CIE 482: Path Planning & Navigation for Mobile Robots

Topic 4: Localization

Lecture 13: Overview of Probabilistic Map-based Localization

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Outline

- Localization & its challenges
- Belief representation
- Basics of probability theory

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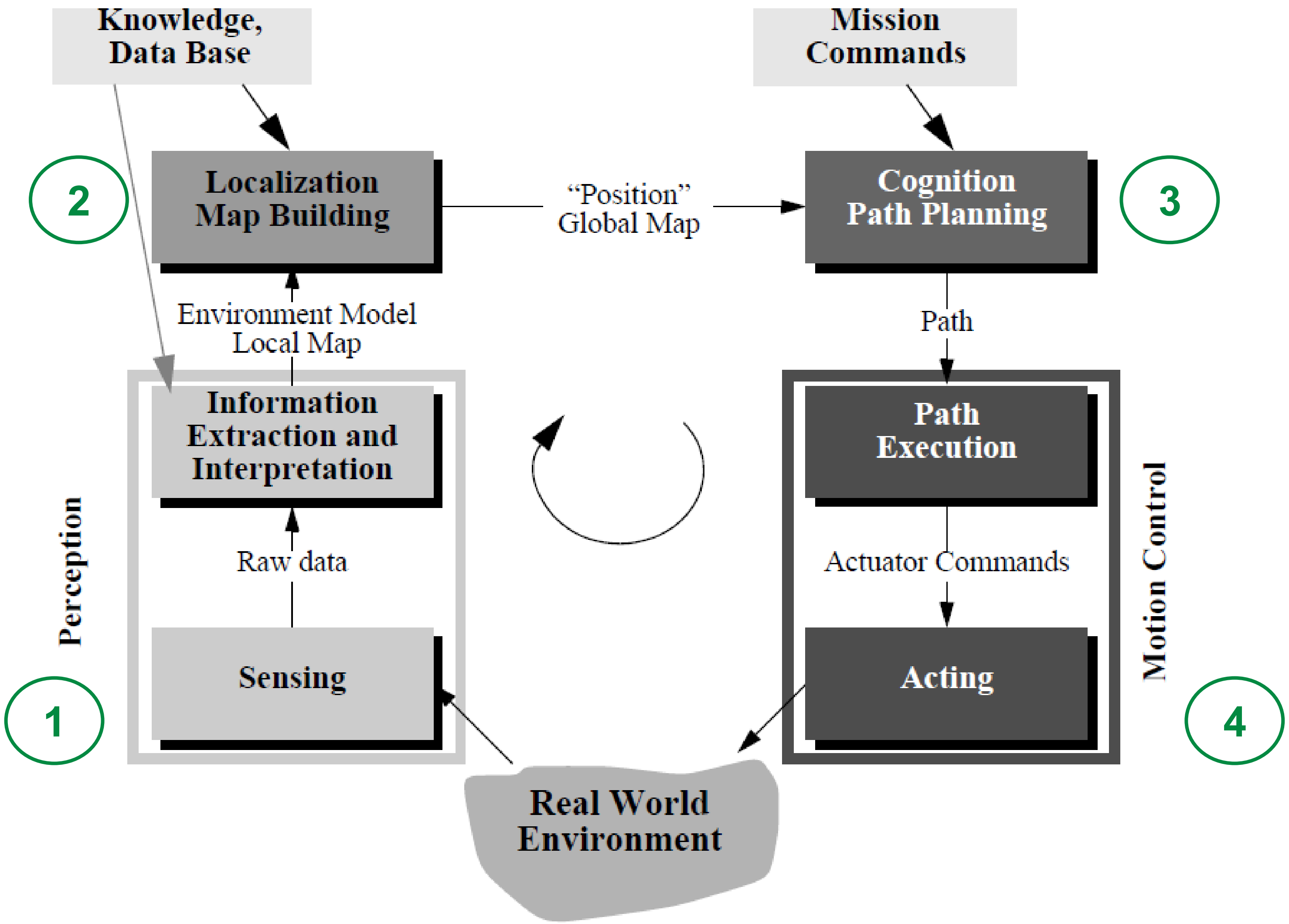
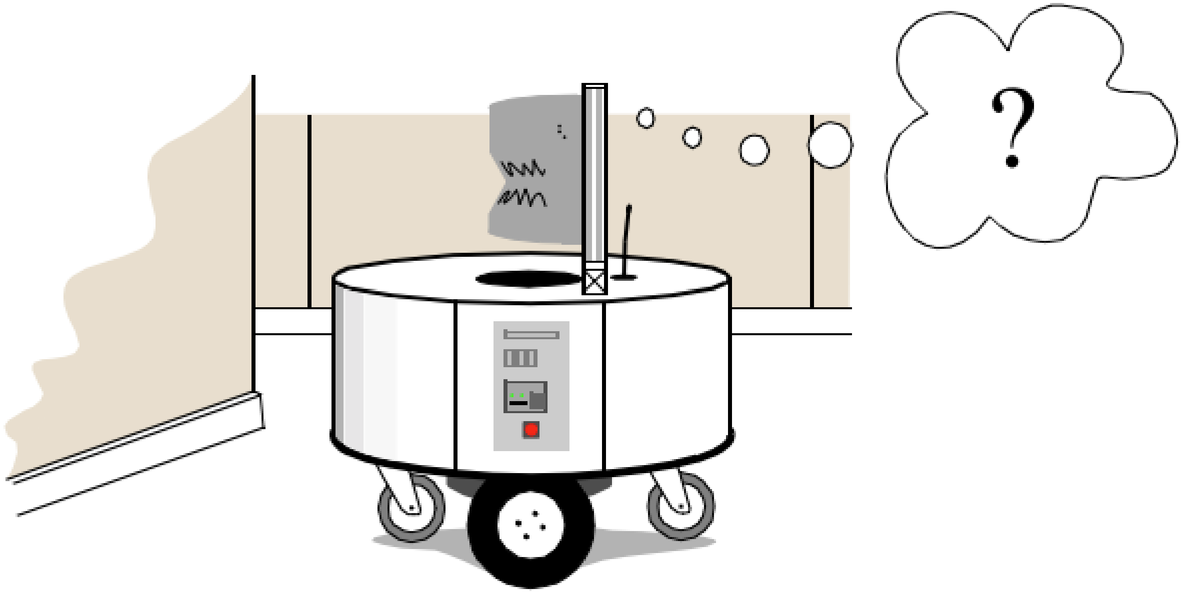
Recall: Robot Navigation Architecture



