \dot{\boldsymbol{H}}_B^I \\ \dot{\boldsymbol{P}}^B \end{pmatrix} = \begin{pmatrix} \mathbf{0} & \chi_{\boldsymbol{H}_B^I} \\ -\chi^*_{\boldsymbol{H}_B^I} & \boldsymbol{J}_{\mathrm{k}}(\boldsymbol{P}^B) \end{pmatrix} \begin{pmatrix} \partial_{\boldsymbol{H}_B^I}\mathcal{H} \\ \partial_{\boldsymbol{P}^B}\mathcal{H} \end{pmatrix} + \begin{pmatrix} \mathbf{0} \\ \boldsymbol{I}_6 \end{pmatrix} (\boldsymbol{W}_{\mathrm{c}}^B + \boldsymbol{W}_{\mathrm{int}}^B)$$

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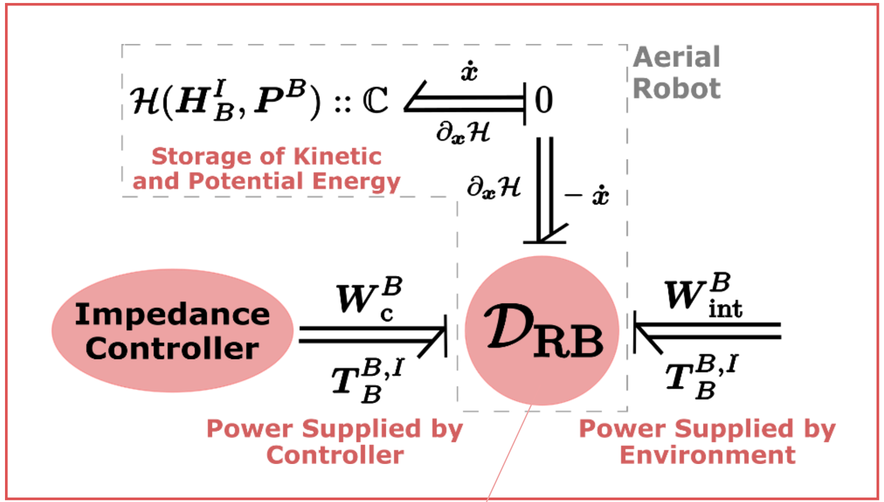
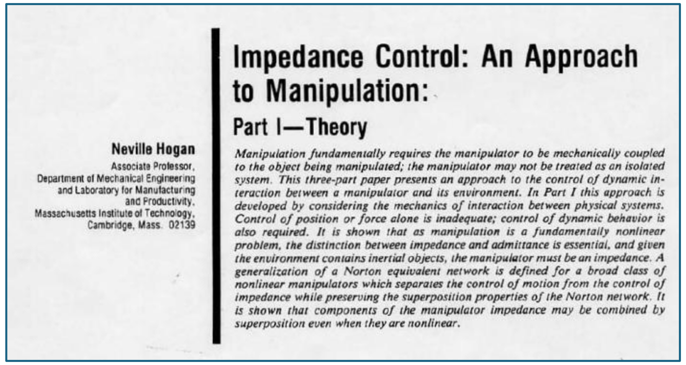


$$\begin{pmatrix} -\dot{\boldsymbol{H}}_B^I \\ -\dot{\boldsymbol{P}}^B \\ \boldsymbol{T}_B^{B,I} \\ \boldsymbol{T}_B^{B,I} \end{pmatrix} = \begin{pmatrix} \mathbf{0} & -\chi_{\boldsymbol{H}_B^I} & \mathbf{0} & \mathbf{0} \\ \chi^*_{\boldsymbol{H}_B^I} & -\boldsymbol{J}_{\mathrm{k}}(\boldsymbol{P}^B) & -\boldsymbol{I}_6 & -\boldsymbol{I}_6 \\ \mathbf{0} & \boldsymbol{I}_6 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{I}_6 & \mathbf{0} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \partial_{\boldsymbol{H}_B^I}\mathcal{H} \\ \partial_{\boldsymbol{P}^B}\mathcal{H} \\ \boldsymbol{W}_{\mathrm{c}}^B \\ \boldsymbol{W}_{\mathrm{int}}^B \end{pmatrix}$$

Impedance Control

- Neville Hogan's idea of impedance control is that instead of commanding exact positions or forces, the robot is controlled to behave like a mechanical impedance:
  - It responds to external forces with a desired relationship between force, position, and velocity, like a spring-damper-mass system.
- The key is shaping the robot's dynamic behavior — its *stiffness*, *damping*, and *inertia* — to match the interaction task needs.

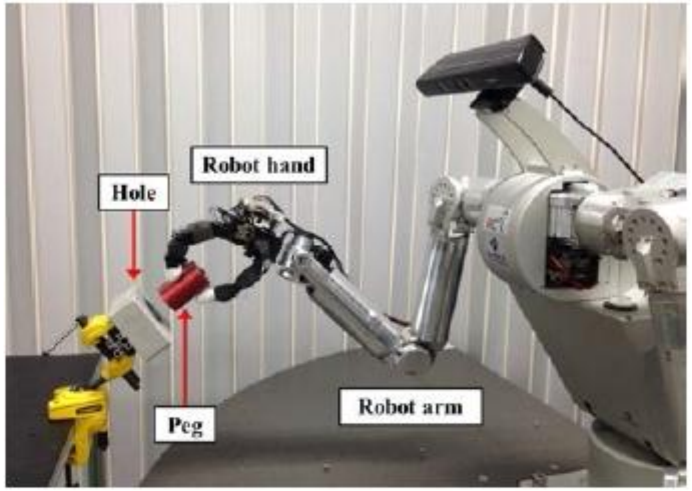


Impedance Control: An Approach to Manipulation:

Part I—Theory

Neville Hogan
Associate Professor,
Department of Mechanical Engineering
and Laboratory for Manufacturing
and Productivity,
Massachusetts Institute of Technology,
Cambridge, Mass. 02139

Manipulation fundamentally requires the manipulator to be mechanically coupled to the object being manipulated; the manipulator may not be treated as an isolated system. This three-part paper presents an approach to the control of dynamic interaction between a manipulator and its environment. In Part I this approach is developed by considering the mechanics of interaction between physical systems. Control of position or force alone is inadequate; control of dynamic behavior is also required. It is shown that as manipulation is a fundamentally nonlinear problem, the distinction between impedance and admittance is essential, and given the environment contains inertial objects, the manipulator must be an impedance. A generalization of a Norton equivalent network is defined for a broad class of nonlinear manipulators which separates the control of motion from the control of impedance while preserving the superposition properties of the Norton network. It is shown that components of the manipulator impedance may be combined by superposition even when they are nonlinear.



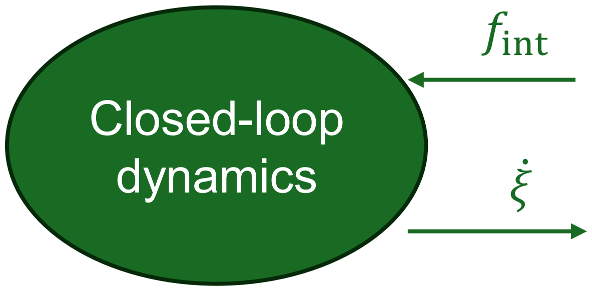
Impedance Control of Mass Damper

- **Equations of motion**
  - $\dot{\xi} = v$
  - $m\dot{v} = -b\,|v|v \; - mg + f_{\text{con}} + f_{\text{int}}$
- **Impedance control law**
  - $f_{\text{con}} = mg \; - K_p\xi \; - K_d v$
- **Closed loop dynamics**

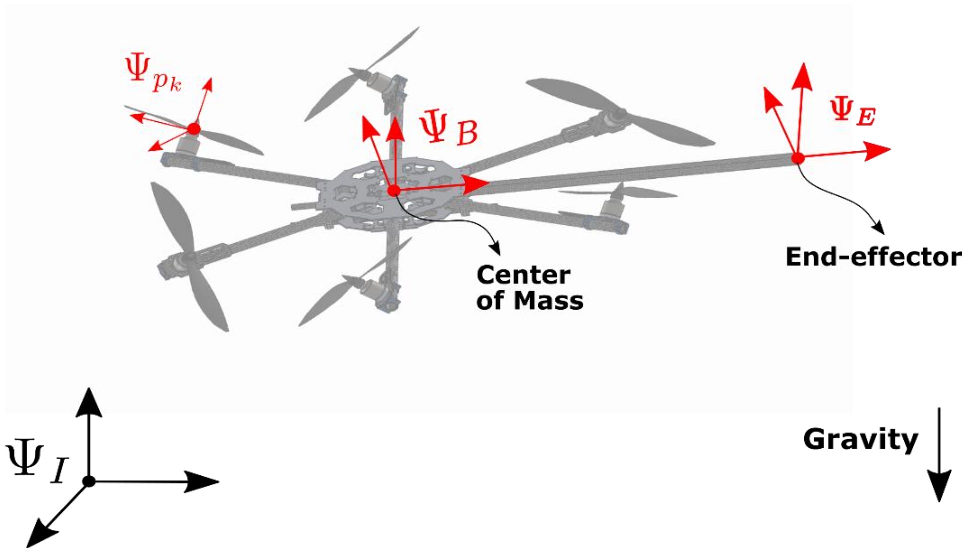
$$m\dot{v} = -b\,|v|v - K_p\xi \; - K_d v + f_{\text{int}}$$

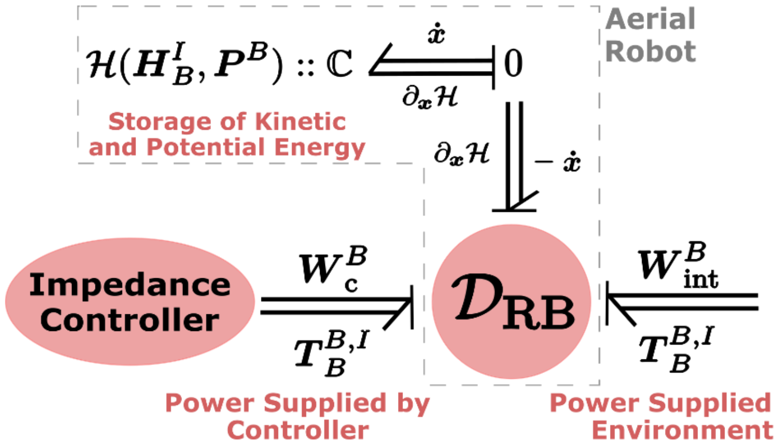
$$\boxed{m\ddot{\xi} + (B(v) + K_d)\dot{\xi} + K_p\xi = f_{\text{int}}}$$

Equivalent mass-spring damper system with respect to $(f_{\text{int}}, \dot{\xi})$



Standard Impedance Control On SE(3)





Port-Hamiltonian dynamics

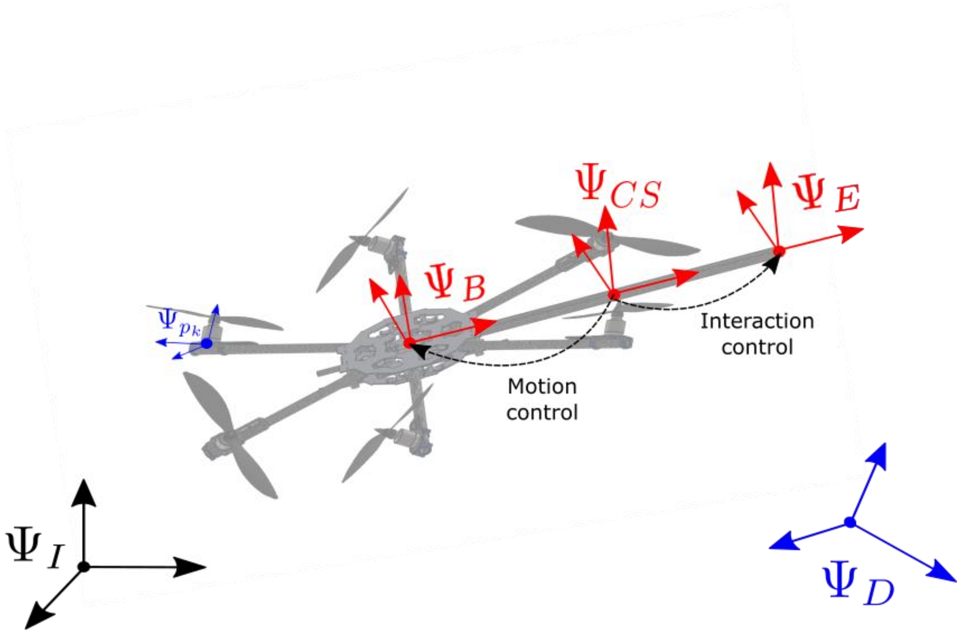
$$\begin{pmatrix}\dot{\boldsymbol{H}}^I_B\\ \dot{\boldsymbol{P}}^B\end{pmatrix} = \begin{pmatrix}\boldsymbol{0} & \chi_{\boldsymbol{H}^I_B}\\ -\chi^*_{\boldsymbol{H}^I_B} & \boldsymbol{J}_{\mathrm{k}}(\boldsymbol{P}^B)\end{pmatrix}\begin{pmatrix}\partial_{\boldsymbol{H}^I_B}\mathcal{H}\\ \partial_{\boldsymbol{P}^B}\mathcal{H}\end{pmatrix} + \begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{I}_6\end{pmatrix}(\boldsymbol{W}^B_{\mathrm{c}} + \boldsymbol{W}^B_{\mathrm{int}})$$

