CIE482: Path Planning & Navigation for Mobile Robots

Assignment 3

Instructions:

- This is a **graded** assignment of 6%.
- No typesetting is allowed, you must write your solutions.
- You can write on the pdf directly with a tablet & electronic pen **or** print the pdf and answer with an ink-pen.
- Good handwriting and neatness makes your work easier to read and understand and is highly recommended.
- Upload your solution as a **single pdf file** on Blackboard by **11:59pm Monday 6 April 2026 (before midnight).**
- Plagiarism will **not tolerated** at all.

Student Name:	
Student ID:	

Question 1:
Expand the expression of a Multivariable Gaussian function

$$p(x) = \frac{1}{(2\pi)^{k/2}\sqrt{\det(\Sigma)}} \exp\left(-\frac{1}{2}(x-\mu)^{\top}\Sigma^{-1}(x-\mu)\right)$$

with diagonal covariance matrix and zero mean variables for the case $x = (x_1, x_2) \in \mathbb{R}^2$. Denote the Covariance matrix by $\Sigma = \begin{pmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{pmatrix}$.

Question 2:

Let $p_1(x) = N(0, \Sigma_1)$ and $p_2(x) = N(0, \Sigma_2)$ be two Gaussian distributions in 2D with diagonal covariance matrices, i.e., $\Sigma_i = \begin{pmatrix} \sigma_{ix}^2 & 0 \\ 0 & \sigma_{iy}^2 \end{pmatrix}$ for $i \in \{1,2\}$.

Show that the product $p_3(x) := \; p_1(x) * p_2(x)$ is another Gaussian distribution? What is the mean and covariance matrix of $p_3(x)$?

Question 3:

For the following multiple-choice questions, choose **one** possible **correct answer** from the options provided.

1. In robot localization, belief about the robot's state is represented as:
A. A deterministic vector
B. A conditional probability distribution
C. A control input
D. A sensor hardware model

2. In a Gaussian distribution, the parameter σ represents:
A. The mean
B. The covariance
C. The standard deviation
D. The marginal probability

3. If $cov(x, y) > 0$, this means that:
A. The two variables vary in opposite directions
B. The two variables vary in the same direction
C. The variables are independent
D. The variables have zero variance

4. If two random variables X and Y are independent, then:
A. $p(x \mid y) = p(y \mid x)$
B. $p(x, y) = p(x) + p(y)$
C. $p(x \mid y) = p(x)$
D. $p(x, y) = 1$

Question 4:
In your own words, provide an explanation for the following questions:

1. **Explain why uncertainty in sensors and actuators makes robot localization a challenging problem.**
2. **Define the Gaussian distribution and explain the roles of the mean and standard deviation in shaping its graph.**
3. **What is covariance? Explain the meaning of positive covariance, negative covariance, and zero covariance.**
4. **Explain the meaning of independence between two random variables. How does independence simplify the expression for conditional probability?**