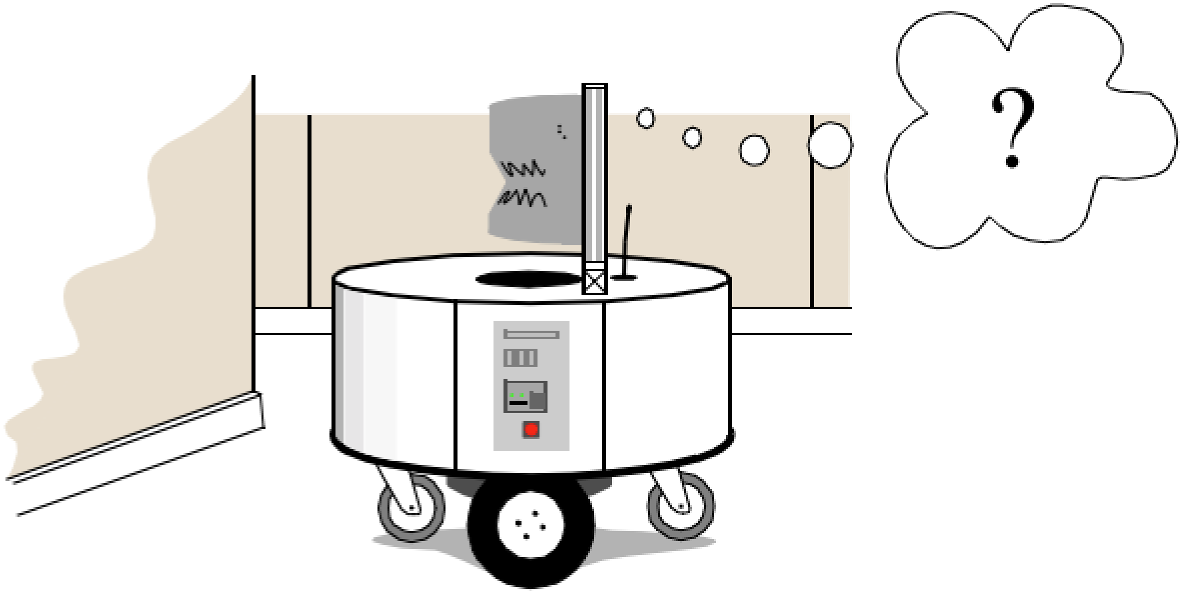
Robot Localization

- Out of these four components of the robot navigation architecture, localization has received the greatest research attention due to its complexity.
- Robot localization focuses on the question:
    - “Where am I ?”



Robot Localization

- Localization is more than knowing the robot's absolute position in Earth's reference frame (using a GPS for instance)
- In general, localization involves:
  1. Identifying the robot's pose relative to a map, which is a model of the (unknown) environment.
  2. Identifying the robot's pose relative to targets, either static or dynamic (for example human targets).



Challenges of localization

- The robot’s sensors and effectors (actuators) play an integral role in any robot localization framework.
- It is because of the inaccuracy and incompleteness of these sensors and effectors that localization poses difficult challenges.
- Challenges:
  - Sensor uncertainty
  - Actuator uncertainty

Challenges of localization - Sensor uncertainty

- Sensor noise induces a limitation on the consistency of sensor readings in the same environmental state. For example:
  - **Magnetometer**
    - Noisy or inconsistent heading readings in the presence of steel beams or other ferromagnetic objects.
  - **GPS**
    - Inconsistent position estimates when operating near tall buildings or under dense tree canopies.
  - **Lidar range finder**
    - In the same indoor hallway, slightly different Lidar readings may appear each time the robot passes by, because reflective or glossy surfaces can cause spurious distance measurements.
  - **Cameras**
    - slight shifts in pixel intensities caused by changes in ambient lighting or sensor thermal noise.

Challenges of localization - Actuator uncertainty

- Mobile robot actuators introduce uncertainty about future state.
- Therefore, the simple act of moving tends to increase the uncertainty of a mobile robot. For example:
  - **Differential Drive Wheel Slippage**
    - Even small differences in floor friction or wheel wear can cause one wheel to slip, which leads to inaccurate wheel odometry estimates of distance traveled and heading.
  - **Quadrotor Motor Thrust Variations**
    - If actuator responses (e.g., motor thrust output) vary because of wear, temperature changes, or imperfect control loops, the vehicle’s actual velocity and altitude deviate from predicted values using inertial odometry.

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Representations in Localization