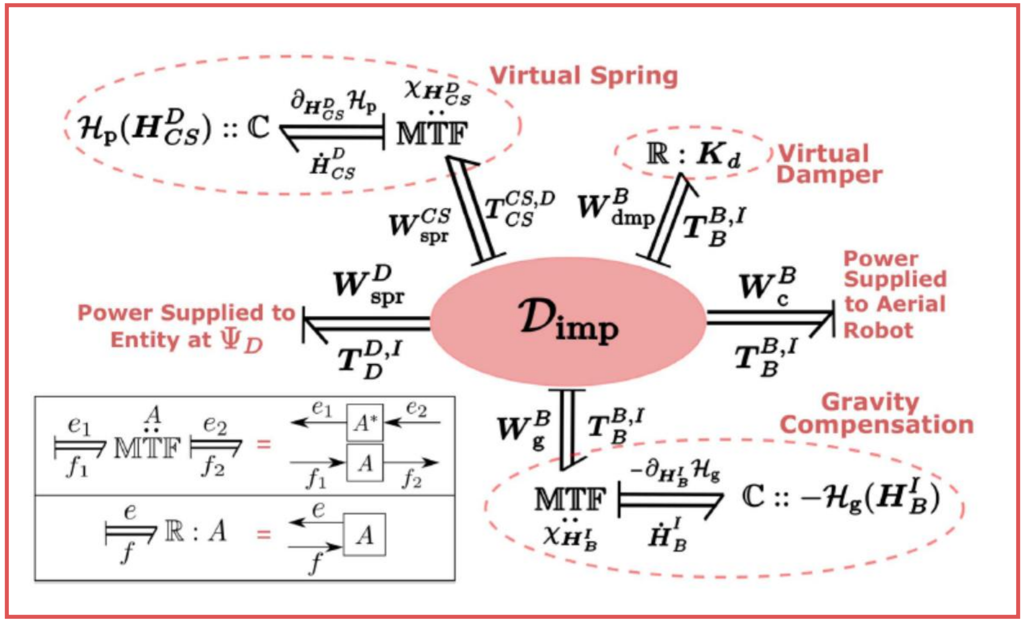
$$\boldsymbol{T}^{B,I}_B = \begin{pmatrix}\boldsymbol{0} & \boldsymbol{I}_6\end{pmatrix}\begin{pmatrix}\partial_{\boldsymbol{H}^I_B}\mathcal{H}\\ \partial_{\boldsymbol{P}^B}\mathcal{H}\end{pmatrix}$$

Energy of aerial robot

$$\mathcal{H}(\boldsymbol{H}^I_B, \boldsymbol{P}^B) = \underbrace{\frac{1}{2}(\boldsymbol{P}^B)^\top \boldsymbol{\mathcal{I}}^{-1}\boldsymbol{P}^B}_{\text{Kinetic}} + \underbrace{m(\boldsymbol{\xi}^I_B)^\top \boldsymbol{g}}_{\text{Potential}}$$

Standard Impedance Control On SE(3)





Energy Damping of Controller

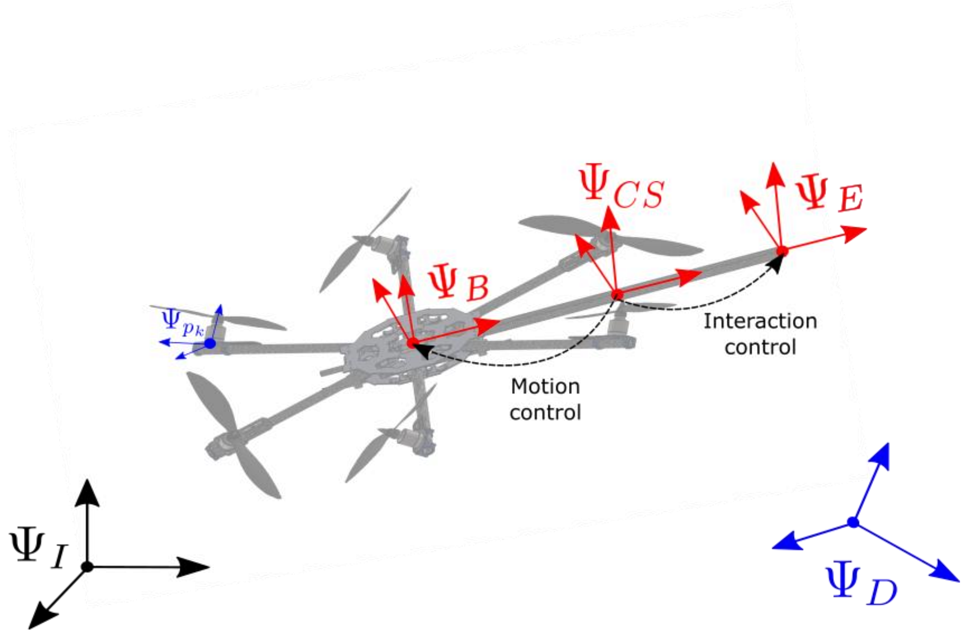
$$\boldsymbol{W}^B_{\text{dmp}} = \boldsymbol{K}_d \boldsymbol{T}^{B,I}_B$$

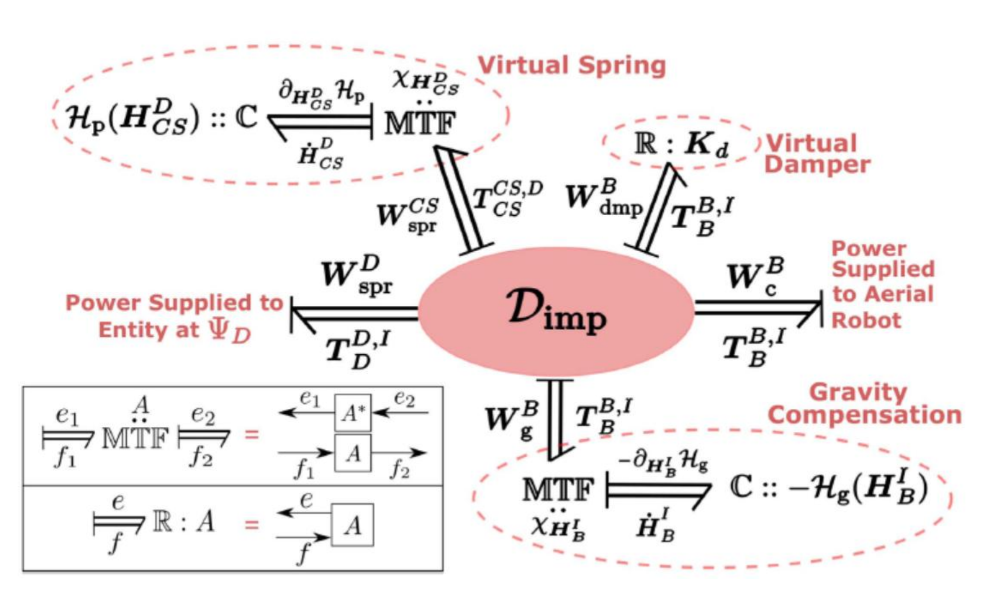
Energy Storage of Controller

$$-\mathcal{H}_g(\boldsymbol{H}^I_B) = -m(\boldsymbol{\xi}^I_B)^\top \boldsymbol{g}$$

$$\mathcal{H}_p(\boldsymbol{H}^D_{CS}) = -\operatorname{tr}(\boldsymbol{G}_o(\boldsymbol{R}^D_{CS} - \boldsymbol{I}_3)) + \frac{1}{4}(\boldsymbol{\xi}^D_{CS})^\top \boldsymbol{K}_t \boldsymbol{\xi}^D_{CS} + \frac{1}{4}(\boldsymbol{\xi}^D_{CS})^\top \boldsymbol{R}^D_{CS} \boldsymbol{K}_t (\boldsymbol{R}^D_{CS})^\top \boldsymbol{\xi}^D_{CS}$$

Standard Impedance Control On SE(3)





Energy Damping of Controller

$$\boxed{W^B_{\mathrm{dmp}} = K_d T^{B,I}_B}$$

Energy Storage of Controller

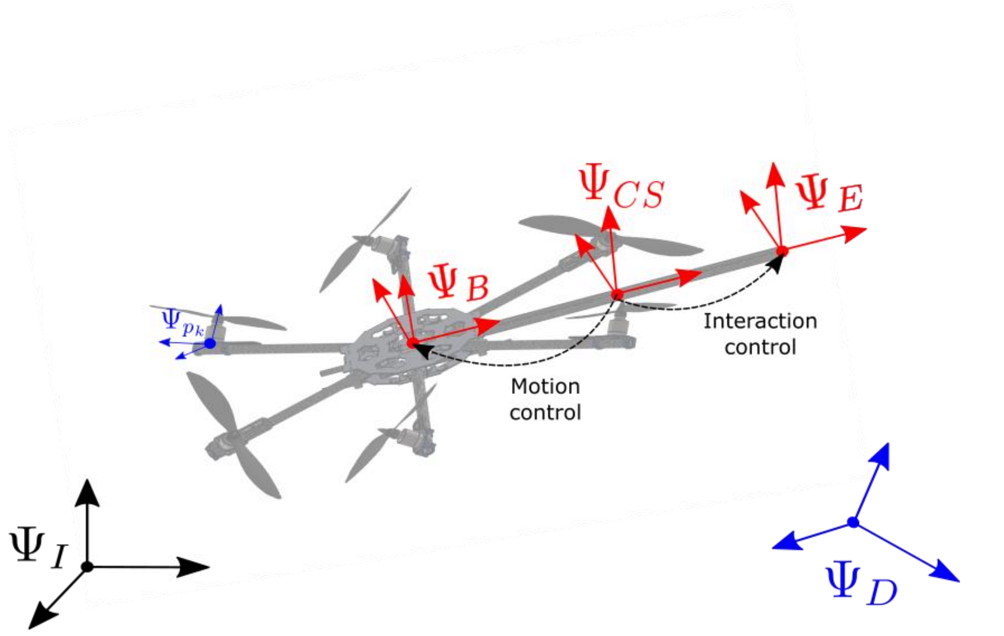
$$\boxed{-\mathcal{H}_g(H^I_B) = -m(\xi^I_B)^\top g}$$

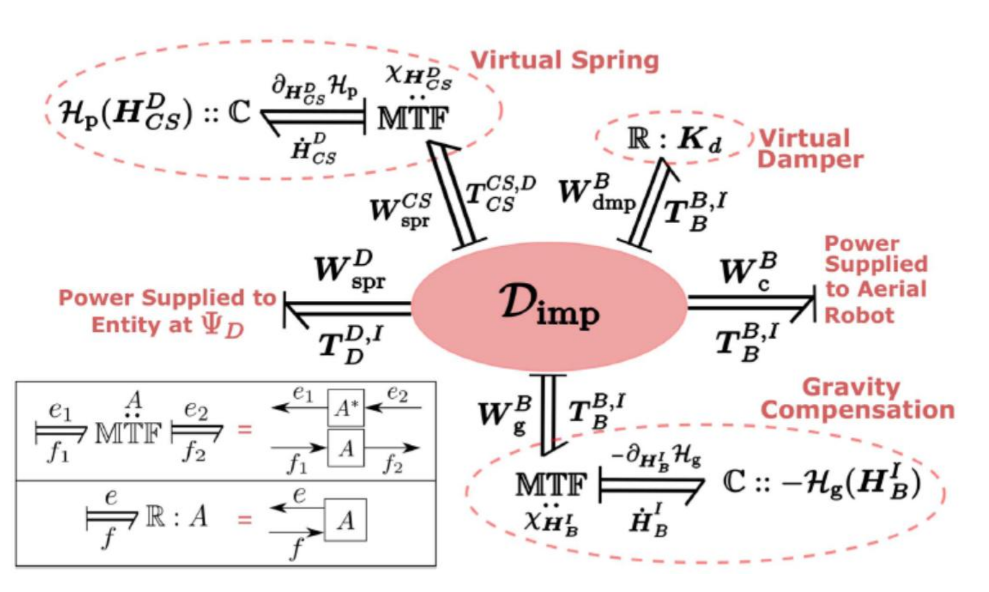
$$\boxed{\mathcal{H}_p(H^D_{CS}) = -\mathrm{tr}(G_o(R^D_{CS} - I_3)) + \frac{1}{4}(\xi^D_{CS})^\top K_t \xi^D_{CS} + \frac{1}{4}(\xi^D_{CS})^\top R^D_{CS} K_t (R^D_{CS})^\top \xi^D_{CS}}$$

$$\begin{pmatrix} T^{CS,D}_{CS} \\ T^{B,I}_B \\ T^{B,I}_B \\ W^D_{\mathrm{spr}} \\ W^B_c \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & -Ad_{H^{CS}_D} & Ad_{H^{CS}_B} \\ 0 & 0 & 0 & 0 & I_6 \\ 0 & 0 & 0 & 0 & I_6 \\ Ad^\top_{H^{CS}_D} & 0 & 0 & 0 & 0 \\ -Ad^\top_{H^{CS}_B} & -I_6 & -I_6 & 0 & 0 \end{pmatrix} \begin{pmatrix} W^{CS}_{\mathrm{spr}} \\ W^B_g \\ W^B_{\mathrm{dmp}} \\ T^{D,I}_D \\ T^{B,I}_B \end{pmatrix}$$

Energy Routing of Controller

Standard Impedance Control On SE(3)





$$\boldsymbol{W}_c^B = -\underbrace{\boldsymbol{Ad}^\top_{\boldsymbol{H}_B^{CS}} \boldsymbol{W}_{spr}^{CS}}_{\text{Virtual Spring}} - \underbrace{\boldsymbol{W}^B_{dmp}}_{\text{Virtual Damper}} - \underbrace{\boldsymbol{W}_g^B}_{\text{Gravity Compensation}}$$

Impedance control law

$$\begin{pmatrix} \boldsymbol{T}_{CS}^{CS,D} \\ \boldsymbol{T}_B^{B,I} \\ \boldsymbol{T}_B^{B,I} \\ \boldsymbol{W}_{spr}^D \\ \boldsymbol{W}_c^B \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & -\boldsymbol{Ad}_{\boldsymbol{H}_D^{CS}} & \boldsymbol{Ad}_{\boldsymbol{H}_B^{CS}} \\ 0 & 0 & 0 & 0 & \boldsymbol{I}_6 \\ 0 & 0 & 0 & 0 & \boldsymbol{I}_6 \\ \boldsymbol{Ad}^\top_{\boldsymbol{H}_D^{CS}} & 0 & 0 & 0 & 0 \\ -\boldsymbol{Ad}^\top_{\boldsymbol{H}_B^{CS}} & -\boldsymbol{I}_6 & -\boldsymbol{I}_6 & 0 & 0 \end{pmatrix} \begin{pmatrix} \boldsymbol{W}_{spr}^{CS} \\ \boldsymbol{W}_g^B \\ \boldsymbol{W}_{dmp}^B \\ \boldsymbol{T}_D^{D,I} \\ \boldsymbol{T}_B^{B,I} \end{pmatrix}$$

Energy Routing of Controller

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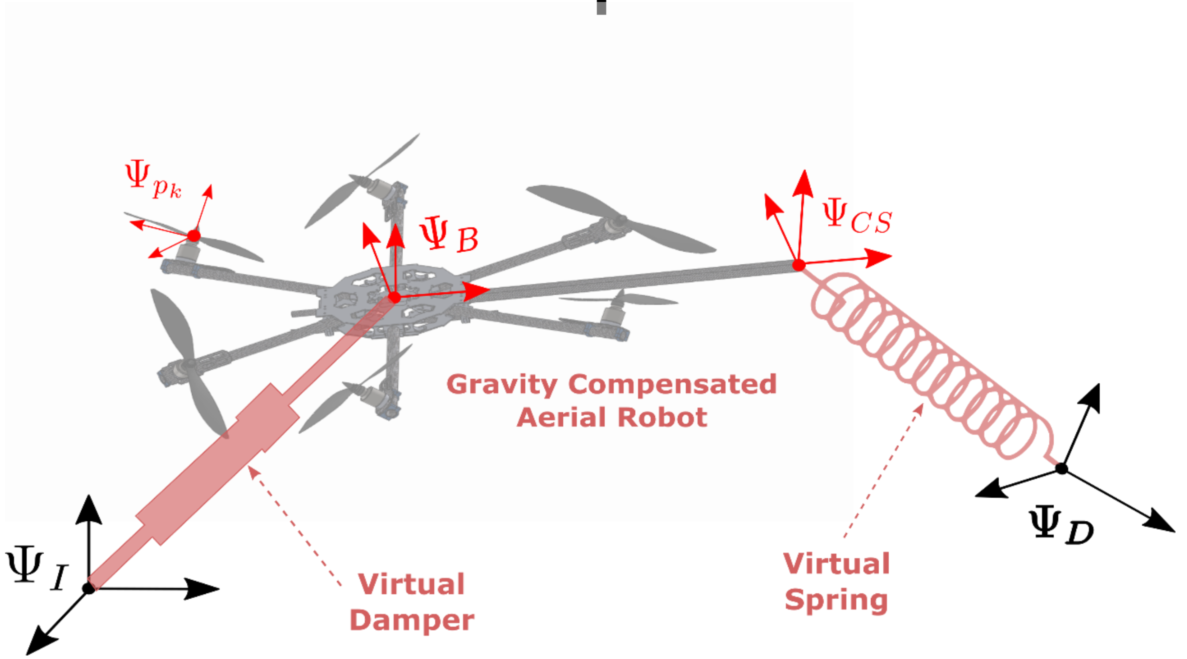

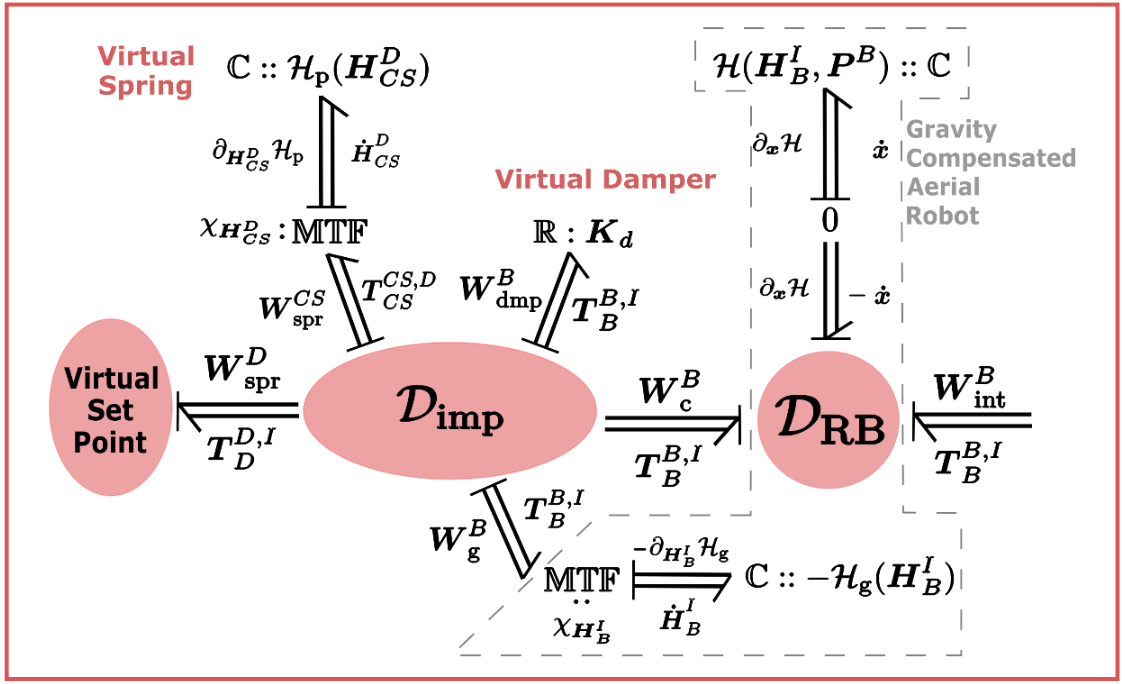

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Advantages

- Passive closed-loop pH system
- Contact-stability guaranteed
- Robust to parametric uncertainties

Outline

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- Importance of Passivity
- Energy Aware Control
- Energy Aware Impedance Control

Passivity and Port-Hamiltonian Systems

A nonlinear affine control system with state $x \in \mathbb{R}^n$ and input-output pair $(u, y) \in \mathbb{R}^m \times \mathbb{R}^m$ in the form

$$\dot{x} = f(x) + g(x)u \tag{1}$$

$$y = h(x) \tag{2}$$

is said to be passive with respect to a non negative state-dependent storage function $V(x)$ if the following inequality is true for any possible input signal $u(t)$ and at any positive time t:

$$V(x(t)) - V(x(0)) \leq \int_0^t y^T(s)u(s)ds. \tag{3}$$

Passivity and Port-Hamiltonian Systems

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Passivity implies stability, which is a property of the autonomous system (i.e., $u = 0$), under the weak conditions that qualify the storage function as a Lyapunov function.

Passivity and Port-Hamiltonian Systems

The input-state-output formulation of a port-Hamiltonian system is given by the following instance of (1–2):

$$\dot{x} = [J(x) - R(x)]\partial_x H(x) + g(x)u, \tag{4}$$

$$y = g^T(x)\partial_x H(x). \tag{5}$$

An immediate property of pH systems (4–5) is that they are passive with respect to the Hamiltonian as storage function:

$$\dot{H} = (\partial_x H(x))^T \dot{x} = -(\partial_x H(x))^T R(x)\partial_x H(x) + y^T u \leq y^T u, \tag{6}$$

Passivity and Port-Hamiltonian Systems

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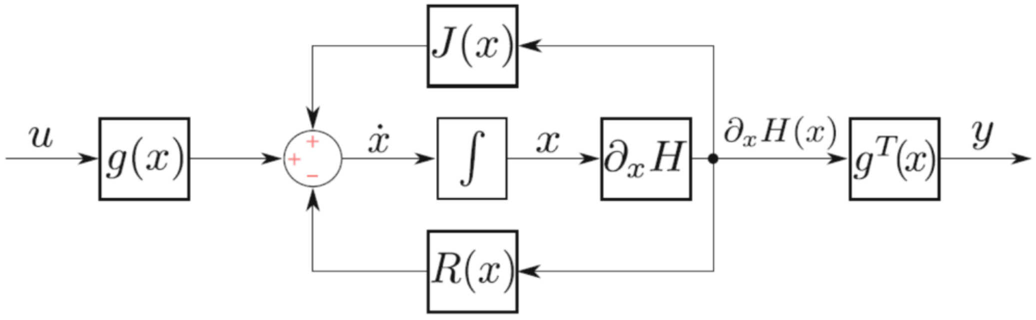
$$\boxed{H(x(t)) - H(x(0)) = \int_0^t y^T(s)u(s)ds - E_d(t) \leq \int_0^t y^T(s)u(s)ds.}$$

Passivity and Port-Hamiltonian Systems

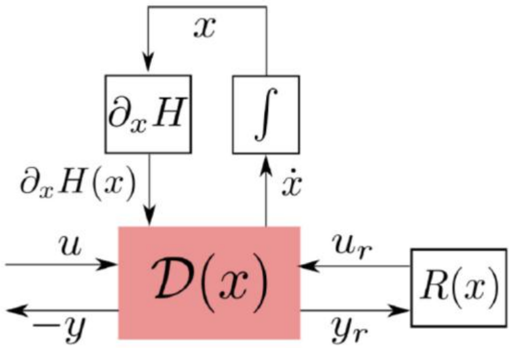
- Port-Hamiltonian system

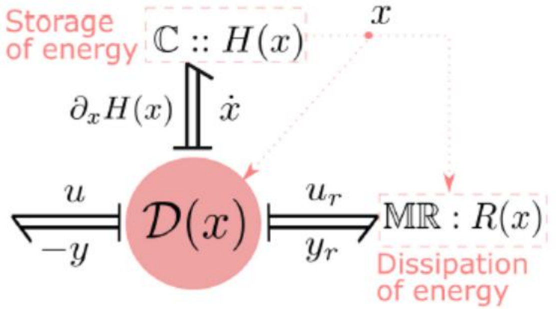
$$\dot{x} = [J(x) - R(x)]\partial_x H(x) + g(x)u,$$
$$y = g^T(x)\partial_x H(x).$$



- Dirac Structure

$$\begin{pmatrix} \dot{x} \\ y_r \\ -y \end{pmatrix} = \begin{pmatrix} J(x) & -I & g \\ I & 0 & 0 \\ -g^T & 0 & 0 \end{pmatrix} \begin{pmatrix} \partial_x H(x) \\ u_r \\ u \end{pmatrix}$$





Passivity-Based Control

- Passivity-based control (PBC) is a set of techniques aiming at designing a controller in such a way that the closed-loop system is passive.
- When it comes to control, port-Hamiltonian theory makes close contact with PBC.
- Port-Hamiltonian systems have a strong property that the form

$$\dot{x} = [J(x) - R(x)]\partial_x H(x) + g(x)u,$$
$$y = g^T(x)\partial_x H(x).$$

is conserved under power-preserving interconnection of pH systems.

The control objective becomes then to design a pH controller such that the desired target pH closed-loop system is achieved !

Passivity-Based Control

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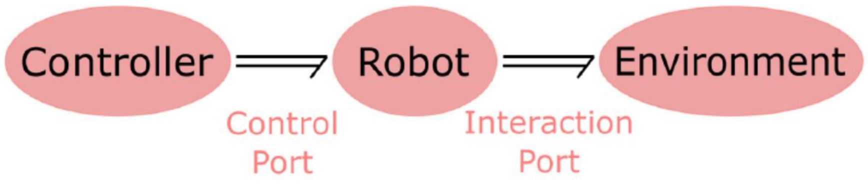
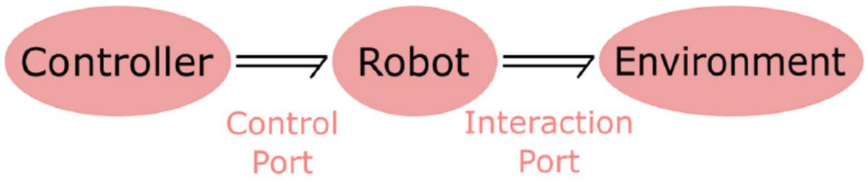
- The control objective becomes then to design a pH controller such that the desired target pH closed-loop system is achieved !
  - Energy Balancing technique (EB-PBC)
  - Interconnection-Damping Assignment technique (IDA-PBC)



Passive Interaction Control

- A robot manipulator can be understood as a pH system with two distinct ports:
  - Control Port
  - Interaction Port
- Along the task it executes, the robot will interact with the environment, whose dynamics is often unknown.
- The interaction will follow physical laws, dependent on the unknown parameters of the environment.

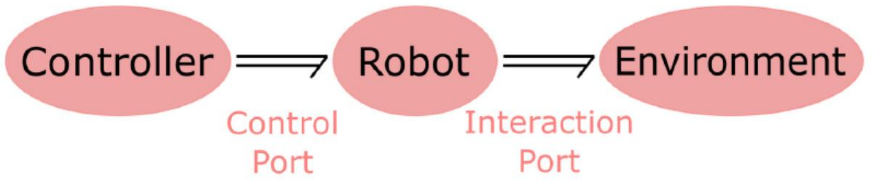


Passive Interaction Control

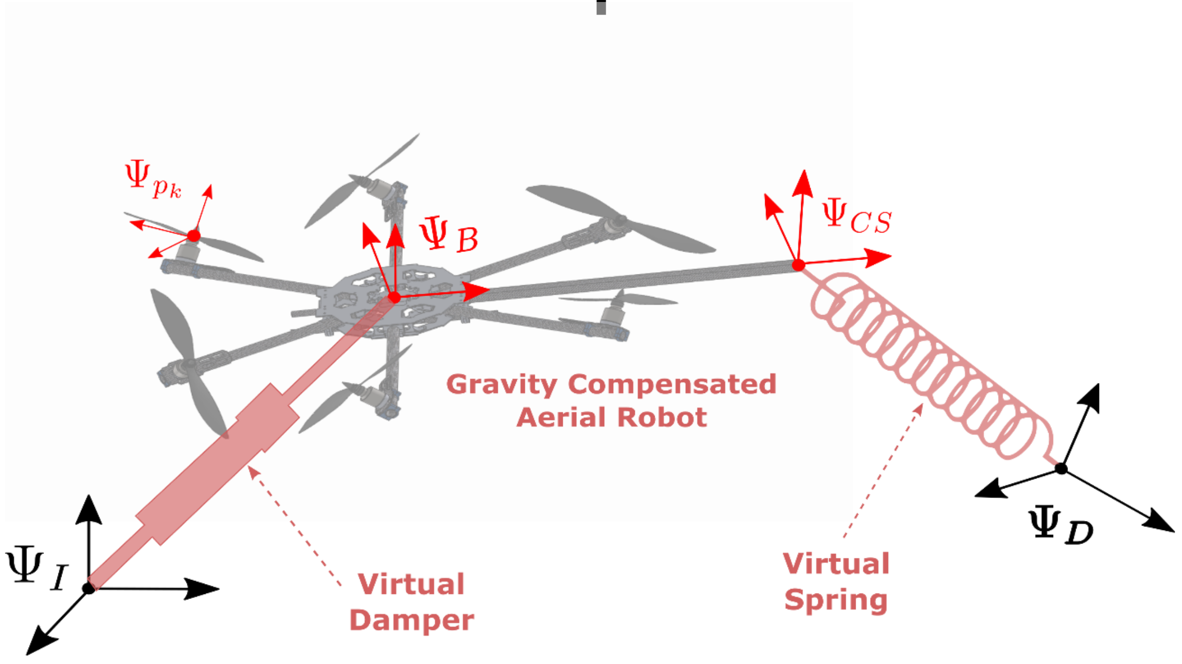
- Control Objective: **Passivity as a must**