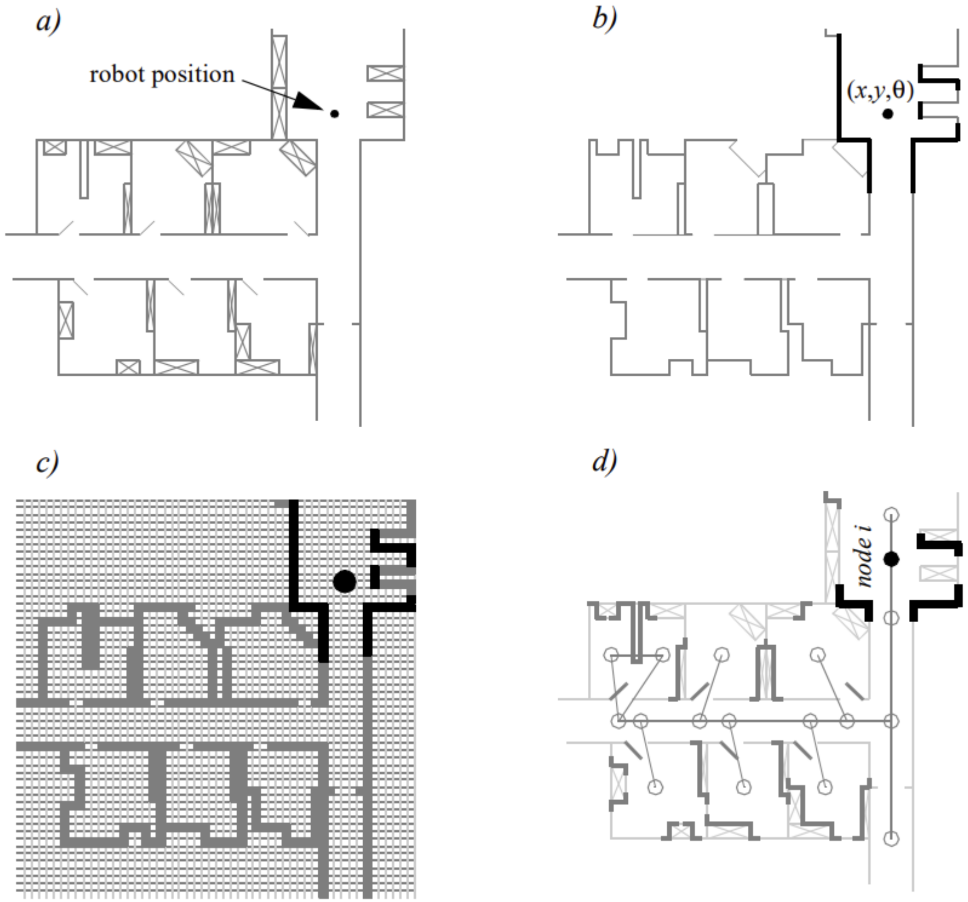
- There are two specific concepts that the robot must represent, and each has its own unique possible solutions. Different choices for representation is the main difference between different localization approaches.
  1. **Map representation:**
     - The robot must have a representation (a model) of the environment, or a map.
  2. **Belief representation:**
     - The robot must also have a representation of its belief regarding its position on the map.

Map Representation

- Given some way of representing the robot's environment, the robot's belief about position is expressed as a single unique point on the map.

real map with walls, doors and furniture

line-based map, around 100 lines



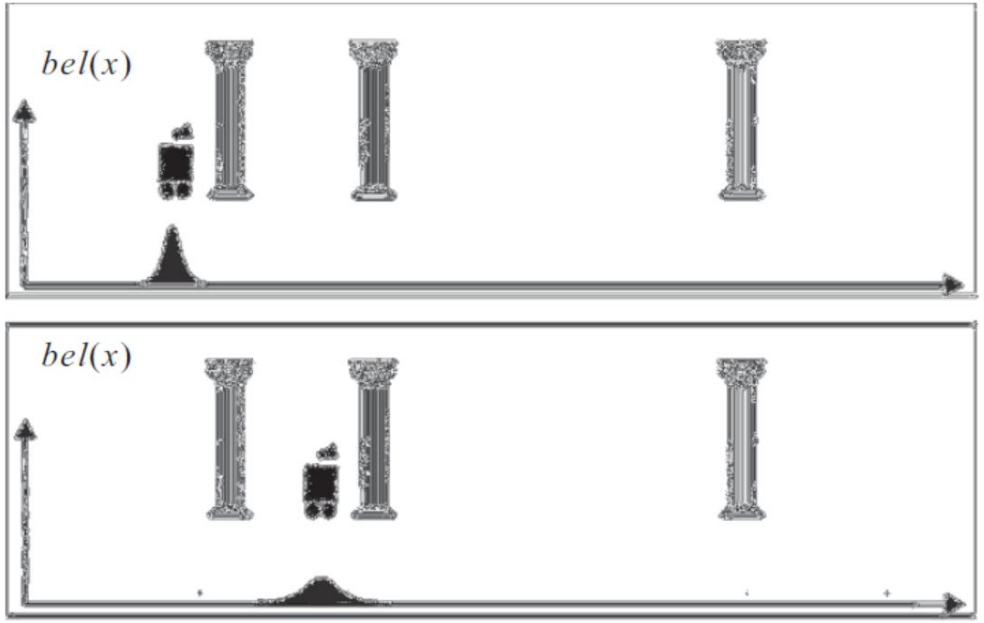
occupancy grid-based map: around 3000 grid cells

topological map using line features and doors, around 50 features

Belief Representation

- The approach we shall follow is that the robot tracks not just a single possible position but also a possibly infinite set of positions.
- In this approach, the belief is modeled by a probability distribution function $bel(x)$.
- In this approach, beliefs are represented by conditional probability distributions:

$$bel(x) = p(x|y)$$



The belief function $bel(x)$ is our best guess of the robot's state x.