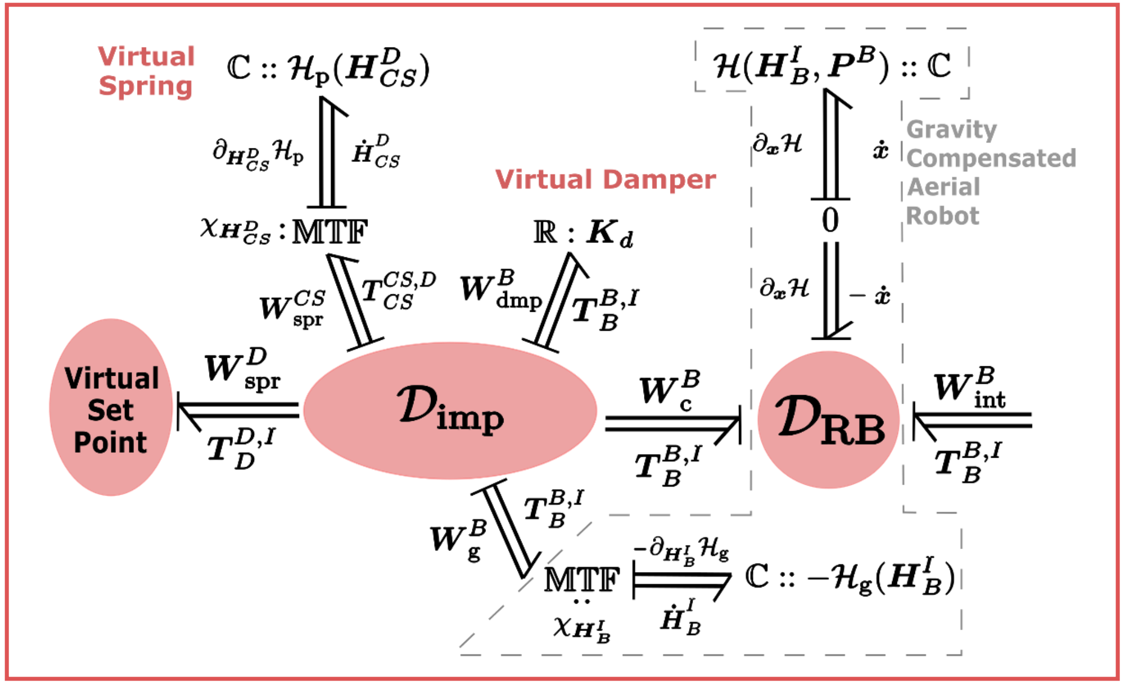
The controlled robot should result in a passive behavior seen from the environment port.

- The motivation behind this claim lies on the fact that if the controlled robot is passive, then the interconnection with any passive environment would indeed result in a stable interconnection



Standard Impedance Control On SE(3)





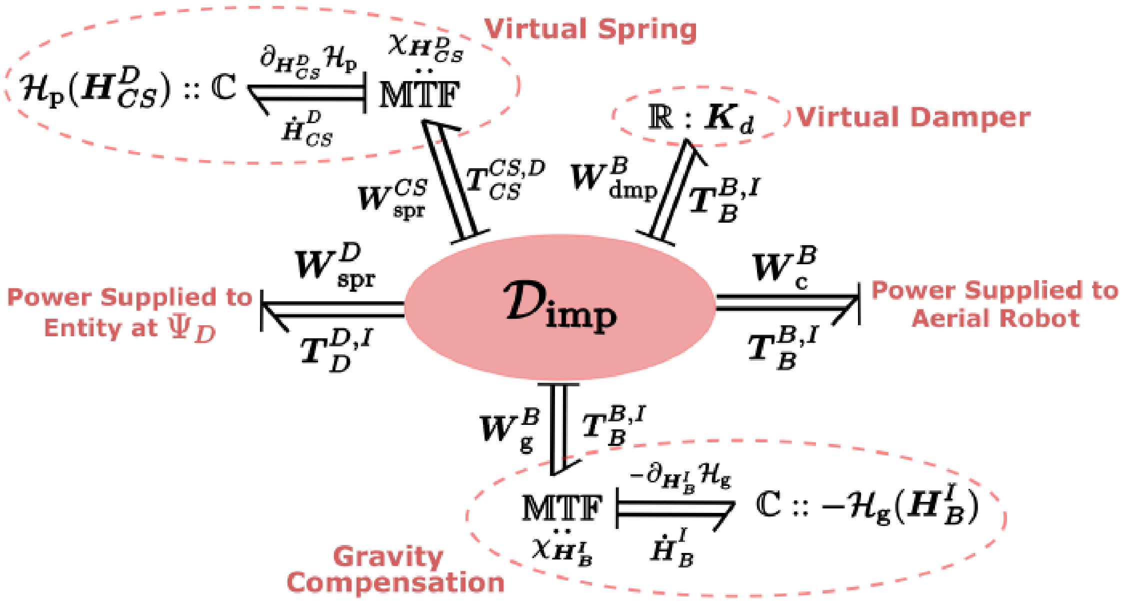
Impedance control law

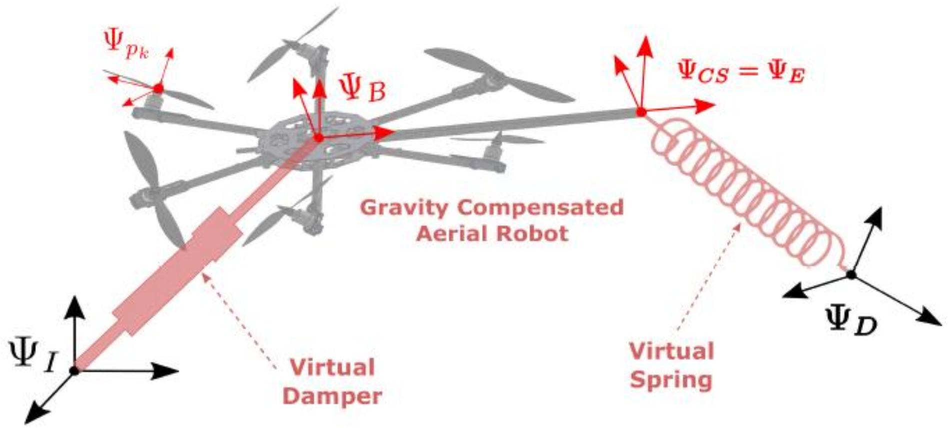
$$W^B_c = -\underbrace{Ad^\top_{H^{CS}_B} W^{CS}_{spr}}_{\substack{\text{Virtual}\\\text{Spring}}} - \underbrace{W^B_{dmp}}_{\substack{\text{Virtual}\\\text{Damper}}} - \underbrace{W^B_g}_{\substack{\text{Gravity}\\\text{Compensation}}}$$

Advantages

- Passive closed-loop pH system
- Contact-stability guaranteed
- Robust to parametric uncertainties

Impedance Control Power Balance





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$$(W_{\text{spr}}^{CS})^\top T_{CS}^{CS,D} + (W_{\text{g}}^{B})^\top T_{B}^{B,I} + (W_{\text{dmp}}^{B})^\top T_{B}^{B,I} + (W_{\text{spr}}^{D})^\top T_{D}^{D,I} + (W_{\text{c}}^{B})^\top T_{B}^{B,I} = 0.$$

$$(W_{\text{c}}^{B})^\top T_{B}^{B,I} = -\dot{\mathcal{H}}_{\text{p}} + \dot{\mathcal{H}}_{\text{g}} - (T_{B}^{B,I})^\top K_d T_{B}^{B,I} - (W_{\text{spr}}^{D})^\top T_{D}^{D,I}.$$

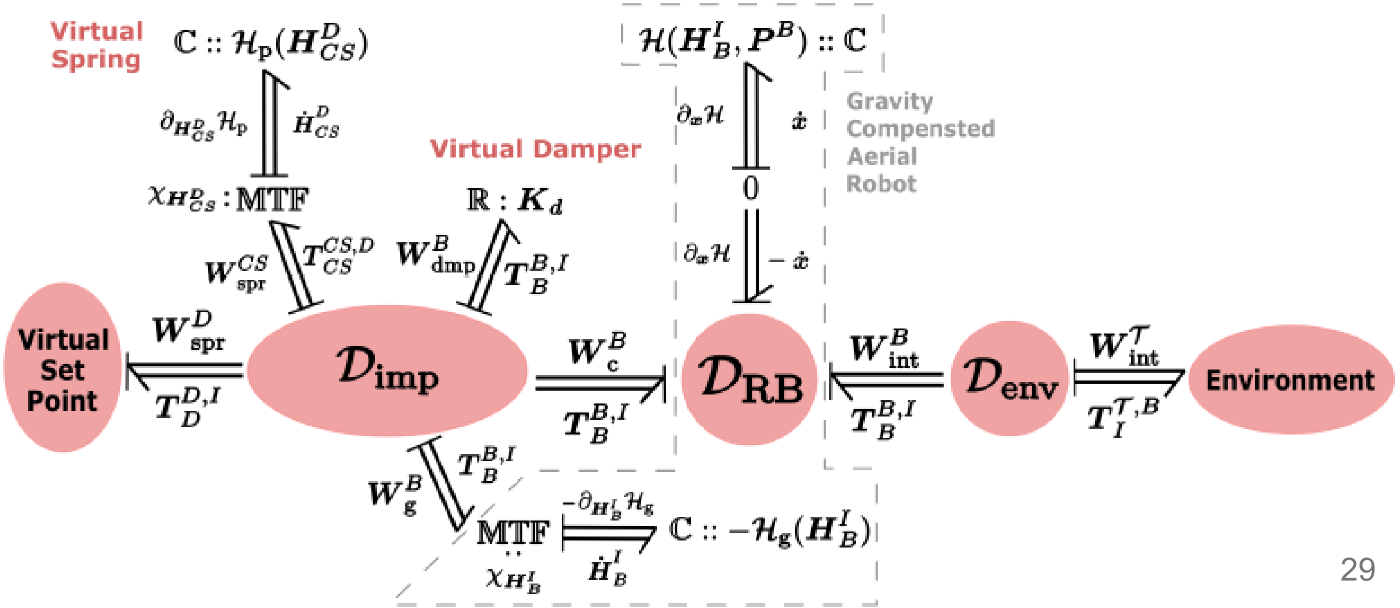
Passivity of closed-loop system

- Passivity for Interactive Robots implies **guaranteed contact stability** with ANY passive environment
- Passivity analysis is **easily performed** by combining the energy balances of the Dirac structures !!

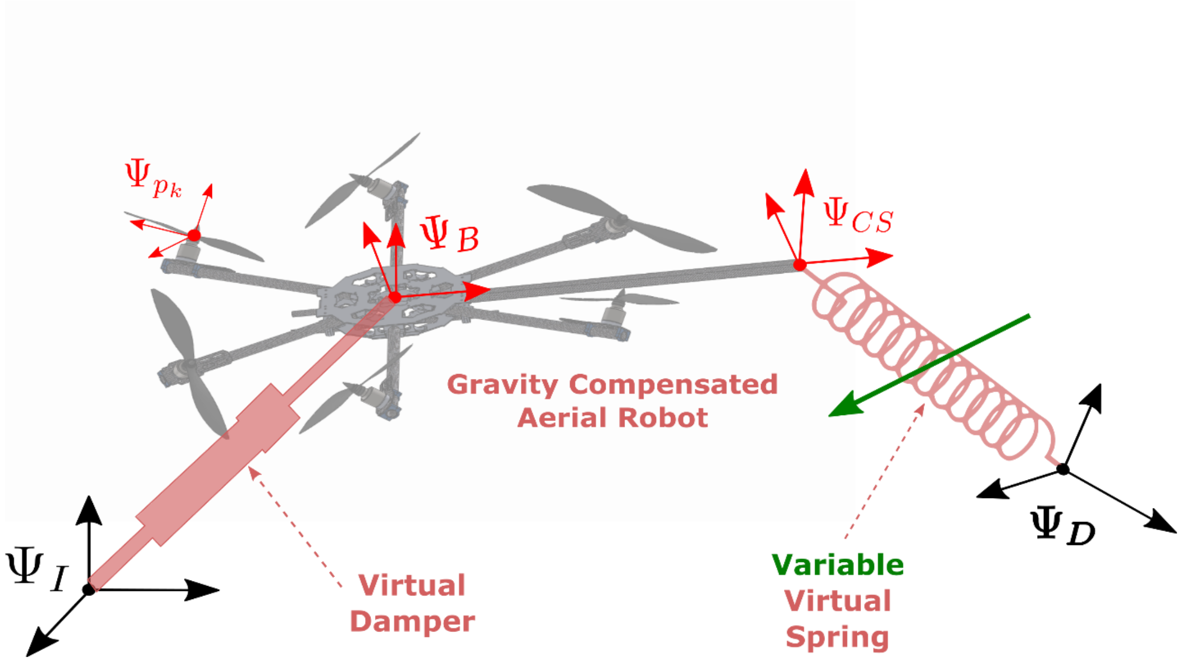
$$\dot{\mathcal{H}}_{\mathrm{g}} + \dot{\mathcal{H}}_{\mathrm{k}} = -\dot{\mathcal{H}}_{\mathrm{p}} + \dot{\mathcal{H}}_{\mathrm{g}} - (\boldsymbol{T}_B^{B,I})^\top \boldsymbol{K}_d \boldsymbol{T}_B^{B,I} - (\boldsymbol{W}_{\mathrm{spr}}^D)^\top \boldsymbol{T}_D^{D,I} + (\boldsymbol{W}_{\mathrm{int}}^B)^\top \boldsymbol{T}_B^{B,I},$$

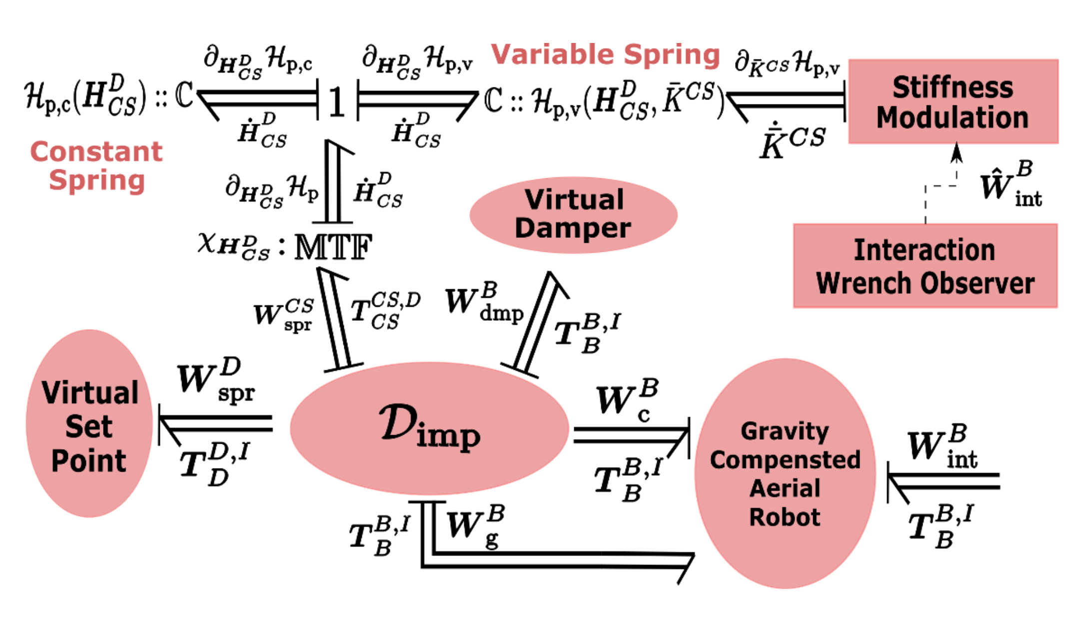
- Under some assumptions:

$$\dot{\mathcal{H}}_{\mathrm{k}} + \dot{\mathcal{H}}_{\mathrm{p}} \leq (\boldsymbol{W}_{\mathrm{int}}^B)^\top \boldsymbol{T}_B^{B,I},$$



Variable Impedance Control On SE(3)





Variable impedance control law

$$\boldsymbol{W}^B_{\mathrm{c}} = -\boldsymbol{Ad}^{\top}_{\boldsymbol{H}^{CS}_B}\left(\boldsymbol{W}^{CS}_{\mathrm{spr,c}} + \boldsymbol{W}^{CS}_{\mathrm{spr,v}}\right) - \boldsymbol{W}^B_{\mathrm{dmp}} - \boldsymbol{W}^B_{\mathrm{g}}$$

Used for regulating the interaction wrench

Drawbacks

- Non-passive controller
- Contact-stability not guaranteed
- Not safe for generic interaction

Variable Stiffness Impedance Controller

Variable impedance control law

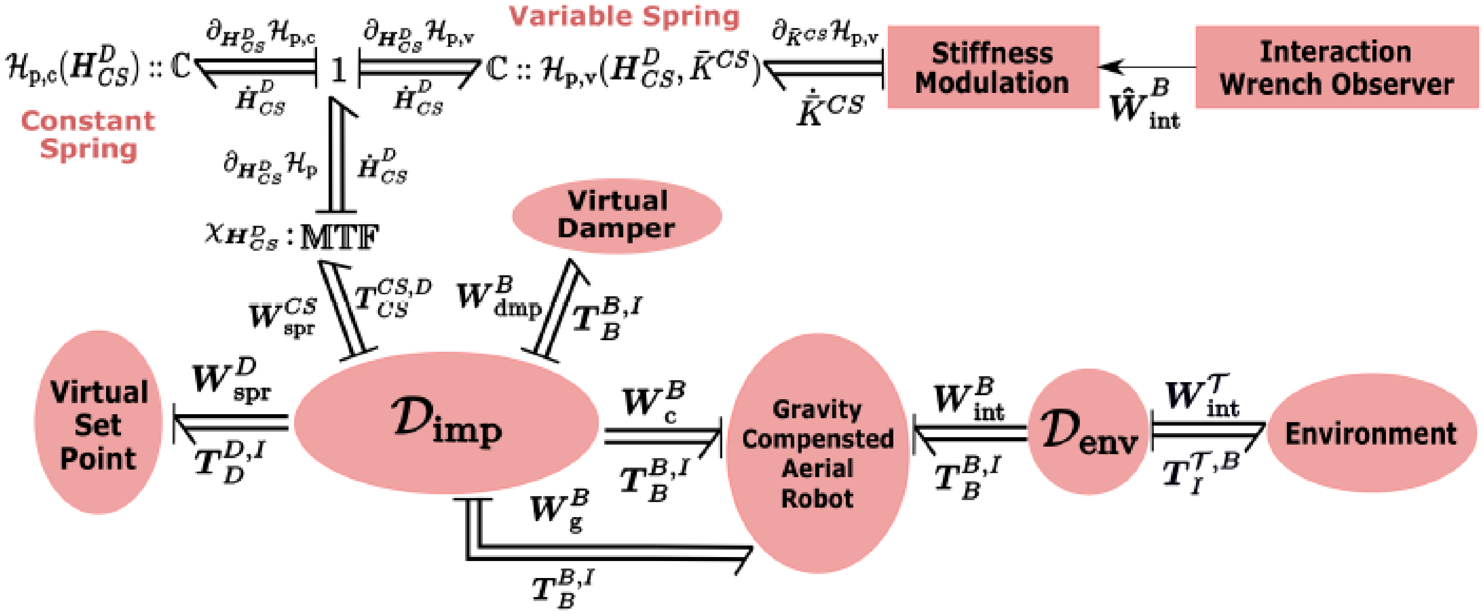
$$\boxed{\boldsymbol{W}_{\mathrm{c}}^{B} = -\boldsymbol{Ad}^{\top}_{\boldsymbol{H}_{B}^{CS}}\left(\boldsymbol{W}^{CS}_{\mathrm{spr,c}} + \boldsymbol{W}^{CS}_{\mathrm{spr,v}}\right) - \boldsymbol{W}^{B}_{\mathrm{dmp}} - \boldsymbol{W}^{B}_{\mathrm{g}}}$$

$$\dot{\mathcal{H}}_{\mathrm{p,v}} = \left\langle \boldsymbol{W}^{CS}_{\mathrm{spr,v}} \middle| \boldsymbol{T}^{CS,D}_{CS} \right\rangle_{\mathbb{R}^6} + \left\langle \partial_{\bar{\boldsymbol{K}}^{CS}} \mathcal{H}_{\mathrm{p,v}} \middle| \dot{\bar{\boldsymbol{K}}}^{CS} \right\rangle_{\mathbb{R}^{6\times 6}},$$

$$\bar{K}^{CS} = Ad^{\top}_{\boldsymbol{H}^{\mathcal{T}}_{CS}} \bar{K}^{\mathcal{T}} Ad_{\boldsymbol{H}^{\mathcal{T}}_{CS}}.$$

$$\dot{\bar{K}}^{\mathcal{T}} = K_{\mathrm{p,w}}(W^{\mathcal{T}}_{\mathrm{des}} - Ad^{\top}_{\boldsymbol{H}^{B}_{\mathcal{T}}} \hat{W}^{B}_{\mathrm{int}}),$$

Problem → Passivity is lost !

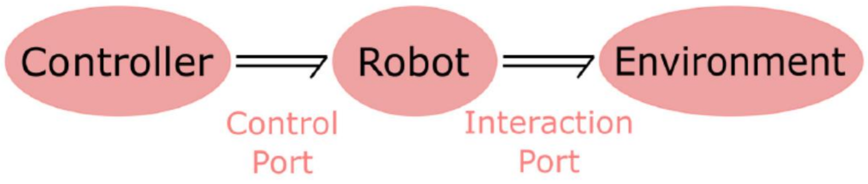


Outline

- Recap last lecture
- Importance of Passivity
- Energy Aware Control
- Energy Aware Impedance Control

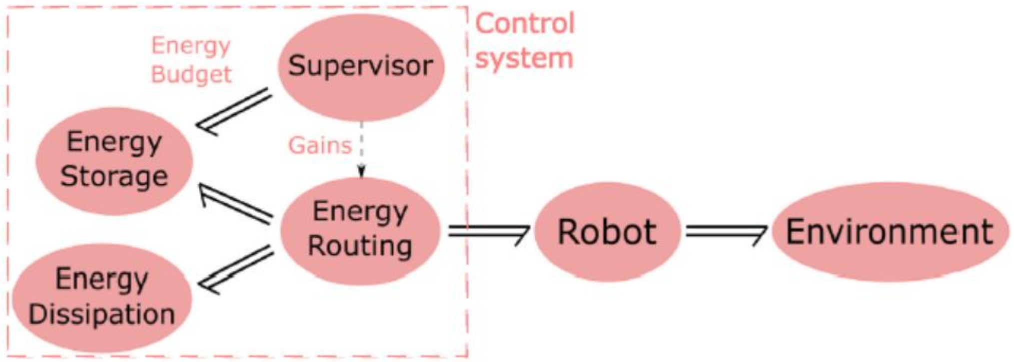
Energy-Aware Control

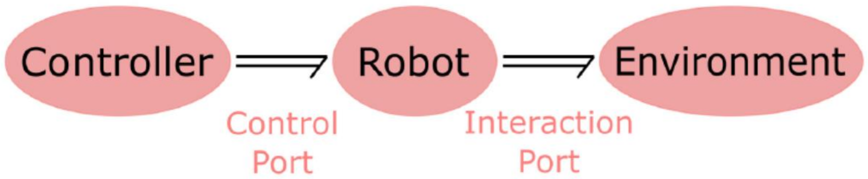
- Designing control systems with passivity as the sole goal is often considered over-conservative and non optimal in terms of performance.
- Some tasks (e.g., periodic locomotion) require a continuous injection of energy in the system to be executed, and cannot be achieved with a pure passive design.



Energy-Aware Control

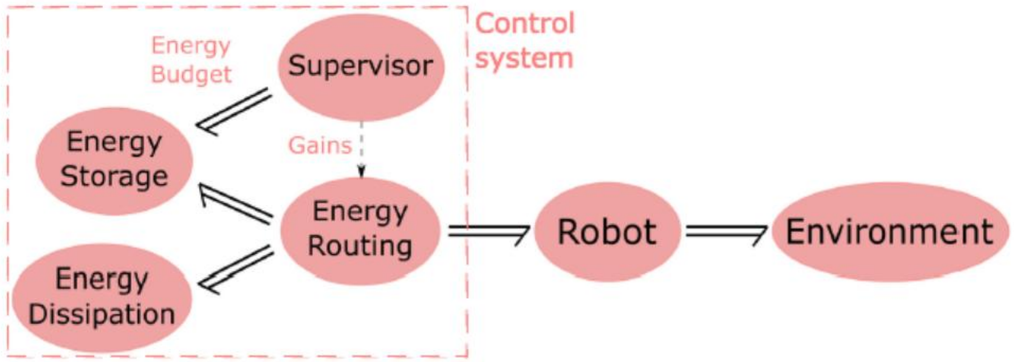
- The *Energy-Aware Control* framework extends the standard passivity-based framework with a set of modules such that the goal is not primarily achieving passivity but being able to take decisions observing the energy flows happening insight the whole control network.

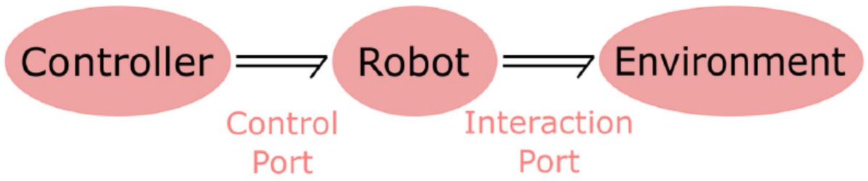




Energy-Aware Control

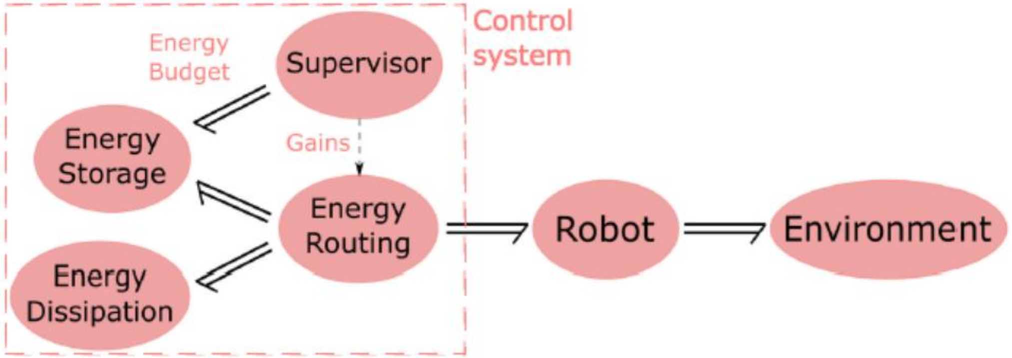
- A fundamental constituent of the energy-aware framework is *energy routing*.
- This technique introduces the possibility to design nontrivial power-preserving interconnections between two or more systems such that the energy could be transferred from one system to another without violating passivity.

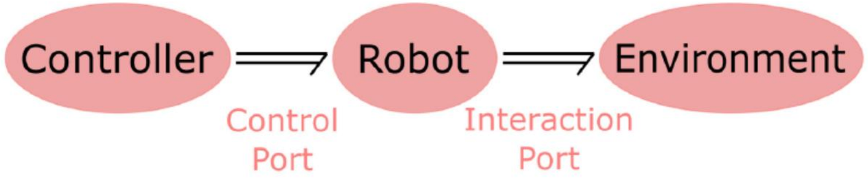




Energy-Aware Control

- Using *energy tanks*, it is possible to passively implement any desired control action according to the available energy budget.
- The energy budget is provided by a high-level control module referred to as the *supervisor*.
- This supervisor module is interconnected to the control system and can provide to it new energy if needed.





Outline

- Recap last lecture
- Importance of Passivity
- Energy Aware Control
- Energy Aware Impedance Control

PROPOSED APPROACH

- A way to **restore the passivity**, violated because of regulating the interaction wrench, is by using **energy routing** and **energy tanks**.
- The basic idea behind energy routing is that energy within the controller is **directed** from certain parts and **stored** in a **virtual tank**.
- This stored energy can then be used to implement **the non-passive control action** of wrench regulation.
- In this way, the **total energy** content of the closed-loop system remains **unaffected**.