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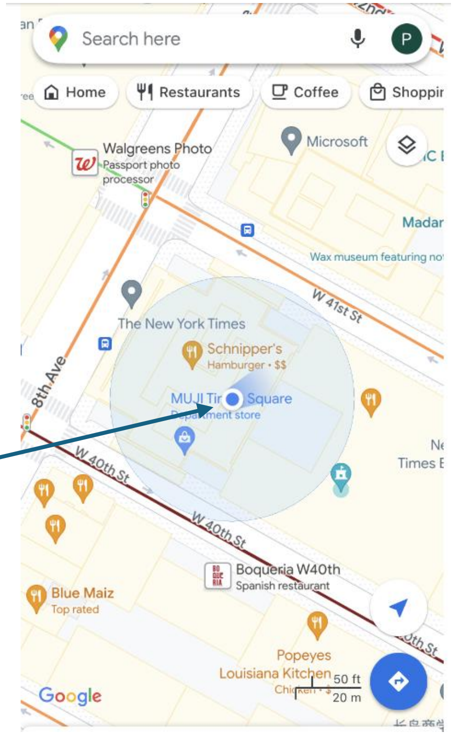

Google Maps
$bel(x)$

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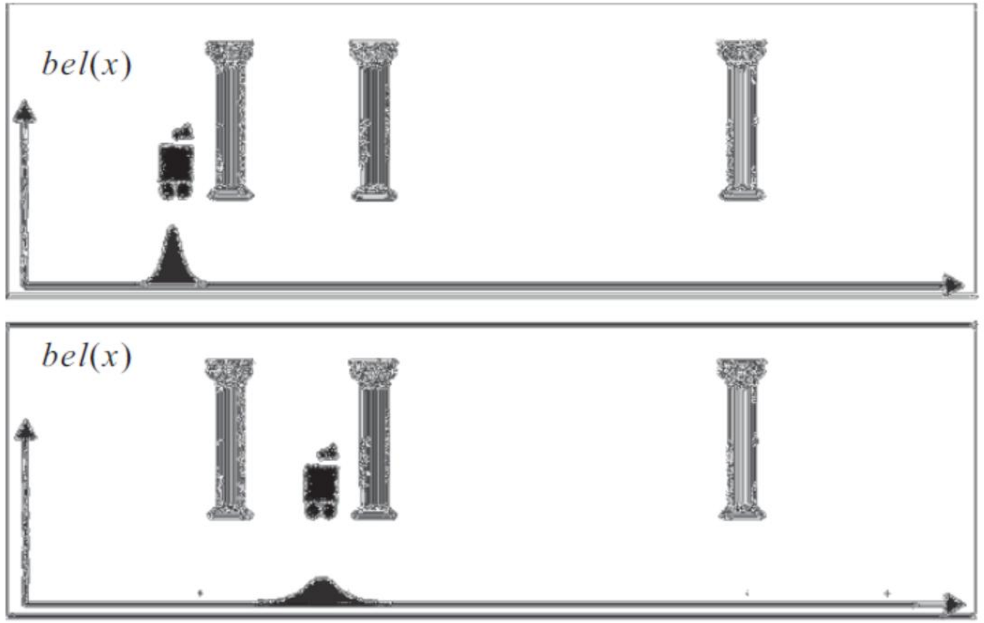
Belief Representation

- The key advantage of using probability distributions for belief representation is that the robot can explicitly maintain uncertainty regarding its position.
- In general, $bel(x)$ is unknown, but the most common (and easiest) approach is to assume it is a Gaussian distribution $\mathcal{N}(\mu, \Sigma)$.