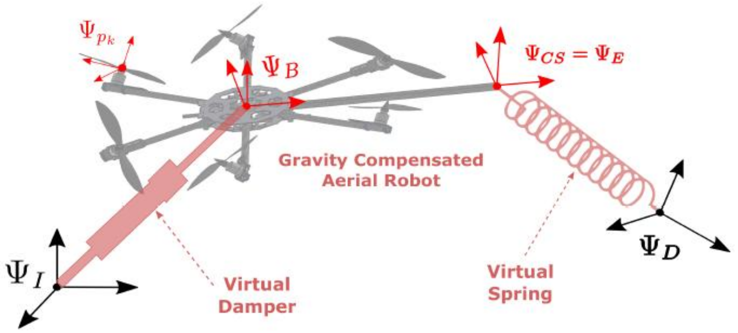
ENERGY ROUTING AND ENERGY TANK

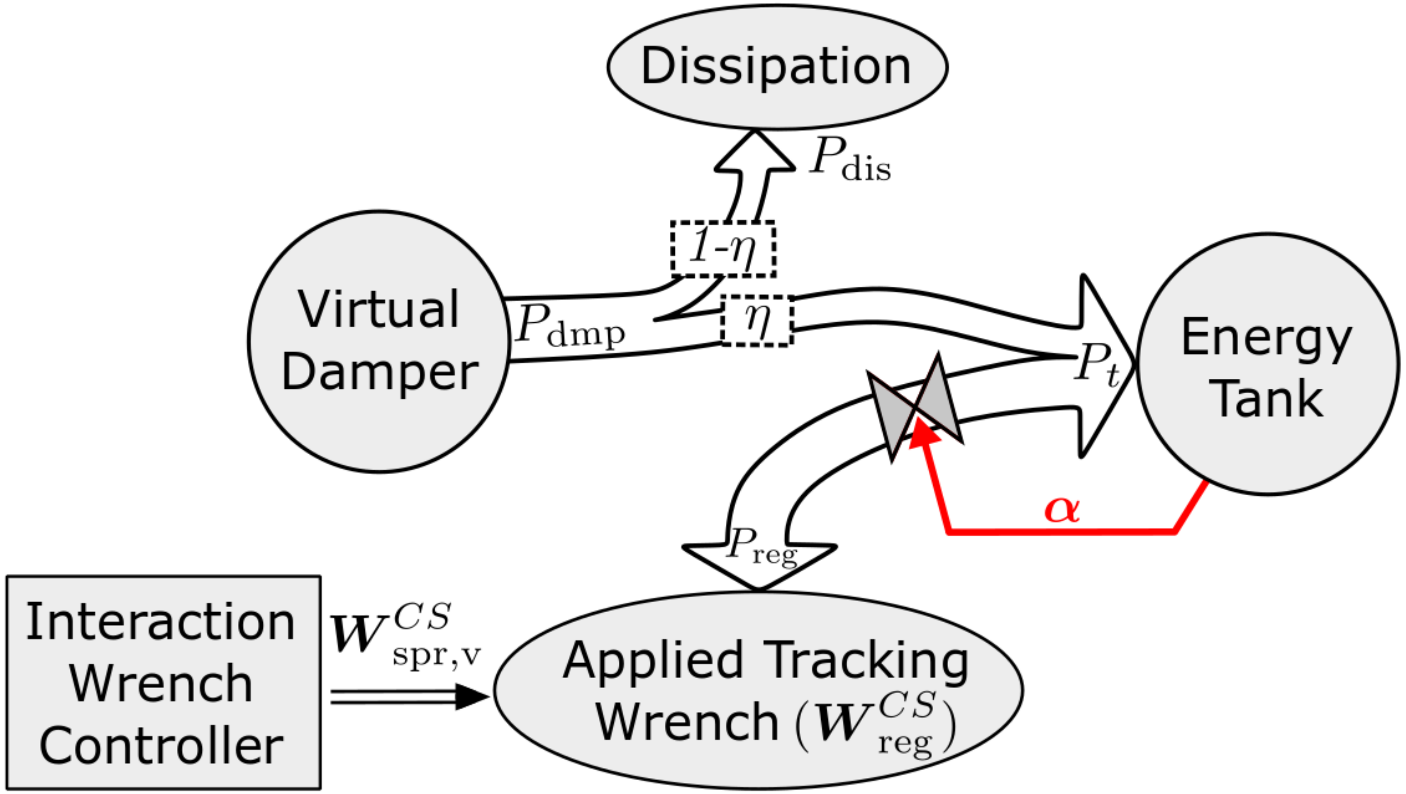
$$\boldsymbol{W}_{\mathrm{c}}^{B} = -\boldsymbol{Ad}_{\boldsymbol{H}_{B}^{CS}}^{\top} \boldsymbol{W}_{\mathrm{spr}}^{CS} - \boldsymbol{W}_{\mathrm{g}}^{B} - \boldsymbol{W}_{\mathrm{dmp}}^{B},$$

$$\boldsymbol{W}_{\mathrm{spr}}^{CS} = \boldsymbol{W}_{\mathrm{spr,c}}^{CS} + \boldsymbol{W}_{\mathrm{reg}}^{CS},$$

$$\boldsymbol{W}_{\mathrm{reg}}^{CS} = \alpha \boldsymbol{W}_{\mathrm{spr,v}}^{CS},$$

- **Enough Energy in Tank → Alpha =1**
- **Energy Budget has to be allocated**





ENERGY ROUTING AND ENERGY TANK

$$W_{\mathrm{spr}}^{CS} = W_{\mathrm{spr,c}}^{CS} + W_{\mathrm{reg}}^{CS},$$

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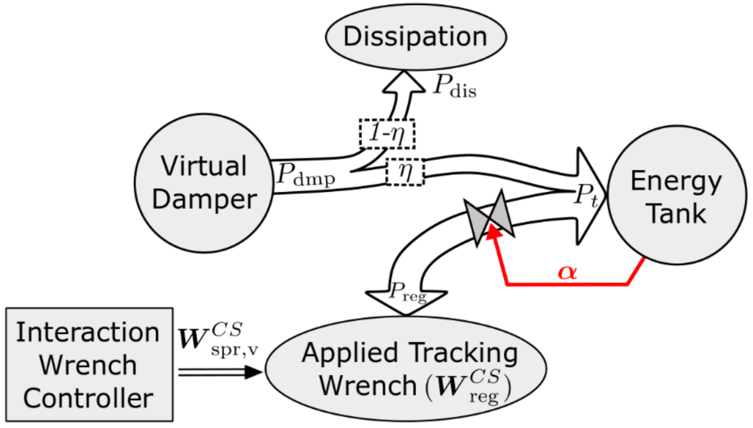
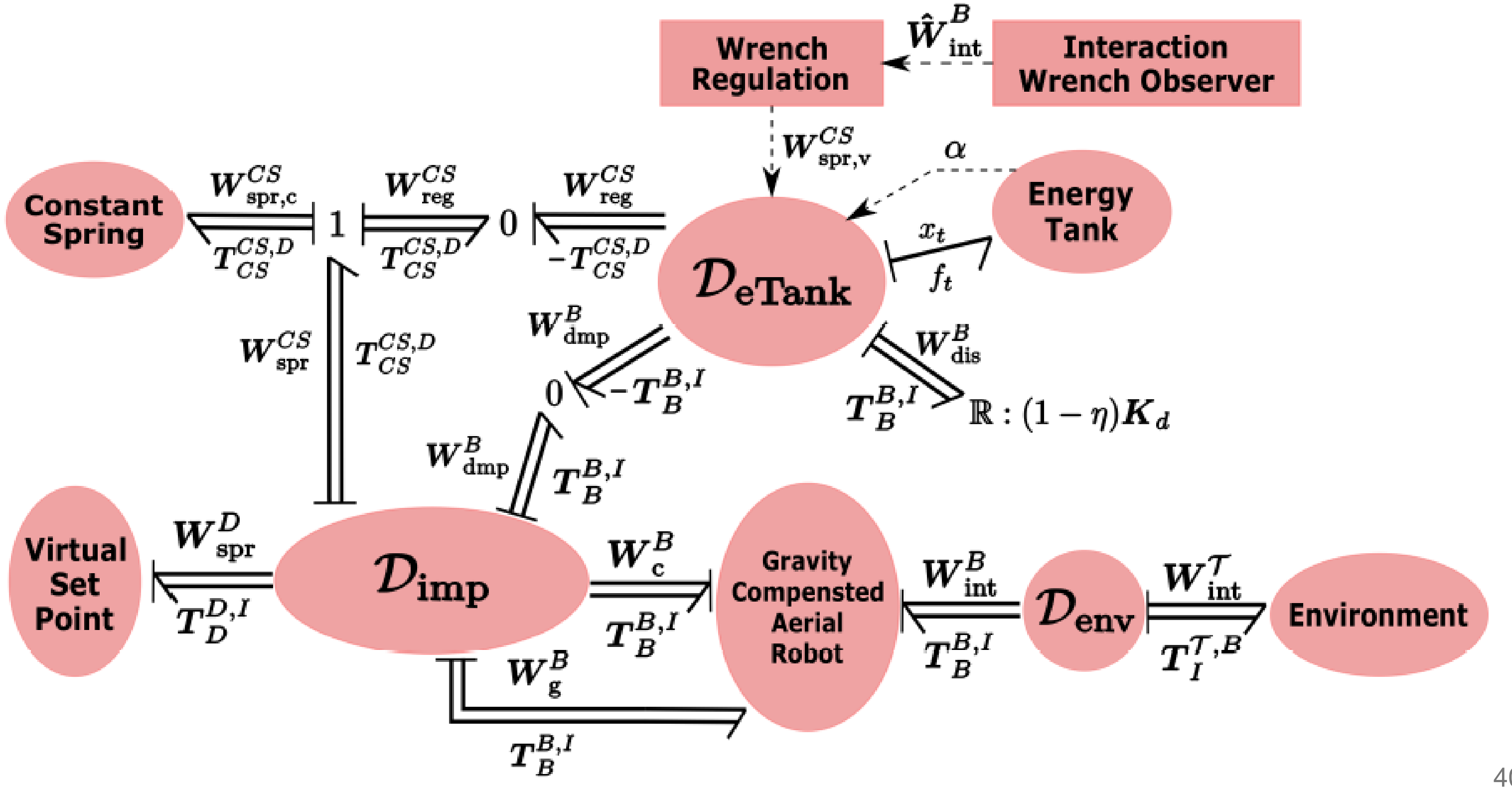
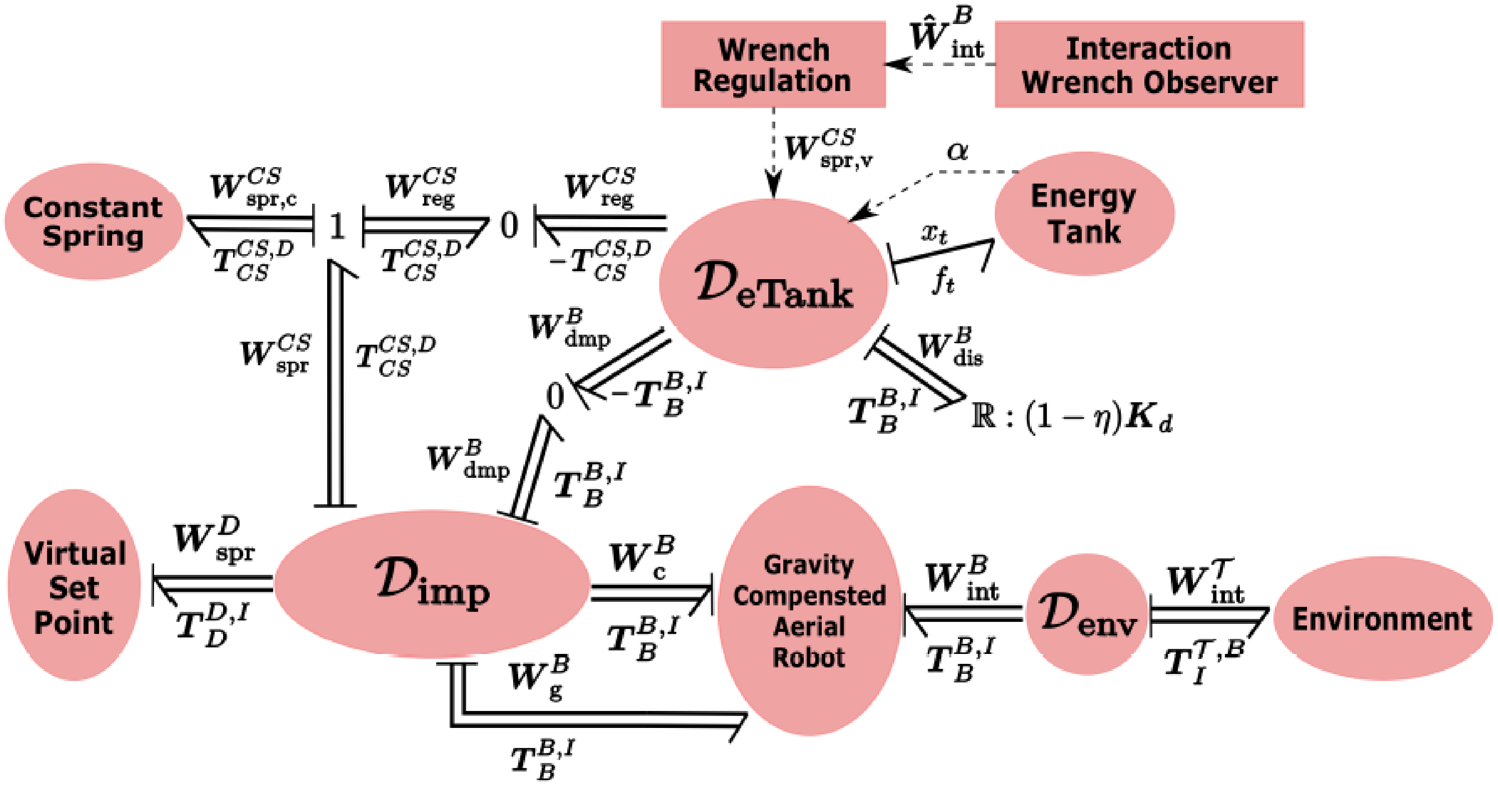
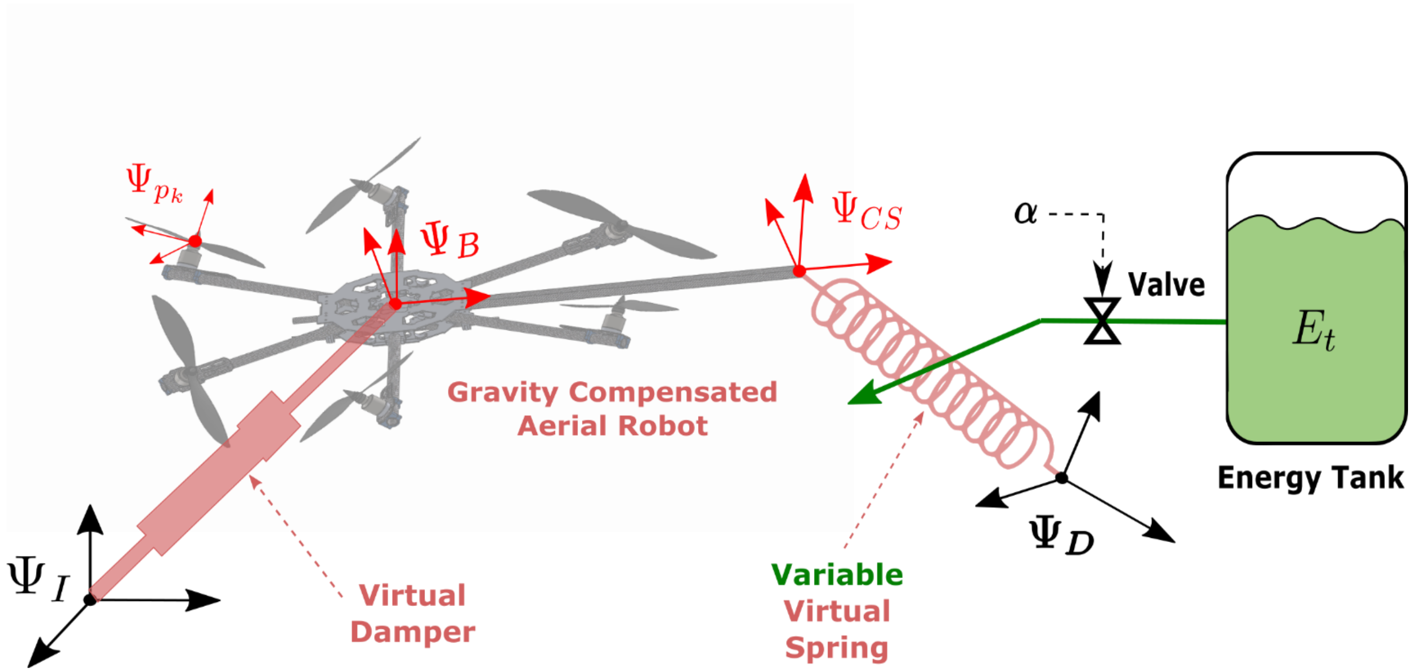


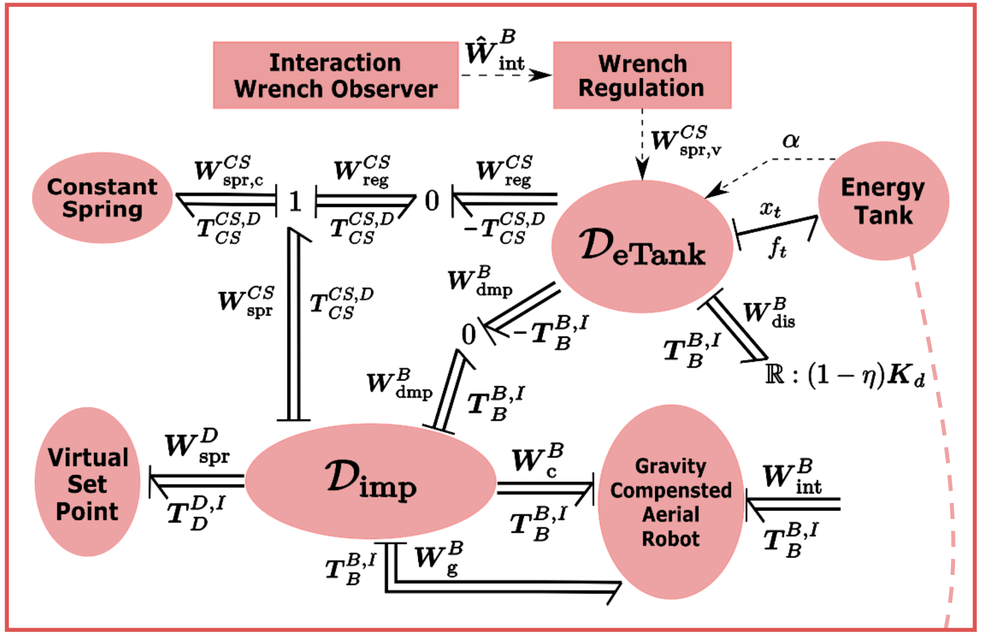


PASSIVITY ANALYSIS
GRAPHICAL STRAIGHTFORWARD – ANALYTICAL NOT NEEDED !



ENERGY AWARE IMPEDANCE CONTROL

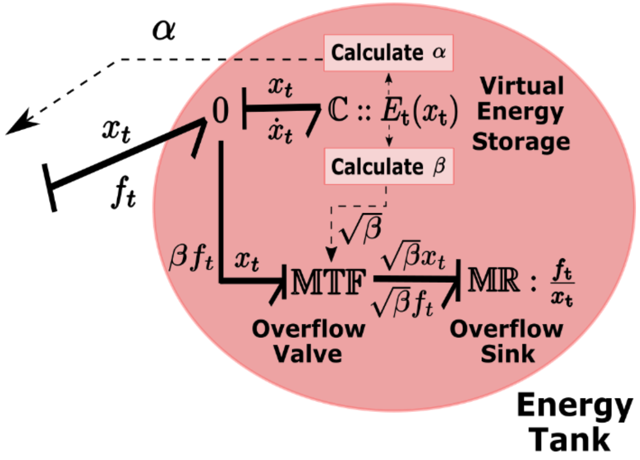




Energy-aware impedance control law

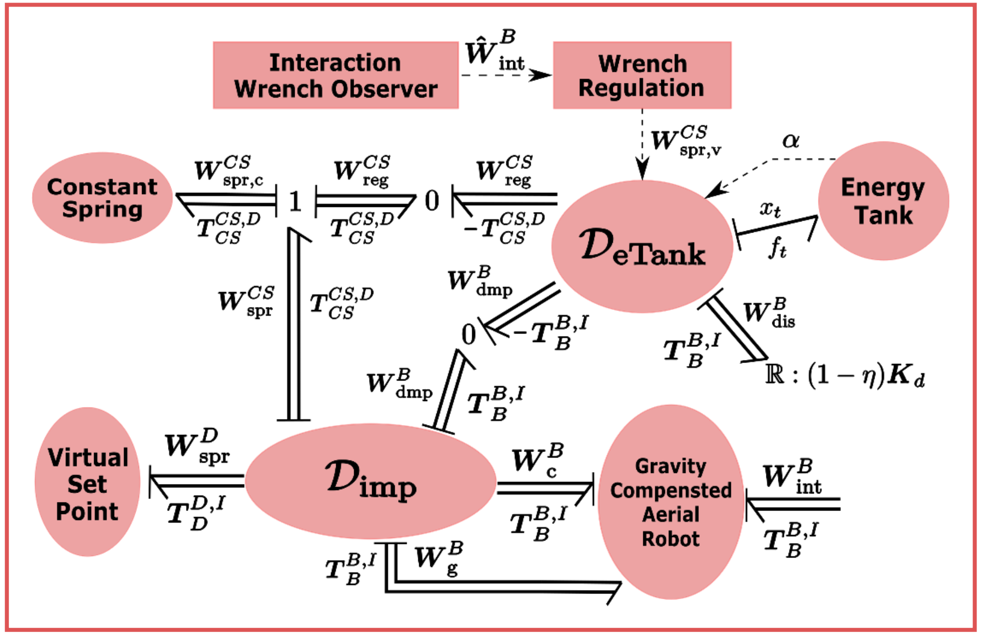
$$\boxed{\boldsymbol{W}_{\mathrm{c}}^{B} = -\boldsymbol{Ad}_{\boldsymbol{H}_{B}^{CS}}^{\top}\left(\boldsymbol{W}_{\mathrm{spr,c}}^{CS} + \alpha \boldsymbol{W}_{\mathrm{spr,v}}^{CS}\right) - \boldsymbol{W}_{\mathrm{g}}^{B} - \boldsymbol{W}_{\mathrm{dmp}}^{B}}$$

Valve gain regulated by energy tank



FEATURES OF ENERGY-AWARE IMPEDANCE CONTROL

- ✓ Interaction wrench regulation
- ✓ Guaranteed contact stability with unknown passive environment
- ✓ SE(3) Geometric consistency
- ✓ High modularity
- ✓ Graphical passivity analysis using bond graphs
- ✓ Embed high-level objectives such as contact-loss stabilization
- ✓ Safe for generic interaction



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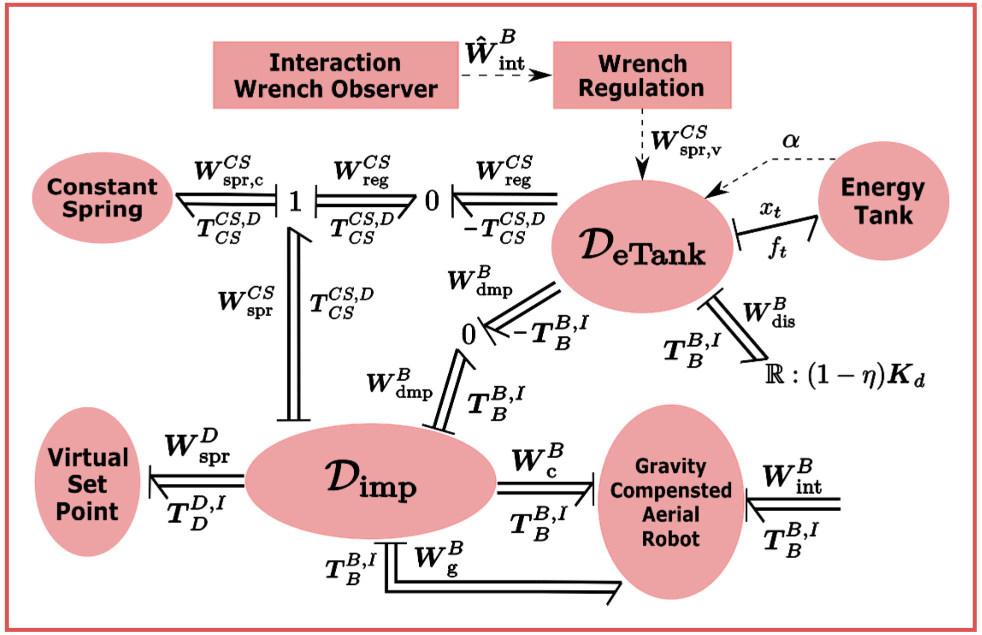
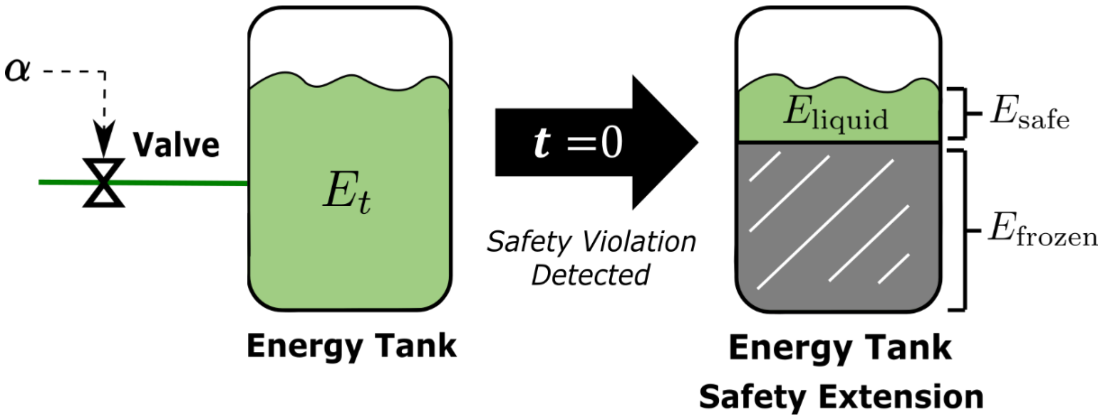
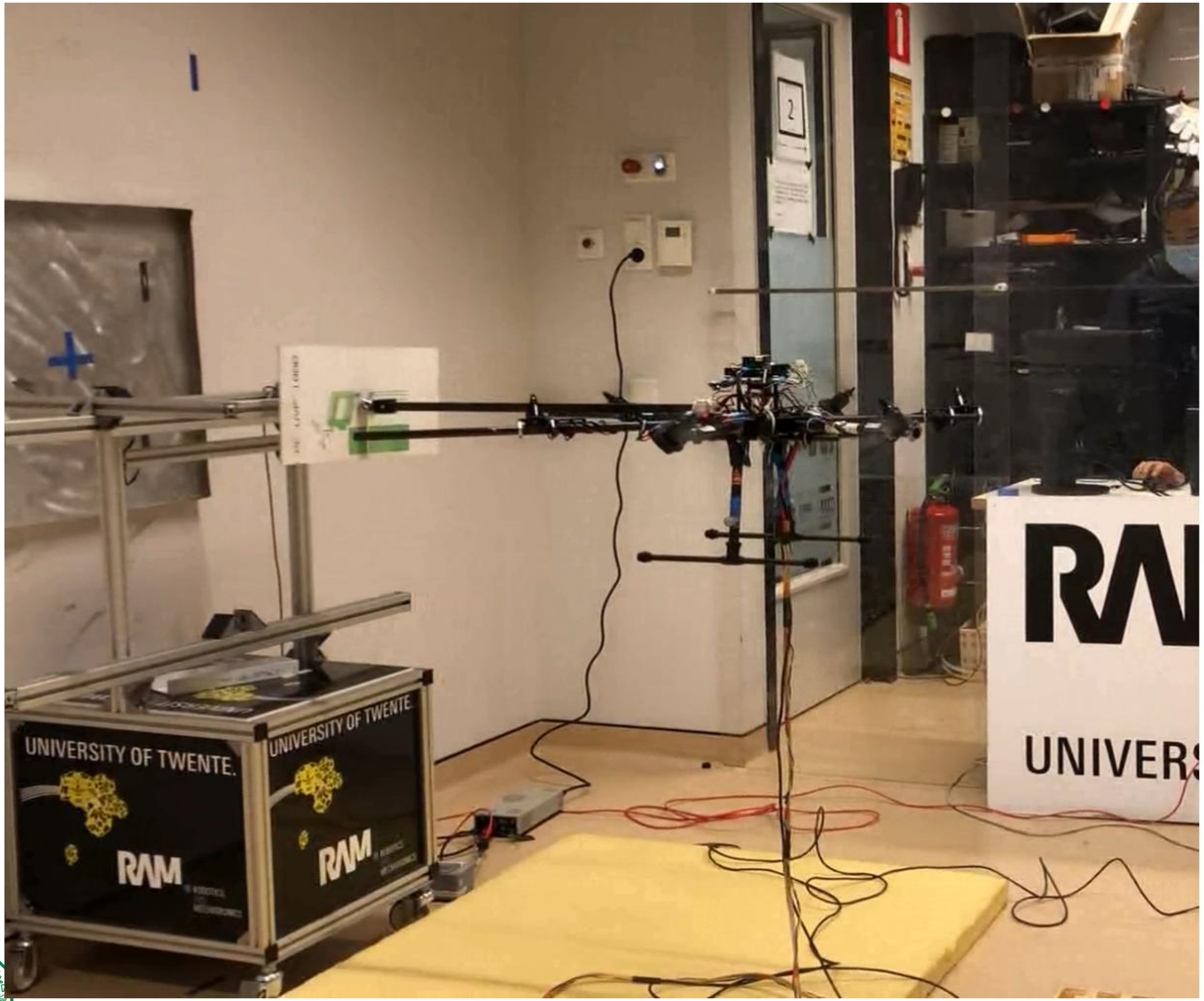
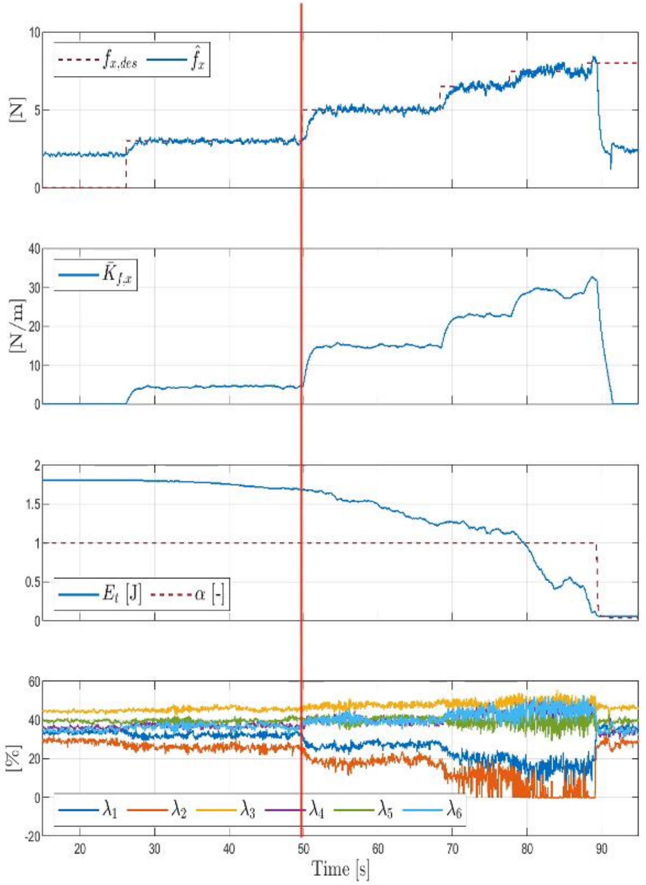