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Random Variable

- A random variable is a function that maps outcomes of a random experiment to real numbers.

$$X: \Omega \rightarrow \mathbb{R}$$

- Ω: is called the sample/state space
- Every value in $\omega \in \Omega$, is assigned a real number $X(\omega)$

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- Example:
  - Discrete $\Omega \coloneqq \{1,2,3,4,5,6\}$
  - X : is a dice roll (top face)

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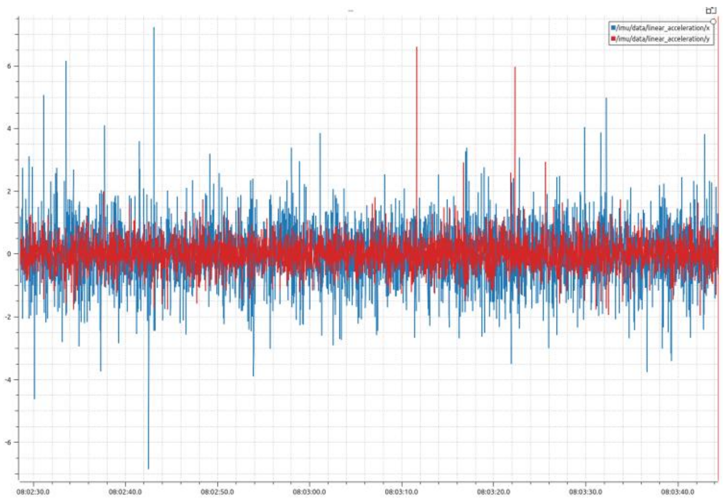
- Example:
  - Continuous $\Omega \in \mathbb{R}$
  - X : is a sensor measurement



Probability Function (Discrete)

- Probability mass function (PMF) is a function of a random variable $p(X = \omega)$ that indicates the probability of the random variable having the value $\omega \in \Omega$, where Ω is discrete.

- Example:
  - $p(X = 1) = p(X = 2) = p(X = 3)$
    $= p(X = 4) = p(X = 5) = p(X = 6) = 1/6$

$$\boxed{\sum_{\omega \in \Omega} p(X = \omega) = 1}$$

Probability Function (Continuous)

- Probability density function (PDF) is a function of a random variable $p(X = \omega)$ that indicates the probability of the random variable having the value $\omega \in \Omega,$ where Ω is continuous.

$$\boxed{\int_{-\infty}^{\infty} p(X = \omega) = 1}$$

Gaussian Probability Density Function

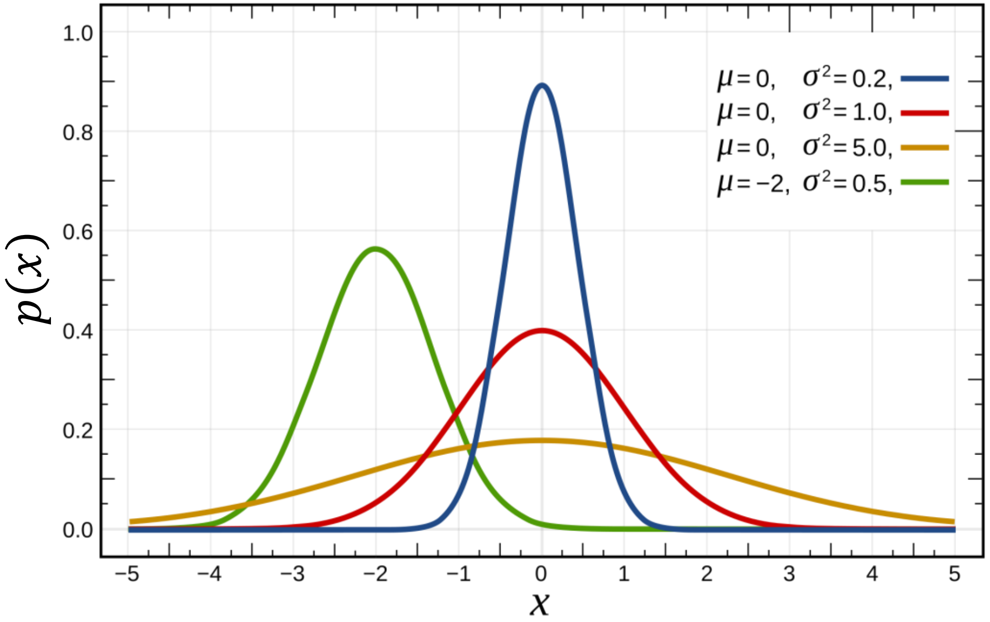
- A common probability density function is the Gaussian distribution

$$p(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(\frac{(x-\mu)^2}{2\,\sigma^2}\right)$$

also called normal distribution, which is commonly abbreviated with

$$p(x) = N(\mu, \sigma^2)$$

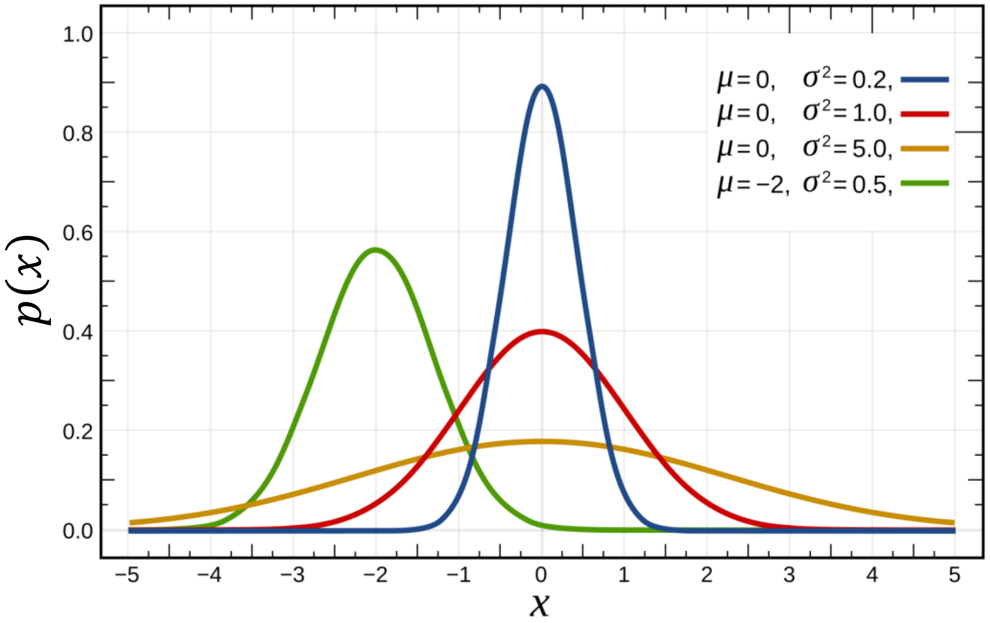
where $\mu \in \mathbb{R}$ & $\sigma^2 \in \mathbb{R}$ denote the mean & variance of the random variable $x \in \mathbb{R}$.



The expression $\boldsymbol{X} \sim \boldsymbol{N}(\boldsymbol{\mu}, \boldsymbol{\sigma^2})$ means that the random variable X has a normal distribution.

Gaussian Probability Density Functions

- The standard deviation σ is a measure of how dispersed the data is in relation to the mean.
- Small σ indicates data are clustered tightly around the mean
- Large σ indicates data are more spread out.



Question: What happens when $\sigma = 0$?

Multivariate Gaussian Distribution

- In case the random variable $x \in \mathbb{R}^k$ is a k-dimensional vector, we will have a multivariate normal distribution characterized by the density function:

$$p(x) = \frac{1}{(2\pi)^{k/2}\sqrt{\det(\Sigma)}} \exp\left(-\frac{1}{2}(x-\mu)^\top \Sigma^{-1}(x-\mu)\right)$$

where $\mu \in \mathbb{R}^k$ & $\Sigma \in \mathbb{R}^{k\times k}$ denote the mean & covariance matrix.