Energy freezing & melting algorithm

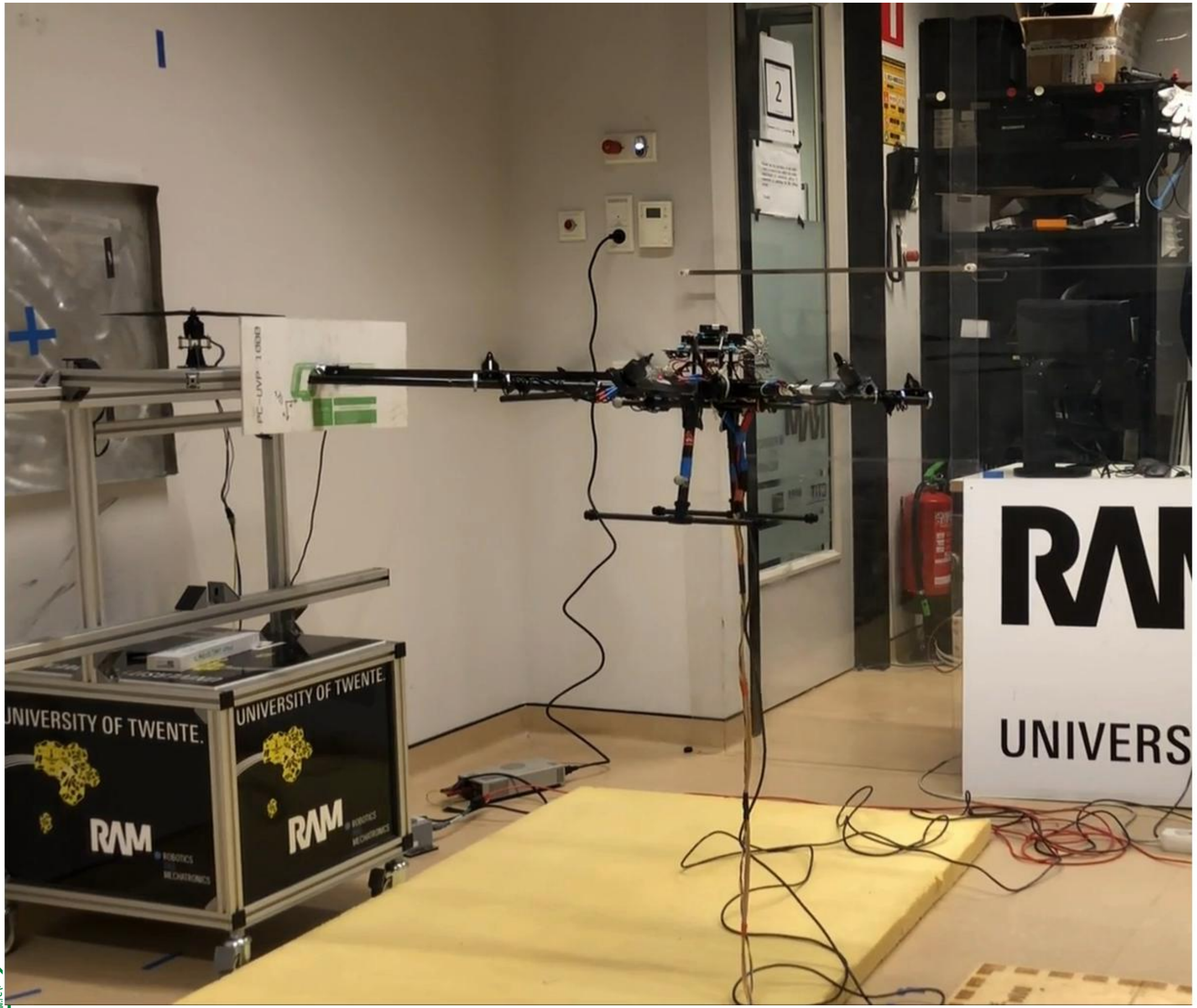


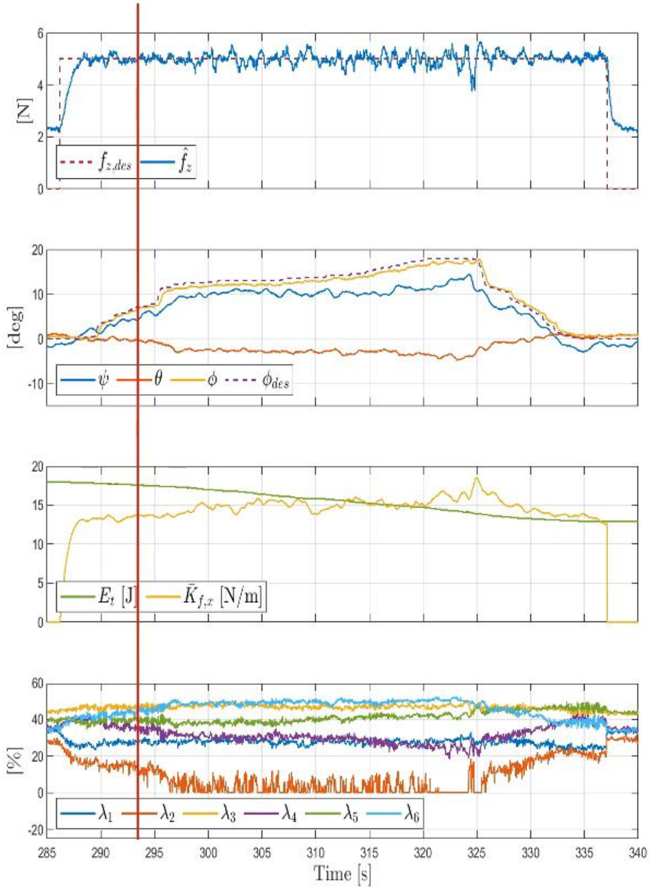
EXP.1: WRENCH REGULATION AND TANK DEPLETION



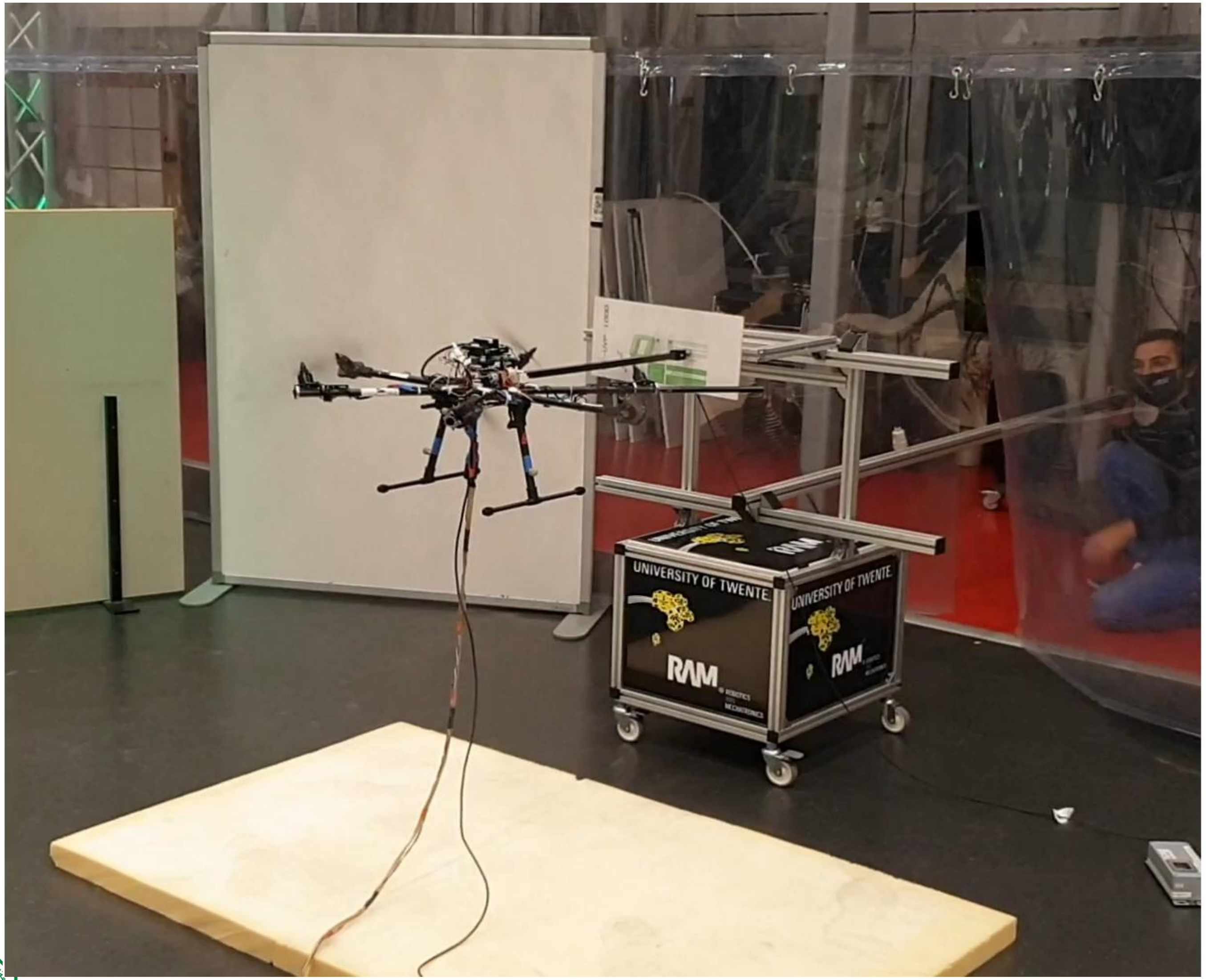


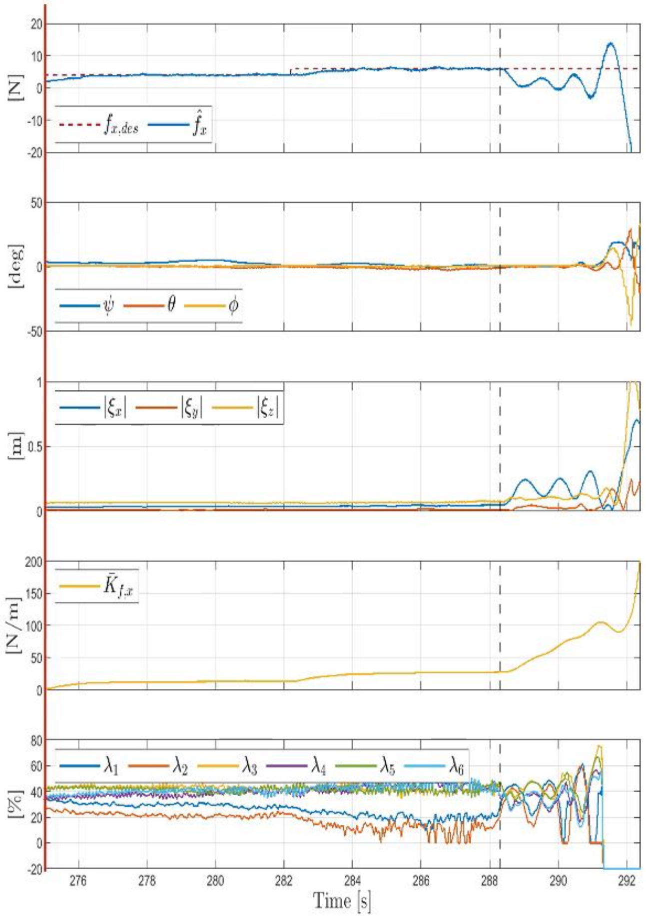
EXP.3: FULL ACTUATION CAPABILITIES





EXP.4: IMPORTANCE OF ENERGY AWARENESS





EXP.5: ENERGY-AWARE CONTACT-LOSS STABILIZATION